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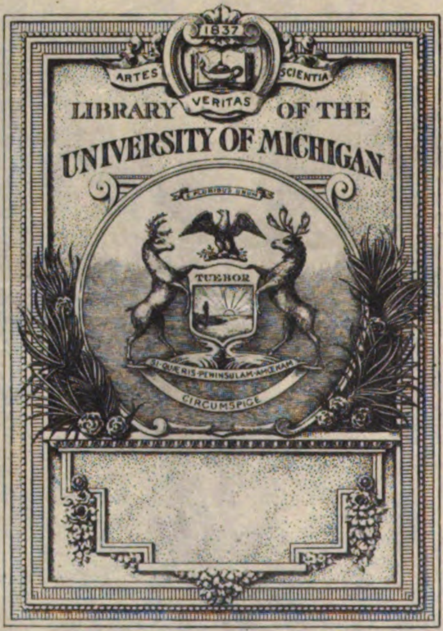

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THERMAL PROCESS TIME FOR CANNED FOOD

By

Charles Olin Ball
Research Laboratory, National Canners Association
Washington, D. C.

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BY CHARLES OLIN BALL

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LIST OF SYMBOLS

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Alphabetical list of symbols used in equations, with definition of the meaning of each and number of the page upon which each first appears

A—Total lethal value of a process, the length of which is to be calculated...... 23

A'—Total lethal value of a process, the length of which is an established condition of the problem...... 53

A''—Total lethal value a process would have if the steam were turned off at the instant the first change in slope of the heating curve takes place.... 65

A_1—The area bounded by the time-temperature curve of the retort during the rise in temperature, the horizontal line representing the initial temperature of the retort, and the vertical line representing the time of reaching holding, or processing, temperature...... 46

A_2—The area, the value of which is given by the product of the quantities, l_1, and m_1...... 46

A_{1h}—Lethal value of the heating portion, only, of a process up to the instant the first change in slope of the heating curve takes place...... 66

A_h—Total lethal value of the heating period of a process, the length of which is to be calculated, when the heating and cooling curves do not have constant slope...... 67
(See note on "change in slope," p. 64)

a—A constant in the equation of the hyperbola...... 14

α—The value of the factor, C, at the point of intersection of a $C:g$ curve of the first division with the horizontal line representing $g = .1°$...... 35

B—The length of process which is to be calculated for the condition that the difference between retort temperature and the initial temperature of the can is I degrees...... 50

b—A constant in the equation of the hyperbola...... 14

β—The value of the factor, C, at the point of intersection of a $C:g$ curve of the second division with the horizontal line representing $g = .5°$...... 35

C—An arbitrary constant, used as a factor in the derived formula for calculating length of process...... 23

C'—The value of the factor, C, when the length of the process to which it applies, is an established condition of the problem...... 53

$C_{.1}$—The value of the factor, C, corresponding to $g = .1°$. $C_{.1} = \alpha = T$ for the first division of $C:g$ curves...... 43

The five following symbols represent the values C would have in particular processes which were terminated when the temperature of the centers of the cans reached the points designated by the respective subscripts, as defined on the pages indicated:

C_b...... 66

C_2...... 70

C_b', C_c', and C_c...... 70

$C:g$—Symbol used to designate the curves by which the factor, C, is plotted against the quantity, g...... 30

c—A constant used in connection with the equation of the hyperbola...... 14

D—A pre-determined length of process for the condition that the difference between retort temperature and the initial temperature of the can is I' degrees...... 50

d—Diameter of the can in cm...... 54

E—A quantity defined by eq. (13)...... 21

$Ei(x)$—A transcendental function defined by Nielsen

as $\displaystyle\int_{\infty}^{-x} \frac{e^{-u}du}{u}$ 21

e—The base of the hyperbolic, or natural, logarithm ($= 2.71828$)...... 13

f_h—Represents the slope of the heating curve. See pp. 13 and 23 for definition. 13

f_{cl}—Represents the slope of the logarithmic part of the cooling curve...... 14

PAGE

The seven following symbols represent slopes of heating and cooling curves for conditions which are defined for each symbol on the page indicated:

f'_h 53
f''_h 54
f_{h1} and f_2 64
f_{h2}, f_{c2} and f_{c3} 70

f_{11}—A factor, the value of which is fixed by the ratio $\frac{L}{d}$ 54

G—A constant in the equation of a thermal death time curve which has been made straight by plotting on paper ruled to the log scale in both directions 72
g—The difference in degrees between retort temperature and the maximum temperature attained by the center of a can during its processing . . 13
H—A constant in the equation of the $C : g$ curve 36
h—The number of minutes required to complete a process after the temperature at the center of the can reaches .1 of a degree below retort temperature 43
I—The difference in degrees between retort temperature and the initial temperature of a can, the length of process for which is to be calculated 50
I'—The difference in degrees between retort temperature and the initial temperature of a can, the length of process for which is known 50
i—A constant used to define certain limits of integration 21
J—A constant in the equation of a thermal death time curve, which has been made straight by plotting on paper ruled to the log scale in both directions 72
j—A factor, which, when multiplied by I, designates the point of intersection of the vertical line representing the beginning of a process with the extension of the straight portion of the semi-log heating curve, when no time is consumed in bringing the retort to holding, or processing, temperature. j has a similar application with reference to the cooling curve when it is multiplied by the quantity, m 22

K—The diffusivity constant $\left(= \frac{\text{thermal conductivity}}{\text{sp. heat} \times \text{density}}\right)$ 54

K' and K''—Diffusivity constants for particular cases 54
k—A factor, which, when multiplied by f_h, gives the value in minutes of the point of intersection of the horizontal line representing the initial temperature of a can, with the extension of the straight portion of the semi-log heating curve, when no time is consumed in bringing the retort to holding, or processing, temperature. k has a similar application with reference to the cooling curve 22
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l_2—The number of minutes separating the heating curves of two similar cans, the initial temperatures of which are not the same 48

M—A symbol substituted in calculations for $\log \frac{W}{H}$ 39

m—The difference in degrees between cooling water temperature and the maximum temperature attained by the center of a can during its processing 13
m_1—The difference in degrees between the initial temperature of the retort and the processing temperature 46

N—A symbol substituted in calculations for $\log \frac{f_h T}{U}$ 39

N'—A symbol substituted in calculations for $\log \frac{f_h T}{U A'}$ 53

n—A symbol substituted for the expression $\frac{a}{z}$ during the integration of the hyperbola 21
P—A symbol constituting part of an empirical expression used in the integration of the hyperbola 21

PAGE

Q—A substituting symbol used to shorten an equation.................... 71

q—The value in degrees of the point of intersection of the straight portion of the semi-log cooling curve extended, with the vertical line representing "steam off." q equals the product, jm.................. 14

R—A symbol constituting part of an empirical expression used in the integration of the hyperbola.. 21

r—The difference in degrees between cooling water temperature and the temperature at which the lethal rate value is assumed to become negligible.. 20

ρ—The percentage of the total lethal value of a process which is represented by the heating portion only, when the slope of the heating curve, as well as of the cooling curve, is constant..................... 65

The five following symbols represent values ρ would have in particular processes, which were terminated when the temperature at the centers of the cans reached the points designated by the subscripts, as defined on the pages indicated:

ρ_b.. 66

ρ_2.. 67

ρ_b', ρ_c', and ρ_c.. 70

S—A constant in the equation of the $C : g$ curve.......................... 36

σ—The value of the quantity, g, at the point of intersection of the extension of the $C : g$ curve of the third division with the vertical line representing $C = 10\omega$.. 36

T—A constant in the equation of the $C : g$ curve.......................... 36

T_1—The initial temperature of the retort.................................. 47

t—The time in minutes required to destroy an organism at the highest temperature attained by the center of the can in a process, the length of which is to be calculated................................ 17

t'—The same as t except that it is applied to a process, the length of which is an established condition of the problem....................... 53

$t_{.1}$—The value of t corresponding to $g = .1°$................................. 43

The five following symbols represent the values t would have in particular processes which were terminated when the temperature of the centers of the cans reached the points designated by the respective subscripts, as defined on the pages indicated:

t_b.. 66

t_2.. 70

t_b', t_c', t_c.. 70

U—The number of minutes required to destroy the organism at retort temperature.. 39

u_h—A symbol substituted for the expression $\frac{y_h}{z}$ during the integration of the heating curve.. 20

u_{ch}—A symbol substituted for $\frac{y_{ch}}{z}$ during the integration of the hyperbola...... 21

V—A substituting symbol used to shorten an equation....................... 69

ϕ—A constant equal to .4 plus the value of the quantity, g, at the point of intersection of the extension of the $C : g$ curve of the second division with the vertical line representing $C = 10\beta$.......................... 35

W—A constant in the equation of the $C : g$ curve.......................... 36

w—A factor, constituting part of an empirical expression used in the integration of the hyperbola.. 22

X and X'—Variable quantities introduced to aid in the solution of eq. (29) for g 39

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x_{ch} and x_{cl}—The variables in the equations of the hyperbola and the semi-log cooling curve respectively, which represent the time in minutes from the turning off of steam.. 14

x_o—The variable in the equation of the thermal death time curve which represents the number of degrees to the highest temperature attained by the center of the can.. 16

x'_o—The variable in the equation of the thermal death time curve which represents the number of degrees to retort temperature................ 39

y_h—The variable in the equation of the heating curve, which represents the number of degrees to retort temperature....................... 13

PAGE

y_{ch}—The variable in the equation of the hyperbola, which represents the sum of "a" degrees and the number of degrees from the highest temperature attained by the center of the can.................... 14

y_{cl}—The variable in the equation of the semi-log cooling curve, which represents the number of degrees to cooling water temperature............. 14

y_o—The variable in the equations of the thermal death time curve, which represents time in minutes.................................... 16

$\frac{y_o}{f_h}$—Symbol used to designate the curves, in which the ratio, $\frac{y_o}{f_h}$ is plotted against g.. 30

α—A constant equal to the value of g at the point of intersection of the extension of the $C : g$ curve of the first division with the vertical line representing $g = 10\alpha$.. 35

ω—The value of the factor, C, at the point of intersection of the $C : g$ curve of the third division with the horizontal line, representing $g = 5$........ 36

z—Represents the slope of the thermal death time curve. See pp. 17 and 23 for definition.. 17

Parenthesized capital letters refer to literature listed on page 76.

Definitions:

An abscissa is distance measured horizontally to the right or left from the y-axis to a point.

An ordinate is distance measured vertically upward or downward from the x-axis to a point.

I

DETERMINING, BY METHODS OF CALCULATION, THE TIME NECESSARY TO PROCESS CANNED FOODS

INTRODUCTION

In commercial canning, when a food is put into a can, it always contains a large number of micro-organisms, which would cause spoilage of the food if they were not subsequently destroyed. Some organisms will not cause spoilage when subjected only to ordinary conditions; and such organisms, it sometimes may not be considered necessary to destroy. When all spores of a certain organism in a can of food have been destroyed, the food is said to be sterile with respect to that organism.

The object in processing canned foods is the attainment of sterility with respect to the most resistant micro-organism present which would bring about spoilage. Therefore, the problem of determining the time necessary to process canned foods consists of determining the time necessary to produce this sterility within the cans. A method of computing this time was described by Bigelow et. al. in 1920 (A), and this is essentially as follows:

Each point on the curves which represent the time-temperature relation at the center of the can during the heating and cooling operations of the process period, is said to represent a lethal rate value. This value, for any point, is the reciprocal of the number of minutes required to destroy all spores of the organism at the temperature represented by the point, and under the conditions obtaining within the food. These values are plotted against time, taken from the abscissæ of the respective points on the time-temperature heating and cooling curves.

A thermal death time curve is a curve showing the time-temperature relation for the death points of the spores of an organism when these spores are present in a certain concentration and are subjected to certain known conditions. These conditions must be carefully controlled. Such curves are determined experimentally in the bacteriological laboratory.

Fig. 1 shows the time-temperature curves of the center of a can during heating and cooling, and the thermal death time curve of an organism. The curves are plotted side by side so that the same scale of degrees applies to all. The abscissæ of the curves represent time. For the thermal death time curve, the abscissa scale, x'', is a scale of the reciprocals of the normal abscissa scale. The broken lines join values on the x-scale of time to values on the x''-scale of reciprocals, which are plotted against each other in the new curve. In this new curve, values from the

x-scale of Fig. 1 are represented as abscissæ, and values from the x'' scale, as ordinates. The new curve is called the lethality curve, and is shown in Fig. 2.

Hereafter, the time-temperature curve representing the period of heating of a can will be designated simply, the "heating curve," and the time-temperature curve representing the cooling period, will be "cooling curve."

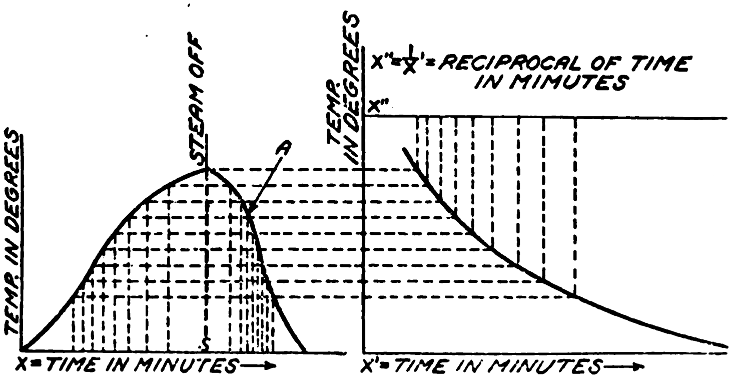

Fig. 1. The inter-relationship between the heating, the cooling, and the thermal death time curves in the calculation of lethality.

The area beneath the lethality curve is equal to unity if the process represented by the heating and cooling curves in Fig. 1 is just sufficient to destroy all spores of the organism; and the time, s, is then the length of process in minutes.

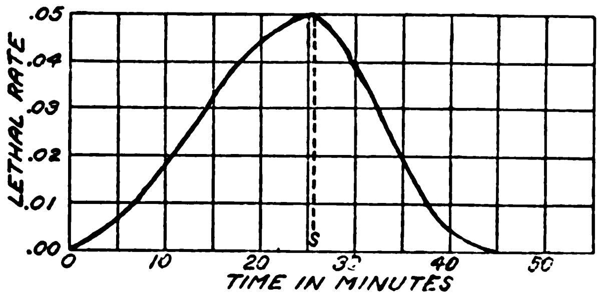

Fig. 2. Lethality curves.

The operation of this method by plotting the lethality curve and determining the area beneath it is a slow and tedious process. The method herein described of calculating process times was derived in order to reduce the time necessary to obtain results, as well as to obtain a basis for coordinating the various factors which enter into the calculation.

The formula, as derived, expresses the area beneath the lethality curve as a function of the properties of the heating, cooling, and thermal death time curves.

Thompson (C) and Williamson and Adams (D) in 1919 published formulæ for calculating the temperature at the center of a cylinder at any time during a period of heating. The difficulties attending the use of such formulæ in connection with canned foods have been pointed out by the originators themselves as well as by others (E, p. 5). On account of the many variable factors which have to be considered it is almost impossible to determine accurately the various constants of the formulæ. In these calculations, the problem is taken up where it is left off by the writers mentioned; but, instead of using time-temperature heating and cooling curves which are calculated by formulæ, curves which have been obtained directly by experiment are used, and simpler equations are applied to them.

From the formulae by Williamson and Adams and by Thompson, we find that, theoretically, the time-temperature heating and cooling curves,—except for the first portion, or lag, of each,—are practically straight line curves on semi-logarithmic paper. These curves were described by Bigelow et al. (A, p. 34). Experimentally, we find that for most products, the heating and cooling curves agree well with this theory.

Likewise, as has also been pointed out by Bigelow (B), the experimentally determined thermal death time curves for the spores of an organism are straight lines when plotted on semi-logarithmic paper.[1]

Simple logarithmic equations may therefore be applied to all of these curves, and upon such equations the derivation of the formula (17a), as described in this paper, is based. These equations, however, are not satisfied by the portion of the heating curve which represents the lag in the rise in temperature for sometime after the heating is begun, and the portion of the cooling curve which represents a corresponding lag in the drop in temperature after cooling is begun.

This deviation in the heating curve will not be considered, because it generally occurs before the temperature at the center of the can has attained a practical sterilizing value. The deviation in the cooling curve, however, occurs at the time of maximum temperature within the can; and therefore at the time of maximum rate of sterilization. It must, therefore, be considered. Rather than use for the cooling curve the complicated expression given by Thompson it has been assumed that, in all cases, the first part of the cooling curve satisfies the equation of a hyperbola until it passes into the logarithmic curve at the point, A,

---

[1] When the work described in this paper was nearing completion, it was found that these are not strictly straight line curves. They are further discussed in this connection on p. 72.

Figs. 3 and 4. Of course, this is not strictly true; not only because the shape of the first part of the cooling curve depends upon the method of cooling used (whether water or air); but also because it depends upon the difference in temperature existing between the center and the margin of the contents of the can when cooling is begun. It has been found, however, that under practical conditions, the latter factor has but little effect upon the shape of the cooling curve when water cooling is used; and these calculations are based upon the conditions of water cooling.

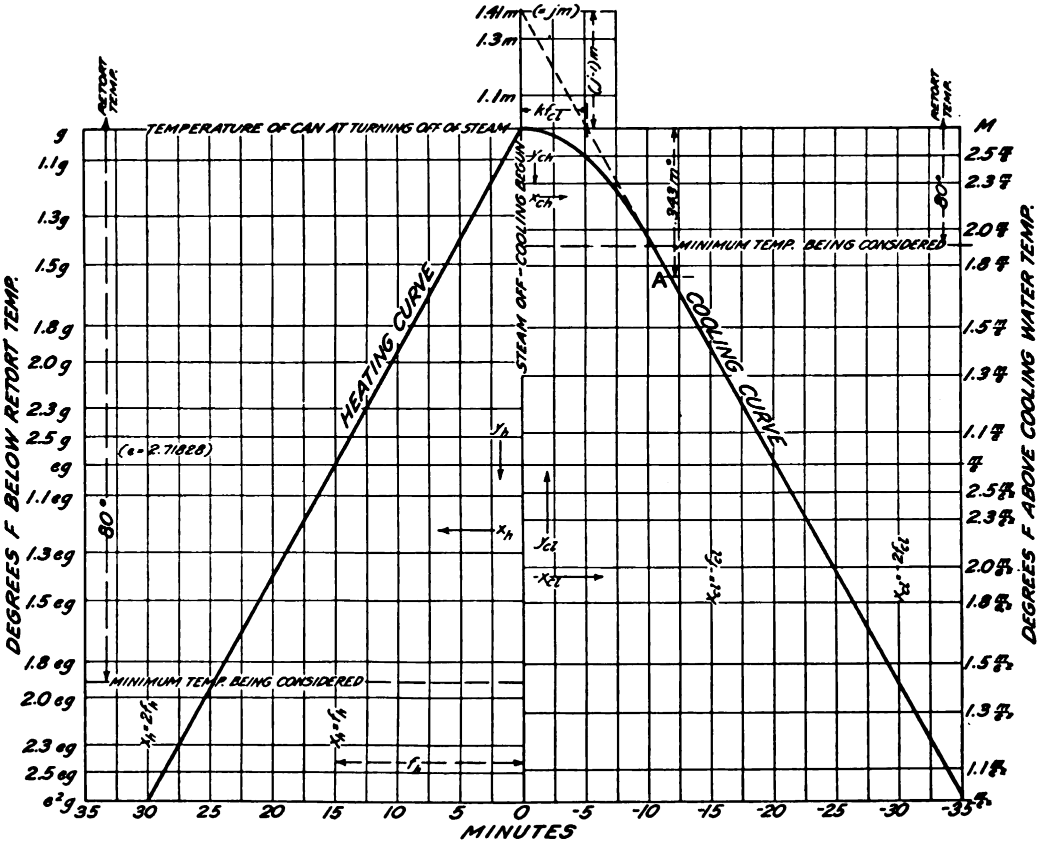

FIG. 3. Heating and cooling curves for a can.

Fig. 3 shows the last part of a theoretical heating curve and the first part of the cooling curve corresponding to it; plotted on semi-logarithmic paper. For the purpose of the problem, it is assumed that the cooling curve is the exact reverse of the heating curve.[1] This is now known to be false in a majority of cases; but it is a convenient assumption upon which to base the calculations. The variations of the cooling curves from this assumed curve are considered in Section VI. In this plot the scale of hyperbolic, or natural, logarithms is used, because the use of natural logarithms simplifies the operations of integration which are to be performed.

The curves in Fig. 3 represent a can, the center of which has attained a temperature g degrees below retort temperature at the time the steam is turned off. It is assumed that the can is plunged immediately into cooling water, the temperature of which is m degrees lower than the maximum temperature reached by the can, or $m + g$ degrees lower than retort temperature; and it is further assumed that the temperature of the cooling water remains constant during the cooling of the can.

Equations of the Curves

To apply an equation to the heating curve the vertical line representing "steam off" is called the y-axis of the curve, and given the value $x_h = 0$.[2] The intersection of this axis with the horizontal line representing retort temperature (which is never reached) is the origin of the curve. y_h increases downward, and therefore measures in degrees the difference between retort temperature and the temperature represented by any point on the curve. Positively x_h increases toward the left, and therefore measures, in minutes, the difference between the end of the process (which is "steam off") and the time represented by any point on the curve.

Under these conditions, the equation of the heating curve is,

$$\frac{x_h}{f_h} = \log \frac{y_h}{g} \tag{1}$$

in which f_h represents the slope of the curve; or, to be more specific, the value of f_h is the number of minutes required for the temperature of the can to increase from [R. T. (retort temperature) $- el$] degrees to [R. T. $- l$] degrees, e being the base of the natural logarithms, and l, a constant having any positive value whatsoever. f_h is easily evaluated either by solving equation (1) or by plotting the curve on semi-log paper and counting the number of minutes required for the curve to pass over one logarithmic cycle. $x_h = f_h$ when $y_h = eg$. This curve intersects the y-axis at the same point as the horizontal line which represents the value, (R. T. $- g)°$; or, $y_h = g$.

---

[1] By the statement that the cooling curve is the exact reverse of the heating curve is meant that the two curves have the same slope; and if plotted on the same sheet of semi-log paper, with the cooling curve ascending instead of descending, they would be parallel to each other. This does not mean, however, that the number of degrees passed over by each curve in any specified equal intervals of time (such as the first five minutes of heating and of cooling) is the same in the two curves. These numbers of degrees for the two curves bear the same ratio to each other as the respective differences between initial and ultimate temperatures.

[2] The subscript h denotes that the symbol to which it is appended refers to the heating curve; the subscript, ch, to the hyperbolic part of the cooling curve; the subscript, cl, to the logarithmic part of the cooling curve; and the subscript, o, to the thermal death time curve.

To apply the equation of a hyperbola to the first part of the cooling curve, the vertical line representing the end of the process is again the y-axis, upon which $x_{ch} = 0$. The origin of the curve is at a point on this axis, "a" degrees higher than the maximum temperature reached by the center of the can. y_{ch} increases downward and x_{ch} increases toward the right. Therefore, the equation is taken as

$$\frac{y_{ch}^2}{a^2} - \frac{x_{ch}^2}{b^2} = 1 \tag{2}$$

in which b is an arbitrary constant, defined as $\sqrt{c^2 - a^2}$; c being the number of units measured along the y-axis between the origin and the focus of the curve.

For the logarithmic part of the cooling curve, the same y-axis is used as for the other two curves, i.e., the vertical line representing the end of the process; the origin is the intersection of this vertical line with the horizontal line representing the temperature of the cooling water (which temperature, also, is never reached by the can). y_{cl} increases upward, and therefore measures in degrees the difference between the cooling water temperature and the temperature represented by any point on the curve. Positively, x_{cl} increases toward the left, but it increases in absolute value negatively toward the right from the y-axis. Therefore the absolute value of x_{cl} measures in minutes the difference in time between "steam off" and that represented by any point on the curve. The equation of the curve is

$$\frac{x_{cl}}{f_{cl}} = \log \frac{y_{cl}}{q} \tag{3}$$

in which f_{cl} represents the slope of the curve, and is found in the same manner as f_h of the heating curve. It is obvious, from equation (3) that, since y_{cl} within the limits of our problem is always less than q, if we give f_{cl} a negative sign, x_{cl} will always be obtained with a positive sign. Under our assumption of the cooling curve being the exact reverse of the heating curve, f_{cl} equals f_h. Therefore we may substitute f_h for f_{cl} in this equation. q is the value in degrees of the point of intersection of the curve with the y-axis, and equals jm, in which j is a constant greater than 1.

The cooling curve may be more easily understood when plotted on squared co-ordinate paper as shown in Fig. 4. In such a plot, the origin,

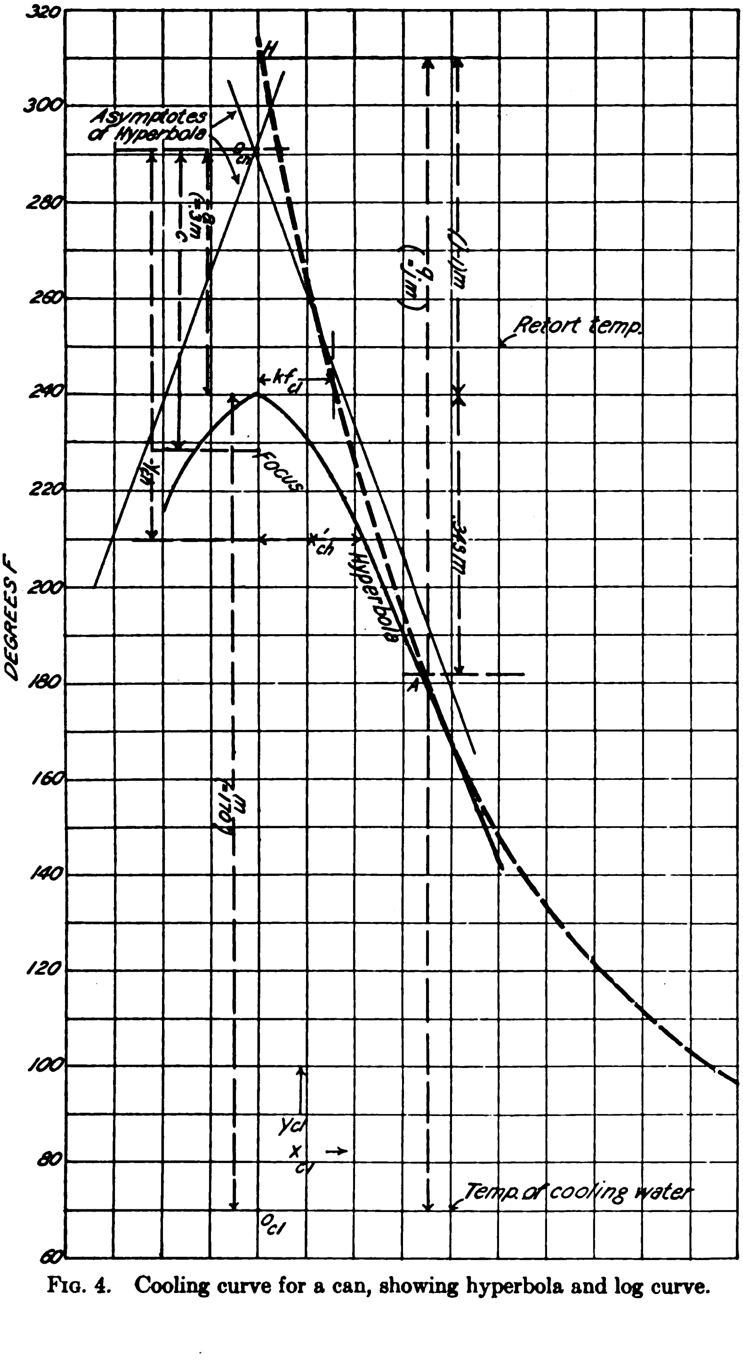

FIG. 4. Cooling curve for a can, showing hyperbola and log curve.

O_{cl}, of the logarithmic curve, as well as that of the hyperbola, O_{ch}, can be shown.

A thermal death time curve is illustrated in Fig. 5. It is plotted on semi-log paper to the scale of natural logarithms. In this chart, time is measured in the direction of ordinates; and temperature, in the direction of abscissæ.

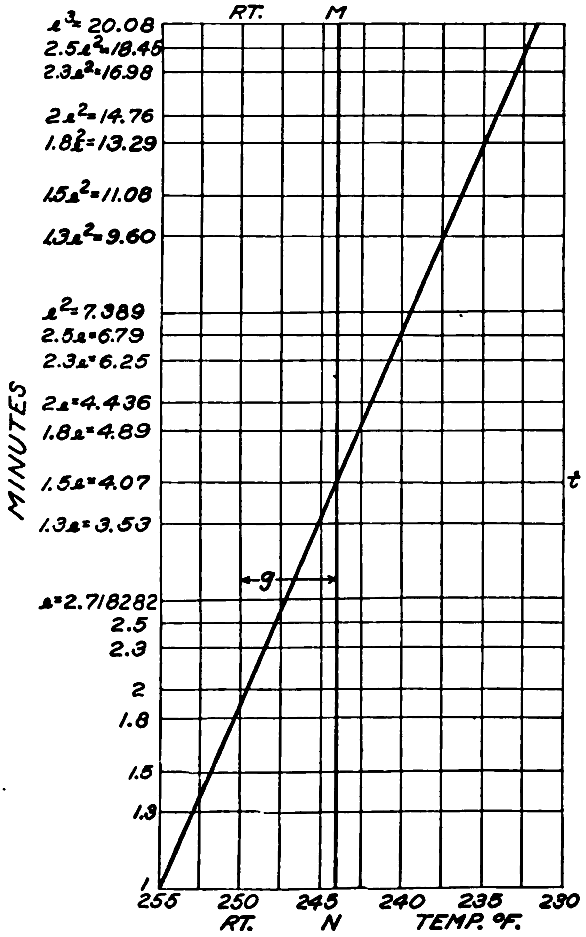

FIG. 5. Thermal death time curve.

In applying the logarithmic equation to this curve, the vertical line representing the highest temperature reached by the center of the can is taken as the y-axis. This axis has the value, $x_o = 0$, and the origin of the curve is at the imaginary point, $x_o = 0$, $y_o = 0$, and cannot be plotted. x_o increases positively toward the right when the slope of the curve is given a positive value, and x_o therefore measures in degrees the difference between the maximum temperature attained by the can and that represented by any other point on the curve. y_o increases upward, and its value is expressed in minutes counted from zero.

The equation of the curve is,

$$\frac{x_o}{z} = \log \frac{y_o}{t} \tag{4}$$

in which z represents the slope of the curve, its value being the number of degrees passed over by the curve in traversing one logarithmic cycle. t is the value in minutes of the horizontal line through the point of intersection of the curve with the y-axis.

Referring again to Fig. 5; if the retort temperature is assumed to be 250° and the maximum temperature reached by a can during a process to be 244°, the value of g (see p. 14) is $250° - 244° = 6°$, and the vertical line, MN, is the y-axis of the thermal death time curve. In this case, $t = 4.07$ min.

The derivation of the formula [17a] will be described not only because of its interest to those engaged in work to which it pertains, but primarily so that anyone contemplating using the formula may judge for himself the amount of dependence which he may safely place in results obtained from it. Only essential points will be included, but since the description is entirely theoretical matter, some may not care to follow it. Those readers are referred directly to the method of applying the formula, the description of which begins on page 30.

II

THE DERIVATION OF THE FORMULA

Along the heating and cooling curves of a can, a strip is taken, which has a constant width of unity, measured in the direction of ordinates. This strip is divided into elements having infinitesimal horizontal width; *dx*. The elements extend across the strip in the direction of ordinates; therefore they are of unit length and the area of an element is $(y + \frac{1}{2}) - (y - \frac{1}{2})\,dx$, which equals *dx*. See Fig. 6. Each element, *dx*, is multiplied by the lethal rate value corresponding to the position of the element on the curve; and the elements are then summed up by integration between limits. The limits of integration found most suitable for this problem are along the *y*-axis; therefore, *dx* must be expressed in terms of *y*.

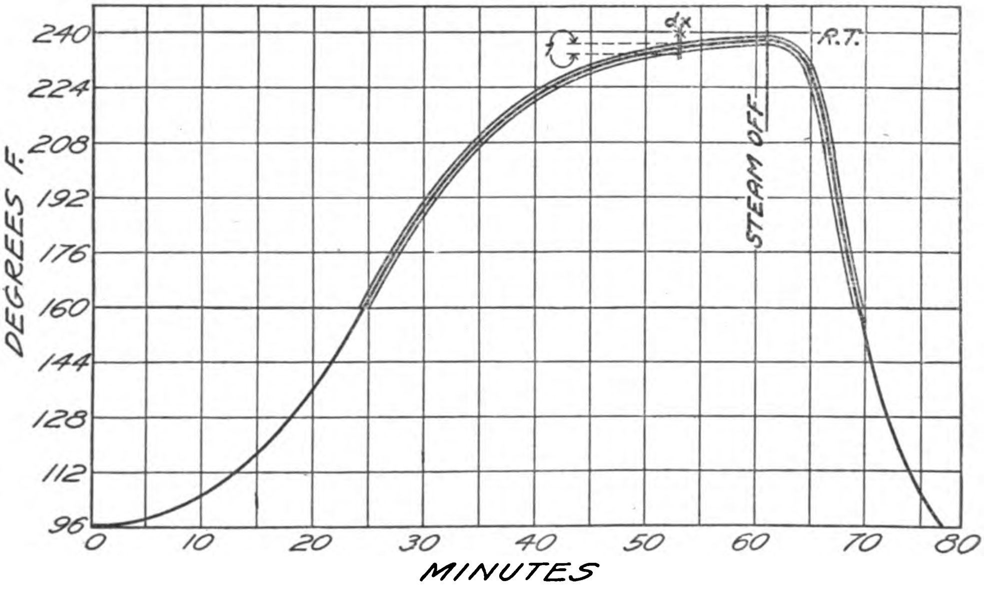

FIG. 6. Heating and cooling curves, showing strip which is integrated.

EXPRESSING *dx* IN TERMS OF *y*

As we have seen, it is necessary to integrate first along a logarithmic curve, then along a hyperbola, and finally along another logarithmic curve. The equations of these curves are equations (1), (2) and (3) respectively.

From equation (1), $x_h = f_h \log y_h - f_h \log g$; and $d(x_h) = f_h \dfrac{d(y_h)}{y_h}$ (5)

From equation (2), $x_{ch} = b\sqrt{\frac{y_{ch}^2}{a^2} - 1}$; and $d(x_{ch}) = \frac{b}{a^2}\frac{y_{ch}d(y_{ch})}{\sqrt{\frac{y_{ch}^2}{a^2}-1}}$ (6)

From equation (3), $x_{cl} = f_{cl}\log y_{cl} - f_{cl}\log q$; and $d(x_{cl}) = f_{cl}\frac{d(y_{cl})}{y_{cl}}$ (7)

Derivation of Factors Expressing Lethal Rate

Now, it is desired to multiply each elementary portion of the area of the strip along the heating and cooling curves by a number which is the reciprocal of the number of minutes required to destroy the organism at the temperature corresponding to that element. This number of minutes in each case is the value of y_o for the same temperature from equation (4) of the thermal death time curve.

From equation (4) $y_o = te^{\frac{x_o}{z}}$ (8)

But $x_o = y_h - g = y_{ch} - a = m - y_{cl}$ (Fig. 3 and Fig. 4), therefore,

$d(x_h)$ [eq. (5)] must be multiplied by the reciprocal of $t(e)^{\frac{y_h - g}{z}}$ (8a)

$d(x_{ch})$ [eq. (6)] must be multiplied by the reciprocal of $t(e)^{\frac{y_{ch} - a}{z}}$ (8b)

and

$d(x_{cl})$ [eq. (7)] must be multiplied by the reciprocal of $t(e)^{\frac{m - y_{cl}}{z}}$ (8c)

The Limits of Integration

It is evident that these reciprocals, or lethal rate values, decrease in magnitude very rapidly as the temperature decreases. The rate of decrease, of course, depends upon the thermal death time curve upon which they are based. It will obviously be of no practical benefit to extend the integration below the points where these quantities become negligible, when compared to the total value of the integral. Since the variation in processing temperatures chosen for use in canning foods shows a certain degree of conformity to the variation in the resistances of the organisms which the processes are intended to destroy; and since the lethal rate value of any given temperature must always be directly dependent upon the resistance of the organism it is seen that a constant interval below retort temperature may consistently be used to establish the lower limit of integration. The interval chosen is 80°, and one is quite safe in assuming that any temperature more than 80° lower than the processing temperature, has, in any case, a lethal rate value which is negligible compared to the lethal value of the entire process.

Therefore, the limits of integration of the heating curve are 80 degrees and g degrees. After a study of various heating curves (principally those of corn) from carefully made runs, it was estimated that the union between the hyperbola and the semi-log curve occurs at a point which is above the initial temperature about .343 times the interval between the initial temperature and retort temperature, expressed in degrees. Theoretically this position is the same for all curves. Thus, considering the cooling curve to be the reverse of the heating curve, we have for the limits of integration of the hyperbola, "a" degrees and $(a + .343\ m)$ degrees; and for the logarithmic part of the cooling curve, $(1 - .343)\ m$ degrees and r degrees, letting r equal $m + g - 80$.

Thus the expressions to be integrated are,

(1) $$\frac{f_h}{t}\int_{g}^{80}\frac{d(y_h)}{y_h e^{\frac{y_h - g}{z}}} \text{ see equation (5)} \tag{8d}$$

(2) $$\frac{b}{a^2 t}\int_{a}^{a+.343m}\frac{y_{ch}}{e^{\frac{y_{ch}-a}{z}}}\frac{d(y_{ch})}{\sqrt{\frac{y_{ch}^2}{a^2}-1}} \text{ see equation (6)} \tag{8e}$$

and

(3) $$\frac{f_{cl}}{t}\int_{r}^{.657m}\frac{d(y_{cl})}{y_{cl}\,e^{\frac{m-y_{cl}}{z}}} \text{ see equation (7)} \tag{8f}$$

INTEGRATION OF THE EXPRESSIONS

From the first expression,

$$\frac{f_h}{t}\int_{g}^{80}\frac{d(y_h)}{y_h e^{\frac{y_h-g}{z}}} = \frac{f_h e^{\frac{g}{z}}}{t}\int_{\frac{g}{z}}^{\frac{80}{z}}\frac{d(u_h)}{u_h e^{u_h}} \tag{9}$$

in which

$$u_h = \frac{y_h}{z}.$$

the function $\frac{e^{-u_h}}{u_h}$ is discontinuous for $u_h = 0$, therefore its integral can have a finite value only between real limits which are both positive, or are both negative. The limits $\frac{80}{z}$ and $\frac{g}{z}$ are both positive.

The integration is accomplished by letting

$$\int_{\frac{g}{z}}^{\frac{80}{z}}\frac{e^{-u_h}d(u_h)}{u_h} = \int_{\infty}^{-(-\frac{80}{z})}\frac{e^{-u_h}d(u_h)}{u_h} - \int_{\infty}^{-(-\frac{g}{z})}\frac{e^{-u_h}d(u_h)}{u_h} \tag{10}$$

which is the difference of two transcendental functions conventionally represented by the symbol $Ei(x)$. Then, the second member of eq. (10) is $Ei(-\frac{80}{z}) - Ei(-\frac{g}{z})$. Each of the two terms represents a universally convergent series, tables of the values of which have been published within certain limits of the variable.

Therefore, the value of the integral for the heating curve is,

$$\frac{f_h e^{\frac{g}{z}}}{t}\left[Ei\left(-\frac{80}{z}\right) - Ei\left(-\frac{g}{z}\right)\right]. \qquad (10a)$$

Expression (2), p. 20, equals $\frac{zbe^{\frac{a}{z}}}{at}\int_{\frac{a}{z}}^{\frac{+.343m}{z}} \frac{u_{ch}d(u_{ch})}{e^{u_{ch}}\sqrt{u_{ch}^2 - (\frac{a}{z})^2}}$

in which

$$u_{ch} = \frac{y_{ch}}{z}.$$

This, after an integration by parts, gives

$$\frac{zbe^{\frac{a}{z}}}{at}\left[\frac{\left(\sqrt{\left(\frac{a+.343m}{z}\right)^2 - \left(\frac{a}{z}\right)^2}\right)}{e^{\frac{a+.343m}{z}}} + \int_{\frac{a}{z}}^{\frac{a+.343m}{z}} \frac{\sqrt{u_{ch}^2 - (\frac{a}{z})^2}}{e^{u_{ch}}}\, d(u_{ch})\right] \qquad (11)$$

Next, the equation

$$Y_o = \frac{\sqrt{u_{ch}^2 - n^2}}{e^{u_{ch}}} \qquad (12)$$

was plotted to a large scale for a number of values of n within the range covered by the problem. The symbol, n, is substituted for $\frac{a}{z}$. A study of these curves led to the following equation, the derivation of which cannot be included here because of space limitations.

$$E = \int_{n}^{n+i} \frac{\sqrt{u_{ch}^2 - n^2}}{e^{u_{ch}}}\, d(u_{ch}) = i[P + w(R - P)] \qquad (13)$$

in which E is defined by the equation; i is the algebraic difference between the limits of integration,(in this case, $i = .343\,\frac{m}{z}$);

$$P = \frac{\sqrt{u_{ch}^2 - n^2}}{e^{u_{ch}}}, \text{ when } u_{ch} = n + i;$$

and

$$R = \frac{\sqrt{u_{ch}^2 - n^2}}{e^{u_{ch}}}, \text{ when } u_{ch} = n + .44;$$

w is a factor, a number of values of which were obtained by solving equation (13) for w, E being given values determined by actually counting squares under the curves plotted from equation (12). After evaluating $n(=\frac{a}{z})$ in terms of $\frac{m}{z}$, w was plotted against $\frac{m}{z}$. E could be evaluated also by using the instrument known as the integraph on the curves of equation (12). The integraph plots the curve of the integral of a function.

Now, any value of E, within the range for which w is plotted against $\frac{m}{z}$, is obtained from equation (13) and E is likewise plotted against $\frac{m}{z}$.

DETERMINATION, IN TERMS OF m AND OF f_h, OF THE VALUES OF a AND b.

It is necessary that we have the factors, a and b, expressed in terms of some known quantity. Their values are obtained in terms of m and f_h by imposing the condition that the hyperbola, at its point of intersection with the semi-log curve, coincides with the latter. In this manner, a is found to equal .300 m; and b, to equal .175 f_h.

In obtaining these values it was necessary to use in the calculations, the values of the constants j and k (see Fig. 3.) The value of the former was obtained in the same manner as was the position of the point of union between the hyperbola and the log curve (p. 20), i.e., by a study of experimentally determined heating curves.* The value of j is taken as 1.41, and substitution of this in the log equation of the heating or the cooling curve, gives $k = .149$ for Brigg's logs and $k = .343$ for natural logs.

Now, substituting our new values for a and b in expression (11) and reducing, gives for the value of the integral for the hyperbola,

$$\frac{f_h}{t}\left[\frac{.33172}{e^{\frac{.343m}{z}}} + \frac{.5833\,ze^{\frac{.300m}{z}}}{m}E\right]. \tag{14}$$

The integration of the second part of the cooling curve is just like that of the heating curve. Expression (8f) gives

$$\frac{f_h}{te^{\frac{m}{z}}}\int_{r}^{.657m}\frac{e^{\frac{y_{cl}}{z}}\,d(y_{cl})}{y_{cl}} \tag{15}$$

the integral of which is

$$\frac{f_h}{te^{\frac{m}{z}}}\left[Ei\left(\frac{.657m}{z}\right) - Ei\left(\frac{m+g-80}{z}\right)\right]. \tag{16}$$

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* It is now realized that this value should have been based upon cooling curves rather than upon heating curves. However, as stated before, adjustment can be made in calculations for variations from the values upon which the derivation of the formula is based. This is discussed on page 70.

TOTAL LETHAL VALUE OF A PROCESS

Combining expressions (10a), (14), and (16), we have the total lethal value, A, which is represented by the area beneath the lethality curve:

$$A = \frac{f_h}{t}\left\{e^{\frac{g}{z}}\left[Ei\left(-\frac{80}{z}\right) - Ei\left(-\frac{g}{z}\right)\right] + \left[\frac{.33172}{e^{\frac{.343m}{z}}} + e^{\frac{.300m}{z}}\frac{.5833z}{m}E\right]\right.$$
$$\left. + \frac{1}{e^{\frac{m}{z}}}\left[Ei\left(\frac{.657m}{z}\right) - Ei\left(\frac{m+g-80}{z}\right)\right]\right\}. \quad (17)$$

The part of the expression included within the braces is considered as an arbitrary constant, C, which must be tabulated. As C is a function of g, m, and z it must be tabulated for all necessary values of these variables. In tabulating, all values used are referred back to Brigg's logs, because the heating, cooling, and thermal death time curves, which are actually used, will always be plotted to the Briggs scale.

The Ei functions are plotted from data obtained in part from the tables of Glaisher, which were published in Phil. Trans., London, Vol. 160, pp. 367–387 (1870). Values needed which were beyond the scope of this table were obtained by plotting the function, $y = \frac{e^{-u}}{u},$ and using the integraph to obtain the curves of the integral function. Table I gives values of C, which were actually calculated. All values of C are referred to Brigg's logs; therefore the symbol, f_h, when used hereafter, will represent the number of minutes on the horizontal scale passed over by a heating curve in crossing one Briggs logarithmic cycle instead of one natural logarithmic cycle. One Brigg's logarithmic cycle along the heating curve extends from the point for which $g = 10\ l$ degrees to the point for which $g = l$ degrees. The quantity, z, also, will hereafter refer to the Briggs cycle instead of to the natural cycle.

In obtaining the final curves for C, preliminary curves were made in which C was plotted in a number of different ways, using in turn, as variables, z, $m + g$, and g. These curves served to eliminate the effect of errors in the calculations.

The final curves for C were found to conform well to straight line curves on logarithmic paper with slight adjustments in the scale. Figures 7a, 7b, and 7c show these curves. The scales along the left-hand margins of the plots are the scales to which the curves are plotted. In using the curves no attention is to be paid to the scale in parentheses shown along the right hand margin of Fig. 7b. This is the original logarithmic scale of the paper, and is given on the plot merely to show the adjustment that was made, so as to facilitate the reproduction of the curves, by any one desiring to plot them to a larger scale.

Table I.

$m + g$	g	$z = 14$	$z = 18$	$z = 22$	$z = 26$
		C	C	C	C
130	0.1	1.6594	1.7725	1.8700	1.9496
	0.25	1.2619	1.3656	1.4585	1.5336
	1.0	.7985	.8906	.9738	1.0445
	5.0	.3827	.4464	.5037	.5592
	10.0	.2771	.3277	.3722	.4140
	15.0	.2279	.2726	.3122	.3475
	20.0	.2000	.2400	.2745	.3078
	25.0	.1810	.2176	.2507	.2814
150	0.1	1.6539	1.7659	1.8629	1.9417
	0.25	1.2490	1.3543	1.4476	1.5254
	1.0	.7884	.8790	.9616	1.0336
	5.0	.3742	.4374	.4952	.5506
	10.0	.2671	.3167	.3612	.4020
	15.0	.2184	.2615	.3000	.3357
	20.0	.1905	.2284	.2635	.2955
	25.0	.1713	.2070	.2391	.2690
160	0.1	1.6510	1.7624	1.8591	1.9374
	0.25	1.2418	1.3484	1.4419	1.5218
	1.0	.7828	.8724	.9556	1.0282
	5.0	.3700	.4329	.4906	.5456
	10.0	.2620	.3115	.3554	.3963
	15.0	.2134	.2558	.2939	.3299
	20.0	.1856	.2231	.2572	.2892
	25.0	.1663	.2015	.2335	.2627
180	0.1	1.6444	1.7543	1.8510	1.9284
	0.25	1.2262	1.3345	1.4289	1.5114
	1.0	.7700	.8589	.9415	1.0165
	5.0	.3610	.4229	.4804	.5349
	10.0	.2508	.2993	.3426	.3830
	15.0	.2029	.2438	.2808	.3167
	20.0	.1748	.2110	.2445	.2755
	25.0	.1558	.1890	.2205	.2490

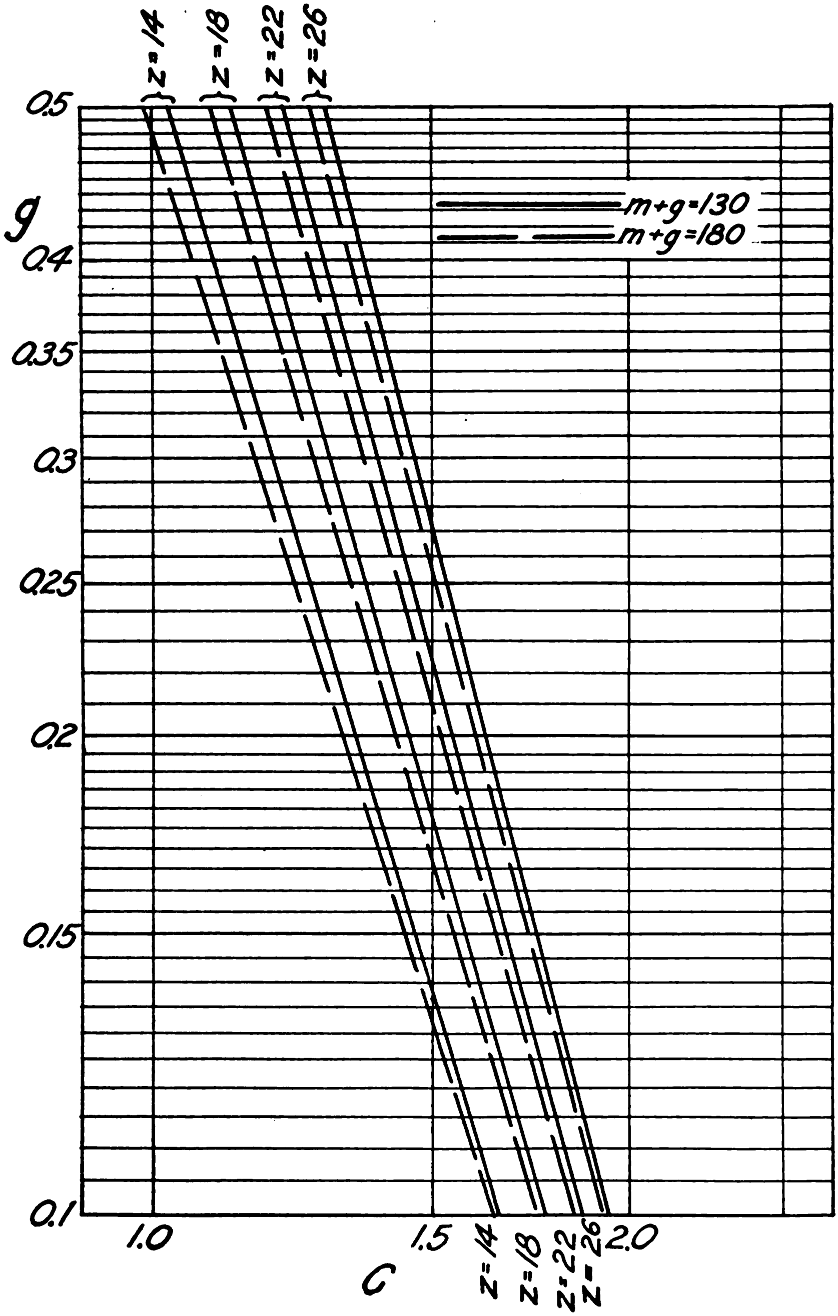

FIG. 7a. The $C : g$ curves.

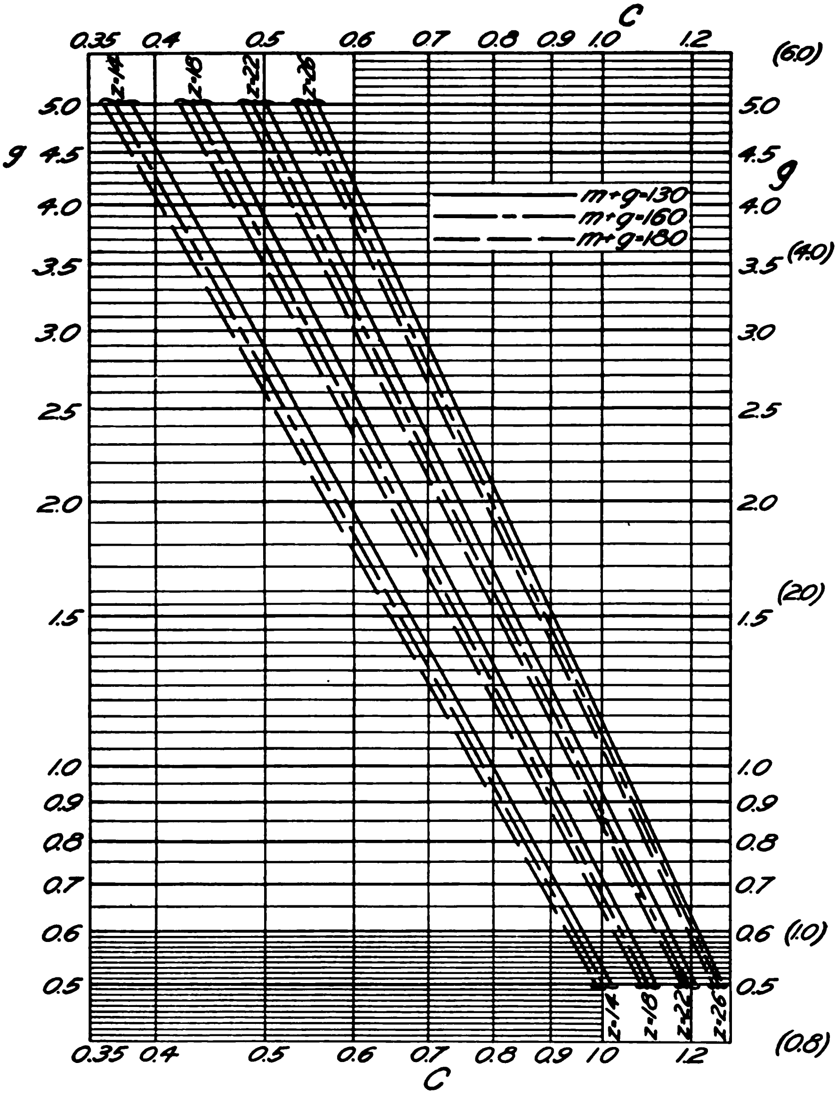

FIG. 7b. The *C* : *g* curves.

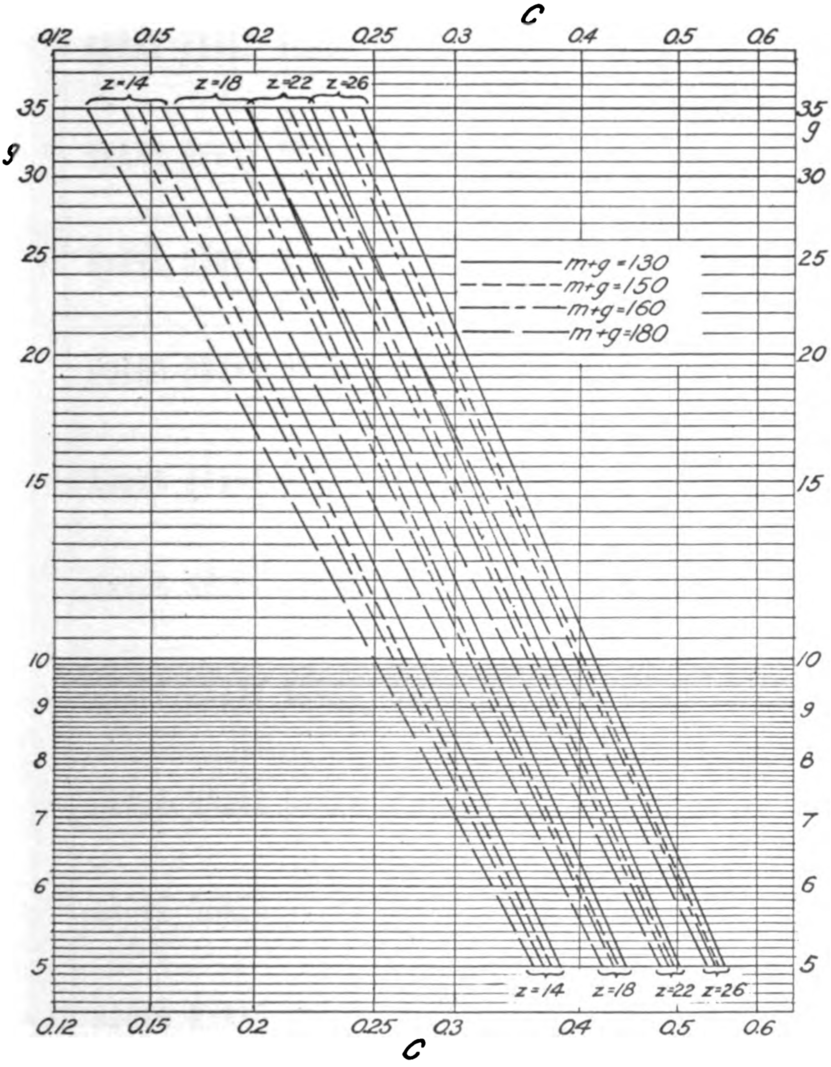

FIG. 7c. The $C:g$ curves.

TABLE IA.—$C:g$ Table

	$z = 14$			$z = 18$			$z = 22$			$z = 26$		
	$m + g$			$m + g$			$m + g$			$m + g$		
	180°	160°	130°	180°	160°	130°	180°	160°	130°	180°	160°	130°
g	C	C	C	C	C	C	C	C	C	C	C	C
.1	1.644	1.651	1.659	1.754	1.762	1.772	1.851	1.859	1.870	1.928	1.937	1.950
.125	1.529	1.538	1.552	1.642	1.652	1.665	1.739	1.749	1.763	1.814	1.824	1.838
.15	1.445	1.454	1.468	1.554	1.565	1.578	1.652	1.662	1.675	1.728	1.739	1.753
.175	1.387	1.398	1.413	1.470	1.483	1.500	1.569	1.581	1.596	1.650	1.660	1.672
.2	1.319	1.332	1.348	1.426	1.438	1.454	1.522	1.534	1.550	1.602	1.612	1.626
.25	1.226	1.242	1.262	1.334	1.348	1.366	1.429	1.442	1.459	1.511	1.522	1.534
.3	1.159	1.172	1.190	1.262	1.272	1.295	1.358	1.370	1.386	1.436	1.447	1.462
.35	1.102	1.117	1.137	1.207	1.220	1.236	1.300	1.312	1.327	1.377	1.390	1.406
.4	1.058	1.066	1.090	1.159	1.173	1.192	1.252	1.263	1.280	1.328	1.341	1.358
.45	1.018	1.032	1.052	1.118	1.132	1.152	1.210	1.224	1.243	1.289	1.303	1.321
.5	.988	1.003	1.021	1.085	1.099	1.116	1.174	1.189	1.208	1.253	1.267	1.285
.55	.958	.973	.990	1.054	1.068	1.087	1.145	1.160	1.176	1.222	1.237	1.253
.6	.930	.946	.962	1.023	1.036	1.057	1.112	1.131	1.149	1.195	1.204	1.222
.65	.904	.920	.935	.998	1.012	1.030	1.087	1.102	1.117	1.163	1.177	1.196
.7	.881	.896	.912	.974	.988	1.004	1.061	1.077	1.096	1.138	1.152	1.170
.8	.840	.854	.870	.931	.946	.963	1.016	1.032	1.049	1.092	1.107	1.122
.9	.802	.817	.832	.892	.908	.925	.978	.993	1.008	1.052	1.065	1.084
1.0	.770	.783	.799	.859	.872	.891	.941	.956	.974	1.016	1.028	1.045
1.25	.702	.717	.731	.788	.801	.819	.868	.883	.899	.941	.954	.971
1.5	.649	.662	.678	.731	.745	.762	.810	.823	.839	.880	.893	.910
1.75	.604	.618	.632	.685	.698	.714	.760	.773	.789	.823	.841	.858
2.0	.570	.581	.596	.647	.660	.675	.720	.733	.748	.786	.799	.816
2.5	.511	.522	.538	.586	.598	.612	.655	.668	.682	.720	.730	.747
3.0	.468	.478	.493	.538	.551	.565	.604	.617	.631	.667	.678	.693
3.5	.433	.443	.457	.501	.512	.527	.564	.576	.590	.624	.636	.651

TABLE IA.—$C:g$ Table—*Continued*

g	$z = 14$			$z = 18$			$z = 22$			$z = 26$		
	$m + g$			$m + g$			$m + g$			$m + g$		
	180°	160°	130°	180°	160°	130°	180°	160°	130°	180°	160°	130°
	C	C	C	C	C	C	C	C	C	C	C	C
4.0	.405	.414	.428	.470	.481	.495	.532	.543	.557	.590	.601	.615
4.5	.381	.391	.403	.444	.454	.468	.503	.514	.528	.561	.571	.584
5.0	.361	.370	.383	.423	.433	.446	.480	.491	.504	.535	.546	.559
5.5	.343	.352	.367	.403	.413	.428	.459	.470	.483	.511	.522	.536
6.0	.329	.337	.352	.386	.397	.411	.440	.451	.466	.490	.502	.517
7.0	.303	.313	.327	.357	.370	.384	.408	.420	.436	.456	.470	.483
8.0	.282	.293	.307	.334	.347	.362	.382	.396	.411	.427	.441	.457
9.0	.265	.276	.291	.314	.328	.343	.361	.374	.390	.404	.418	.434
10.0	.251	.262	.277	.299	.312	.328	.343	.355	.372	.383	.396	.414
11.0	.239	.250	.264	.284	.297	.313	.327	.341	.357	.367	.381	.398
12.0	.228	.239	.254	.272	.285	.301	.313	.327	.343	.352	.366	.383
13.0	.218	.230	.244	.262	.275	.291	.302	.316	.332	.339	.353	.370
14.0	.210	.222	.236	.252	.266	.282	.291	.305	.321	.327	.341	.359
15.0	.203	.213	.228	.244	.256	.273	.281	.294	.312	.317	.330	.348
16.0	.196	.208	.222	.236	.250	.265	.273	.287	.304	.306	.321	.339
18.0	.184	.196	.210	.222	.236	.251	.257	.271	.289	.290	.304	.322
20.0	.175	.186	.200	.211	.223	.240	.244	.257	.275	.276	.289	.308
22.0	.166	.177	.191	.201	.214	.230	.233	.248	.264	.263	.279	.296
24.0	.158	.170	.183	.192	.206	.221	.223	.238	.254	.252	.268	.285
26.0	.152	.163	.177	.185	.199	.214	.215	.229	.246	.243	.258	.275
28.0	.146	.157	.171	.178	.191	.207	.207	.221	.238	.234	.248	.267
30.0	.141	.152	.165	.172	.185	.200	.200	.214	.231	.227	.241	.259
32.0	.136	.147	.160	.167	.180	.195	.195	.208	.224	.220	.235	.252
34.0	.132	.142	.156	.162	.175	.189	.188	.203	.219	.214	.229	.245
36.0	.128	.138	.152	.157	.170	.185	.183	.198	.213	.209	.223	.240

III

THE APPLICATION OF THE FORMULA

The formula is worked in the form $\frac{f_h C}{t} = A$; (17a) or, for the condition of sterility,

$$\frac{f_h C}{t} = 1. \tag{17b}$$

Therefore, for sterility,

$$\frac{t}{f_h} = C. \tag{18}$$

To apply:

From the heating curve, we know f_h (which has been referred to Briggs logs by multiplying the original value from Fig. 3 by 2.3026). We find a value of g corresponding to a certain value of t on the thermal death time curve, and to a certain value of C on the $C : g$ curves, which values of t and C will satisfy equation (18). This value of g, referred to the heating curve, gives the length of process necessary.

The solution of the equation may be effected by either of two methods —the one entirely graphical, the other, a combination of equations and graphs. The graphical method will be described first.

DETERMINING g BY GRAPH

(1) Draw a semi-log curve parallel to the thermal death time curve, and in such a position that its point of intersection with any vertical line has a y-value which is $\frac{1}{f_h}$ times the y-value of the point of intersection of the thermal death time curve with the same vertical line. This curve, owing to the nature of the logarithmic scale, represents $\frac{y_o}{f_h}$ plotted against temperature. It is best to draw this curve on a sheet separate from that upon which the thermal death time curve is plotted.

(2) Express the scale in the direction of abscissæ in terms of g; i. e., giving the vertical line representing retort temperature the value zero, count from this in the sense of decreasing temperature.

(3) Referring now to the $C : g$ curves, Fig. 7, find the curve corresponding to the particular values of z and $m + g$ obtaining in the problem. If the curve is not plotted for the particular values desired, interpolate to find the position of the curve. Place a straight edge—preferably a transparent one—to mark the curve. The section of this curve which

will intersect the "$\frac{y_o}{f_h}$" curve [step (1)] can be estimated by a hasty observation of the two curves.

(4) Draw to a large temperature scale, a small portion; say the portion extending over about four degrees; of the "$\frac{y_o}{f_h}$" curve on a separate sheet of semi-log paper. See Fig. 10.

(5) On the same sheet, and using the same scale for g, plot the corresponding portion of the $C:g$ curve, transferring four or five points. The intersection of the two curves gives the value of g desired.

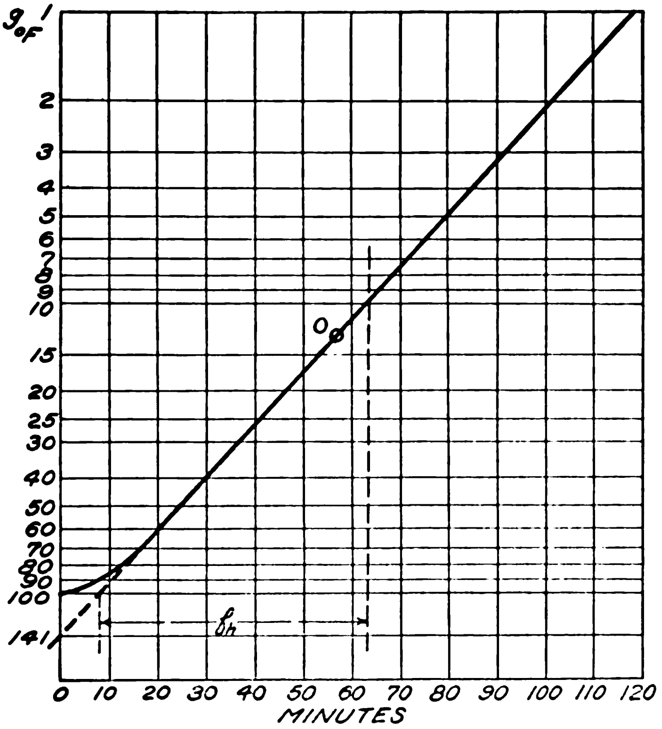

Fig. 8. Theoretical heating curve.

(6) Refer this value of g to the heating curve to obtain the length of process.

Example:

Suppose that a No. 2 can of corn gives the heating curve shown in Fig. 8, and a certain organism when studied under the conditions existing in the corn during processing, gives the thermal death time curve shown as AB in Fig. 9.

From Fig. 8, it is seen that the initial temperature of the can is 100° F below the retort temperature, which we shall assume to be 250° F. Also, from Fig. 8, $f_h = 55$ min.

From Fig. 9, z is found to be equal to 18°. The curve, *CD*, is the "$\frac{y_o}{f_h}$" curve described in step (1), p. 30, and its position is found by locating the point, *E*, on the thermal death time curve at which $y_o = f_h = 55$, and dropping a perpendicular from this point to the horizontal line representing $y_o = 1$. The curve, *CD*, is then drawn, parallel to *AB*. It generally happens when the curve, *CD*, is constructed on the same sheet as curve, *AB*, that an insufficient length of *CD* can be plotted on the scale as given for the thermal death time curve. In such a case, the curve, *CD*, should be moved in the direction of ordinates to the position, *HK*, and the logarithmic scale should be changed correspondingly. Thus, in Fig. 9, the scale on the right hand margin refers to the curve, *HK*, which is now the "$\frac{y_o}{f_h}$" curve of our problem.

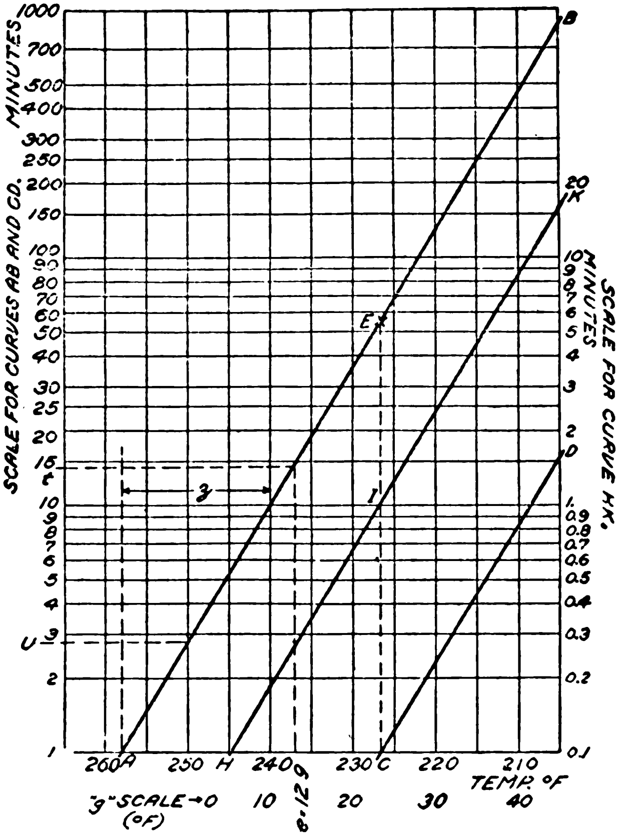

FIG. 9. Thermal death time curve and "$\frac{y_o}{f_h}$" curve.

We shall assume the temperature of the cooling water to be 70° F.[2] Therefore, $m + g = 250° - 70° = 180°$. Now we have secured the data, $z = 18$ and $m + g = 180$, which is all we need in addition to the curves we have already plotted. The $C : g$ curve, which we must choose, is determined by these values.

To determine approximately the position on the HK curve, Fig. 9, of its intersection by the $C : g$ curve [see step (3), p. 30]; let us, in order that we may have a well defined method of procedure, assume any four values of g, such as, 5, 10, 15, 20, and tabulate them, together with their approximate corresponding values from the "$\frac{y_0}{f_k}$" and $C : g$ curves. Take these values from the two curves:

TABLE II

g	$\frac{y_0}{f_k}$	C
5	0.096	0.423
10	0.182	0.299
15	0.345	0.244
20	0.653	0.211

This shows at a glance that the two curves cross between the vertical lines $g = 10$ and $g = 15$; probably at about $g = 12$ or $g = 13$.

Re-plot according to step (4), p. 31 the "$\frac{y_0}{f_k}$" curve to a large scale between the vertical lines, $g = 11$, and $g = 15$. See Fig. 10. This is done by merely drawing a straight line between the two points, M, $(g = 11, \frac{y_0}{f_k} = .207)$; and N, $(g = 15, \frac{y_0}{f_k} = .345)$; taken from the HK curve, Fig. 9.

Now plot on the same sheet, the $C : g$ curve. [See step (5), p. 31]. Five points taken from the original curve, Fig. 7 are:

TABLE III

g	C	Designated on Fig. 10 by—
11	.2855	P
12	.2737	Q
13	.2628	R
14	.2527	S
15	.2438	T

---

[2] It is well, in all general cases, to assume a low temperature for the cooling water, for lower temperature means shorter time for cooling; so results from calculations have a small margin of safety if based upon temperatures lower than those actually used.

The point of intersection, *O*, now gives the value of g for which we have been seeking. Refer this value, $g = 12.9$ back to the heating curve, Fig. 8. It tells us that the length of process necessary is 57 minutes.[1] [See step (6), p. 31].

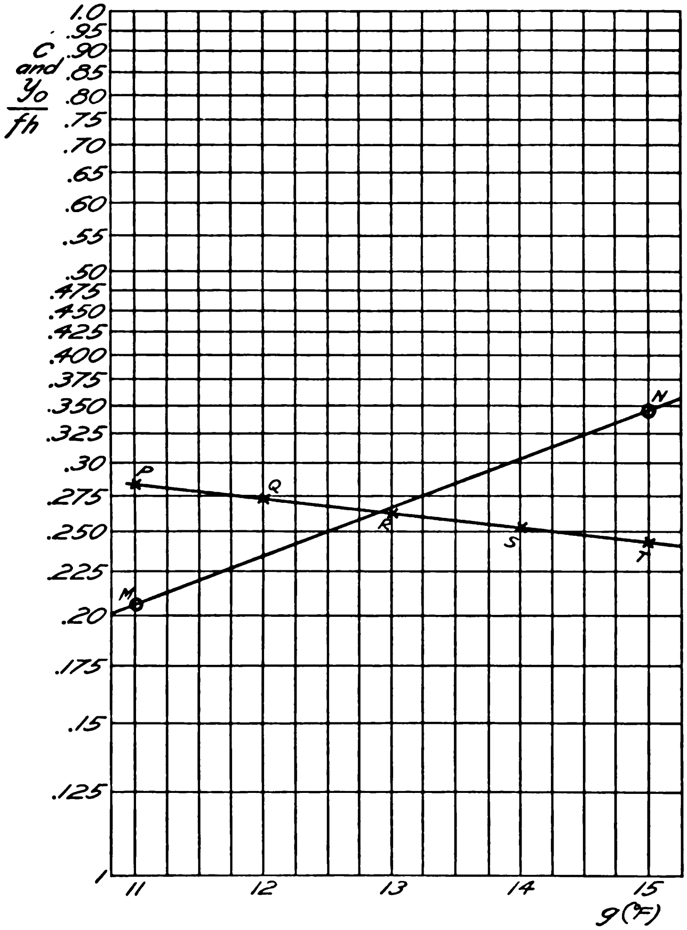

FIG. 10. Intersection of the $C : g$ and the "$\frac{y_o}{f_h}$" curve.

If the preliminary inspection of the curves made in the manner shown in Table II reveals the fact that $g < 2°$, the values chosen for column 1, Table III will be expressed in fractions of degrees.

The reader may find that time would be saved, in making calculations, by taking the values of *C*, which are to be placed in the third column of Table II and in the second column of Table III from a table rather than from the curves of Figures 7a, 7b, and 7c. Table IA is therefore given to be used for this purpose. If greater accuracy is desired than can be obtained by the use of Table IA, a similar table using smaller intervals of g and of $m + g$ may be constructed by referring to the curves.

---

[1] It is not meant here to recommend this as a process for corn in No. 2 cans. This is merely a hypothetical case, used to illustrate the method.

Although the explanation of this method is necessarily a little long, it is seen that the steps are all extremely simple and are easily performed; so that, after the heating curve and the thermal death time curve have been plotted, a result can be obtained in a very few minutes.

DETERMINING g BY EQUATION AND GRAPH[1]

The $C : g$ curves plotted in Fig. 7, are not in reality true log curves; and in order to plot them as straight lines on log paper and at the same time make them represent the $C : g$ relation with sufficient accuracy for the purpose of the problem, it was necessary to change the scale, thus breaking the curves at two points. The curves are broken at the horizontal lines $g = 0.5°$ and $g = 5°$.

By inspection, equations were applied to the curves, which represent them accurately as plotted:

All curves from $g = .1$ to $g = .5$, satisfy the equation,

$$\frac{\log \frac{g}{.1}}{\log \frac{\frown}{.1}} = \log \frac{C}{\alpha} \tag{19}$$

in which

α = a constant = the value of C at the point of intersection of the curve with the horizontal line, $g = .1$; and

$\frown$ = a constant = the value of g at the point of intersection between the curve and the vertical line, $C = 10\alpha$.

All curves from $g = .5$ to $g = 5$ satisfy the equation,

$$\frac{\log \frac{g + .4}{.9}}{\log \frac{\phi}{.9}} = \log \frac{C}{\beta} \tag{20}$$

in which

β = a constant = the value of C at the point of intersection of the curve with the horizontal line, $g = .5$; and

ϕ = a constant = .4 + the value of g at the point of intersection between the curve and the vertical line, $C = 10\beta$.

---

[1] The reader who does not care to follow the derivation of the equation is referred to the method of its application, p. 41, and Table VI.

All curves above $g = 5$ satisfy the equation,

$$\frac{\log \frac{g}{5}}{\log \frac{\sigma}{5}} = \log \frac{C}{\omega} \tag{21}$$

in which

ω = a constant = the value of C at the point of intersection of the curve with the horizontal line, $g = 5$; and

σ = a constant = the value of g at the point of intersection of the curve with the vertical line, $C = 10\omega$.

These equations may be generalized by putting in the form,

$$\frac{\log \frac{g + S}{H}}{\log \frac{\mathrm{W}}{\mathrm{H}}} = \log \frac{C}{T} \tag{22}$$

in which the values of the constants, S, H, W, and T, are given in the following table:

Table IV

From—	g = .1	g = .5	g = 5
To—	g = .5	g = 5	g = ∞
S =	0	.4	0
H =	.1	.9	5
W =	$\frown$	ϕ	σ
T =	α	β	ω

Table V may now be formed, giving the values of S, H, W and T which correspond to different values of $m + g$, of z, and of g; and by means of this table, the use of Fig. 7 may be eliminated, and the labor of solving equation (18) be still further reduced. The values of S and H which can be obtained directly from Table IV are not given in Table V.

TABLE V

1	2	3	4	5	6	7	8
		Division 1 $g = .1$ to $g = .5$		Division 2 $g = .5$ to $g = 5$		Division 3 $g = 5$ to $g = \infty$	
$m + g$	z	W	T	W	T	W	T
130	14	.00004968	1.6594	.01344	1.0210	.03683	.3827
	16	.00004012	1.7160	.01143	1.0700	.03232	.4150
	18	.00003204	1.7725	.01001	1.1160	.02912	.4464
	20	.00002495	1.8230	.00898	1.1625	.02684	.4760
	22	.00002087	1.8700	.00801	1.2076	.02528	.5037
	24	.00001720	1.9135	.00716	1.2475	.02425	.5333
130	26	.00001411	1.9496	.00626	1.2845	.02341	.5592
135	14	.00005145	1.6580	.01355	1.0178	.03815	.3806
	16	.00004120	1.7144	.01154	1.0667	.03347	.4128
	18	.00003287	1.7708	.01012	1.1126	.03020	.4442
	20	.00002580	1.8216	.00907	1.1597	.02790	.4740
	22	.00002160	1.8682	.00807	1.2040	.02645	.5017
	24	.00001760	1.9115	.00721	1.2443	.02540	.5315
135	26	.00001440	1.9477	.00633	1.2817	.02456	.5570
140	14	.00005303	1.6566	.01366	1.0145	.03960	.3787
	16	.00004240	1.7134	.01165	1.0636	.03480	.4107
	18	.00003372	1.7695	.01023	1.1097	.03140	.4421
	20	.00002670	1.8200	.00917	1.1567	.02920	.4719
	22	.00002215	1.8667	.00814	1.2013	.02770	.4995
	24	.00001806	1.9096	.00728	1.2415	.02660	.5291
140	26	.00001470	1.9457	.00639	1.2787	.02580	.5549
145	14	.00005480	1.6552	.01379	1.0113	.04135	.3766
	16	.00004360	1.7119	.01176	1.0601	.03640	.4085
	18	.00003462	1.7680	.01035	1.1069	.03280	.4398
	20	.00002760	1.8182	.00926	1.1538	.03055	.4697
	22	.00002270	1.8648	.00822	1.1980	.02905	.4973
	24	.00001853	1.9076	.00735	1.2383	.02787	.5269
145	26	.00001501	1.9438	.00646	1.2757	.02700	.5525
150	14	.00005644	1.6539	.01391	1.0081	.04349	.3742
	16	.00004470	1.7100	.01188	1.0575	.03826	.4065
	18	.00003559	1.7659	.01047	1.1043	.03456	.4374
	20	.00002890	1.8160	.00935	1.1512	.03200	.4675
	22	.00002353	1.8629	.00832	1.1949	.03028	.4952
	24	.00001910	1.9064	.00742	1.2360	.02910	.5244
150	26	.00001542	1.9417	.00653	1.2724	.02799	.5506
155	14	.00005850	1.6522	.01407	1.0047	.04570	.3723
	16	.00004620	1.7086	.01201	1.0534	.04045	.4039
	18	.00003660	1.7642	.01059	1.1009	.03640	.4350
	20	.00002942	1.8146	.00946	1.1476	.03375	.4649
	22	.00002392	1.8612	.00840	1.1914	.03183	.4929
	24	.00001950	1.9040	.00750	1.2323	.03047	.5219
155	26	.00001570	1.9396	.00660	1.2695	.02960	.5479

1	2	3	4	5	6	7	8
		Division 1 $g = .1$ to $g = .5$		Division 2 $g = .5$ to $g = 5$		Division 3 $g = 5$ to $g = \infty$	
$m + g$	z	W	T	W	T	W	T
160	14	.00006073	1.6510	.01423	1.0018	.04824	.3700
	16	.00004805	1.7070	.01218	1.0500	.04285	.4018
	18	.00003777	1.7624	.01073	1.0980	.03862	.4329
	20	.00003035	1.8124	.00958	1.1440	.03560	.4623
	22	.00002455	1.8591	.00850	1.1881	.03376	.4906
	24	.00001987	1.9020	.00760	1.2290	.03220	.5190
160	26	.00001608	1.9374	.00667	1.2660	.03104	.5456
165	14	.00006280	1.6493	.01440	0.9980	.05123	.3680
	16	.00004920	1.7046	.01227	1.0465	.04560	.3990
	18	.00003883	1.7603	.01084	1.0945	.04130	.4301
	20	.00003145	1.8105	.00968	1.1403	.03755	.4598
	22	.00002530	1.8573	.00860	1.1842	.03480	.4880
	24	.00002055	1.9000	.00769	1.2253	.03318	.5164
165	26	.00001641	1.9361	.00675	1.2627	.03232	.5427
170	14	.00006530	1.6477	.01457	0.9945	.05460	.3657
	16	.00005090	1.7024	.01242	1.0430	.04870	.3966
	18	.00004012	1.7583	.01096	1.0914	.04400	.4279
	20	.00003253	1.8086	.00980	1.1366	.03965	.4570
	22	.00002613	1.8553	.00870	1.1805	.03635	.4854
	24	.00002110	1.8980	.00778	1.2220	.03455	.5137
170	26	.00001682	1.9329	.00684	1.2595	.03370	.5400
175	14	.00006810	1.6458	.01475	0.9906	.05830	.3633
	16	.00005280	1.7002	.01255	1.0392	.05220	.3940
	18	.00004150	1.7562	.01109	1.0880	.04720	.4253
	20	.00003366	1.8064	.00991	1.1336	.04190	.4542
	22	.00002700	1.8531	.00881	1.1768	.03790	.4829
	24	.00002170	1.8960	.00788	1.2180	.03592	.5110
175	26	.00001725	1.9306	.00693	1.2560	.03510	.5374
180	14	.00007155	1.6444	.01491	0.9876	.06242	.3610
	16	.00005500	1.6980	.01270	1.0360	.05595	.3920
	18	.00004308	1.7543	.01122	1.0845	.05083	.4229
	20	.00003487	1.8045	.01003	1.1295	.04420	.4519
	22	.00002800	1.8510	.00892	1.1738	.03949	.4804
	24	.00002237	1.8940	.00798	1.2145	.03730	.5081
180	26	.00001772	1.9284	.00703	1.2525	.03650	.5340

An equation, expressing the relation between g and t may be constructed by an inspection of Fig. 9. Taking as the y-axis of the thermal death time curve the vertical line representing the retort temperature (in this case 250°), the equation of the curve is

$$\frac{x'_o}{z} = \log \frac{y_o}{U} \qquad (23)$$

in which

U = a constant = the value of y_0 at the intersection of the curve with the vertical line, $x'_o = 0$; i.e., U = the number of minutes required to destroy the organism at retort temperature.

z = a constant, which represents the slope of the curve, as defined for equation (4).

Substituting g for x'_o and t for y_o in equation (23),

gives
$$\frac{g}{z} = \log \frac{t}{U}. \tag{24}$$

Now, from equation (22),
$$C = \log^{-1}\left[\frac{\log \frac{g+S}{H}}{\log \frac{W}{H}} + \log T\right] \tag{25}$$

and from equation (24),
$$t = \log^{-1}\left(\frac{g}{z} + \log U\right); \tag{26}$$

substituting these values in equation (18),
$$\log^{-1}\left(\frac{g}{z} + \log U\right) = f_h \log^{-1}\left[\frac{\log \frac{g+S}{H}}{\log \frac{W}{H}} + \log T\right]. \tag{27}$$

Log [eq. 27] is
$$\frac{g}{z} + \log U = \log f_h + \frac{\log \frac{g+S}{H}}{\log \frac{W}{H}} + \log T \tag{28}$$

$$\log \frac{g+S}{H} = \left[\log \frac{W}{H}\right]\left[\frac{g}{z} - \log \frac{f_hT}{U}\right] \tag{29}$$

Let
$$\log \frac{W}{H} = M \tag{30a}$$

and
$$\log \frac{f_hT}{U} = N \tag{30b}$$

$$\log \frac{g+S}{H} = \frac{M\ (g - Nz)}{z}. \tag{31}$$

By placing the left hand member of equation (31) equal to X, and the right hand member equal to X', we obtain the two equations,
$$g = 10^X H - S \tag{32}$$

and
$$g = z\left(N + \frac{X'}{M}\right) \tag{33}$$

from which we have
$$X = \log\left(\frac{zN}{H} + \frac{S}{H} + \frac{z}{HM} X'\right). \tag{34}$$

Now, to complete the calculation we must return again to plotting curves. Equation (34) is plotted as curve AB in Fig. 11. This curve remains the same for all calculations and can be easily reproduced to a larger scale. If we plot X' against the expression, $\frac{zN}{H}+\frac{S}{H}+\frac{z}{HM}X'$,

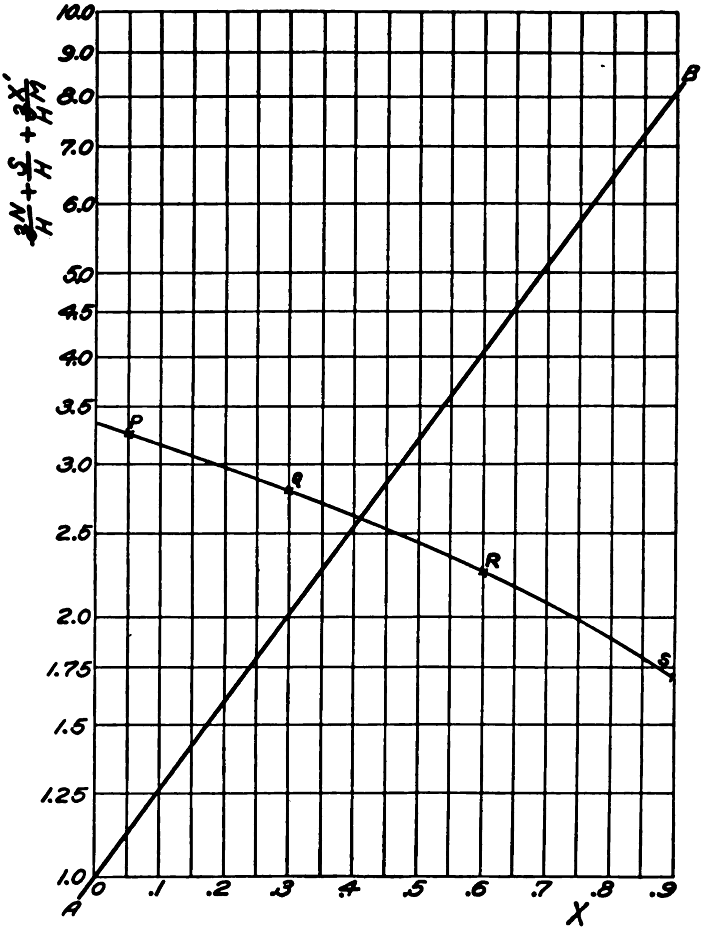

Fig. 11. Plot of equation (34).

on the same sheet, using the X-scale as the X'-scale, the intersection of the two curves gives the value of X, which, when substituted in equation (32) or equation (33) gives the value of g which is sought.

Table VI shows the entire method of procedure in condensed form:

In solving for g in this manner, it is well to assume at least four or five values for X' to be placed in column 5, Table VI, in order to have the curve well defined. Before beginning the solution it is necessary

TABLE VI

1	2	3	4	5	6	7	8	9
Obtain from								
Heating curves	Tables IV and V	Slide rule calculations		Assumed values	Slide rule	Simple addition	Plot X' (column 5) against $\frac{zN}{H}+\frac{S}{H}+\frac{zX'}{HM}$ (column 7) and obtain the value of X from the plot. (Fig. 11.)	Slide rule
z	S	$\frac{S}{H}$	$\frac{z}{HM}$	X_1'	$\frac{zX_1'}{HM}$	$\frac{zN}{H}+\frac{S}{H}+\frac{zX_1'}{HM}$		$\frac{X}{M}$
		$\frac{z}{H}$						
fh	W	$\frac{W}{H}$		X_2'	$\frac{zX_2'}{HM}$	$\frac{zN}{H}+\frac{S}{H}+\frac{zX_2'}{HM}$		$N+\frac{X}{M}$
		M	$\frac{zN}{H}$					
$m+g$	H	$\frac{fh}{U}$	$\frac{zN}{H}+\frac{S}{H}$	X_3'	$\frac{zX_3'}{HM}$	$\frac{zN}{H}+\frac{S}{H}+\frac{zX_3'}{HM}$		
U	T	N		X_4'	$\frac{zX_4'}{HM}$	$\frac{zN}{H}+\frac{S}{H}+\frac{zX_4'}{HM}$		$g=z(N+\frac{X}{M})$

$M = \log\frac{W}{H}$ N $= \log fh\frac{T}{U}$

to assume in which of the three divisions of the $C:g$ curves the value of g will fall, and the values of S, W, H, and T, will be chosen accordingly. The limits of the values of X in the three divisions are as follows:

	Min. value of X	Max. value of X
1st Div. [$g = .1$ to $g = .5$]........	0	.70
2nd Div. [$g = .5$ to $g = 5$.........	0	.78
3rd Div. [$g = 5$ to $g = \infty$]........	0	.90

Therefore, in solving within any division, values of X' beyond the limits given above need not be assumed. The assumed values, which will be entered in column 5, Table VI, are to be plotted against the corresponding calculated values of column 7. If the value of g obtained in column 9 falls outside the limits of the division for which it was calculated, it shows that the division assumed was not the proper one; and the value obtained will indicate which of the other divisions should have been chosen.

The fact that one is working in the wrong division may be revealed before the entire calculation is completed. For instance, a positive indication that the division which has been chosen is too high is given when the values calculated for column 7 are less than unity. If the curve, which is plotted according to the instructions given in column 8, intersects the curve, AB, Fig. 11, at a point for which the value of X is greater than the maximum value given on page 41 for the division chosen, it indicates that this division is too low. A positive indication that the division chosen is too low is given if the values of all four expressions in column 7 are greater than 5.0 for division 1, 6.0 for division 2, or 7.9 for division 3. The test for the condition, $g < .1°$ will be found on page 44.

Example:

An example may be taken with the same given conditions as were assumed in the previous example on page 31; that is, the heating curve of Fig. 8; the thermal death time curve, AB, of Fig. 9; processing temperature 250°; cooling water, 70°, $m + g = 180°$, $z = 18°$, and $f_h = 55$ minutes. U, from Fig. 9, equals 2.78 minutes.

Assuming that the calculation should fall under the second division, the values of S and H from Table IV are .4 and .9, respectively, and those of W and T from Table V are .01122 and 1.0845, respectively. Now, the operations indicated in Table VI, column by column, give

$$\frac{S}{H} = .4444 \qquad \frac{z}{HM} = -10.5$$

$$\frac{z}{H} = 20 \qquad \frac{zN}{H} = 26.6292$$

$$\frac{W}{H} = .01247 \qquad \frac{zN}{H} + \frac{S}{H} = 27.0736$$

$$M = \log\frac{W}{H} = 8.09587 - 10$$

$$= -1.90413$$

$$\frac{f_h}{U} = \frac{55}{2.78} = 19.78$$

$$N = \log\frac{f_hT}{U} = 1.33146$$

From page 41, it is seen that the assumed values of X^1_1, X^1_2, X^1_3, and X^1_4 must all lie between 0 and .78. Therefore, they should be taken as follows:

$$X^1_1 = .05 \qquad X^1_2 = .20 \qquad X^1_3 = .50 \qquad X^1_4 = .78$$

$$\text{Then } \frac{zX^1_1}{HM} = -.525 \quad \frac{zX^1_2}{HM} = -2.1 \quad \frac{zX^1_3}{HM} = -5.25 \quad \frac{zX^1_4}{HM} = -8.19$$

and the values of the four expressions in column 7 are 26.5486, 25.9736, 21.5236, and 18.8836, respectively. All four of these values are greater than 5. This indicates, as stated on page 41, that the division chosen is too low. The problem must therefore be solved in division 3.

The solution is as follows:

From Table IV, $S = 0$ and $H = 5$.
From Table V, $W = .05083$ and $T = .4229$.

Carrying the calculations through the scheme as outlined in Table VI and as illustrated above for the second division; when the four values for column 5 are assumed to be .05, .3, .6, and .9, respectively, the corresponding values for column 7 are 3.23055, 2.77893, 2.23699, and 1.69506. Plotting these four values on Fig. 11 against the corresponding values of X^1 gives the curve, *PS*, defined by the four points, *P*, *Q*, *R* and *S*. This curve intersects the curve *AB* at the point for which *X* equals .41.

$$\frac{X}{M} = -.205736 \qquad (M = -1.993)$$

$$N + \frac{X}{M} = .922464 - .205736 = .717728$$

$$g = z\left(N + \frac{X}{M}\right) = 18 \times .717728 = 12.92$$

When this value is referred to the heating curve, Fig. 8, it gives the process of 57 minutes. It is noted that this is the same result as that obtained by the previous method of calculation.

USE OF FORMULA WHEN g IS LESS THAN .1°

The reader has probably noticed that in the methods given, no provision has been made for the case in which g becomes less than .1°. It is recognized that the difference between the lethal rate values of two temperatures separated by .1 of 1 degree F. is too small to be considered in calculations of this nature. Therefore, it is assumed that after the center of a can has reached the temperature .1° below retort temperature, the lethal rate is constant for the remainder of the process.

On this assumption, the formula as given in equation (17b), may be expanded to

$$\frac{f_h C_{.1}}{t_{.1}} + \frac{h}{t_{.1}} = 1 \qquad (35)$$

in which

h = the number of minutes required to complete the process after the temperature of the can reaches (RT − .1°).

$C_{.1}$ = the value of C corresponding to g = .1°, and

$t_{.1}$ = the value of t corresponding to g = .1°.

A transposition of equation (35) gives

$$h = t_{.1} - f_h C_{.1}. \qquad (36)$$

If, in solving for g by the first method, it is found that $g < .1°$; the value $C_{.1}$ taken from the $C : g$ curves, and that of $t_{.1}$ taken from the thermal death time curve, substituted in equation (36), give the value of h. The length of process required equals the time taken to bring the temperature of the can to .1° below retort temperature; plus h minutes.

If, in solving for g by the second method, it is found that $g < .1°$; the value of $C_{.1}$ is taken from Table V, in which it is given as T in column 4. A sure and quick test for determining whether or not g is less than .1° is to first calculate the value of the product, $f_h C_{.1}$. If this is less than the value of $t_{.1}$, then $g < .1°$.

On the whole, the second method will probably be found the more convenient method to use.

IV

THE APPLICATION OF THE FORMULA TO VARIOUS RELATIONSHIPS

Many simple relations exist between results calculated by means of the formula, [equation (18)], for varying conditions; such as different lengths of time taken to bring the retort to holding temperature, different initial temperatures, and different retort temperatures. The present section will deal with these relations, and all considerations will continue to be based upon the assumption that all heating, cooling, and thermal death time curves agree with the theoretical curves described in the introduction.

Of course, there are many conditions which cause the curves, with which we have to deal, to deviate from these theoretical curves; and these deviations introduce difficulties, which must be overcome before the calculations can assume much practical value. Certain of these difficulties will be considered in Section V. Some of the relationships presented in Section IV, however, hold true under practically any conditions; others will apply to all cases with only slight alteration; and it is thought that they all have a value, which, from a practical standpoint, may be greater than the actual solution of the formula as presented in section III. A knowledge of them will certainly reduce the labor necessary to make calculations, when a great many are to be made.

EFFECT OF VARYING THE TIME TAKEN TO BRING THE RETORT TO PROCESSING TEMPERATURE

The time taken to bring a retort to processing temperature after the steam has been turned on is time during which heat is entering the can, and therefore this period must have some time value as a part of the process. This value may be expressed in per cent of the actual length of time consumed. That is, this period of increasing the temperature of the retort, will shorten the length of time necessary to process a can of food after the retort has reached processing temperature, by a certain percentage of the period.

Thompson (*C*) presents a formula which gives the change in temperature at the center of a can in a bath, the temperature of which increases at a uniform rate. A theoretical study of the effect of varying the time taken to bring the retort to processing temperature might be made on the basis of this formula. An experimental method was pursued by the writer, however, with very satisfactory results.

The study was made by conducting heat penetration runs with boiling water, using No. 2 cans of corn and of 4 per cent starch solution. The lengths of time taken to bring the bath to boiling temperature varied from 0 minutes to 66 minutes. An accurate record was made of the rate of change of temperature of the bath while this was approaching processing temperature. Curves representing these rates of change were plotted, as shown in Fig. 12, curve *BCD*; and the area beneath each curve was determined between the starting point and the boiling point; (area A_1, Fig. 12).

This area varies, between the different runs, from 47 to 66 per cent of the product of the length of the period [l_1 minutes, (Fig. 12)], and the difference between the initial temperature of the bath and the processing temperature. The product, $l_1 m_1$, it is readily seen, represents the area, A_2, which lies beneath the time-temperature curve of the bath when the processing temperature remains constant for a period of l_1 minutes. In terms of process time, however, the value of this period of increasing temperature, referred to a like period of constant processing temperature, is less than indicated by the preceding percentages. It was found to vary between 82 and 91 per cent of the value as indicated by the area beneath the curve.

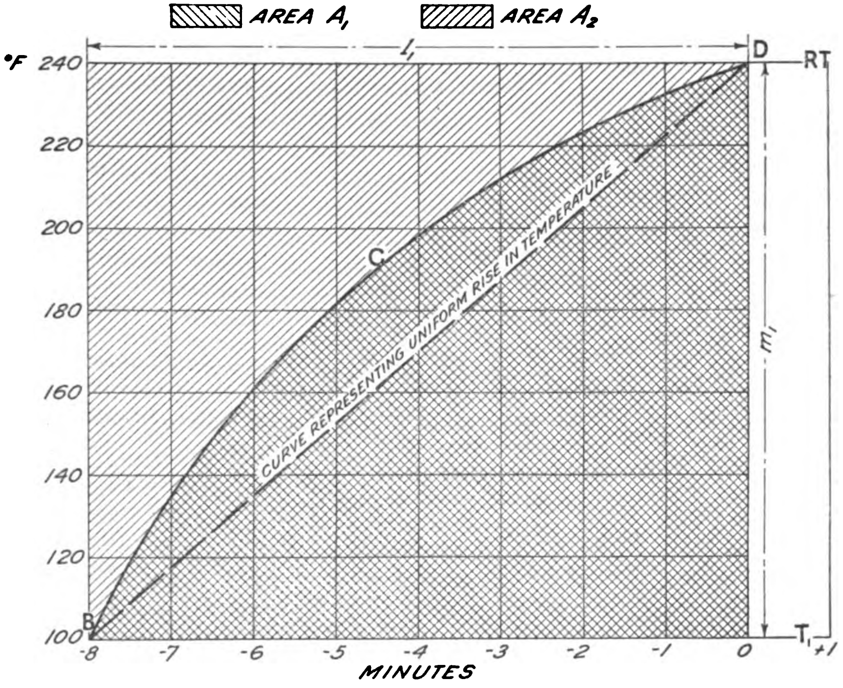

Fig. 12. Time-temperature curve for a retort previous to the nominal beginning of the process period.

Therefore, it may be said that the time taken to bring a retort to processing temperature has a value in terms of process time equal to about $\frac{.85A_1l_1}{A_2}$ minutes. As a basis of calculation it may be considered that the temperature of the retort rises uniformly from T_1 to RT. Then the retort time-temperature curve from T_1 to RT is a straight line, $A_1 = \frac{1}{2} A_2$, and the value of the period, l_1, in terms of process time is

$$.85 \times \frac{A_2}{2A_2} \times l_1 = .42\, l_1 \text{ minutes.} \qquad (37)$$

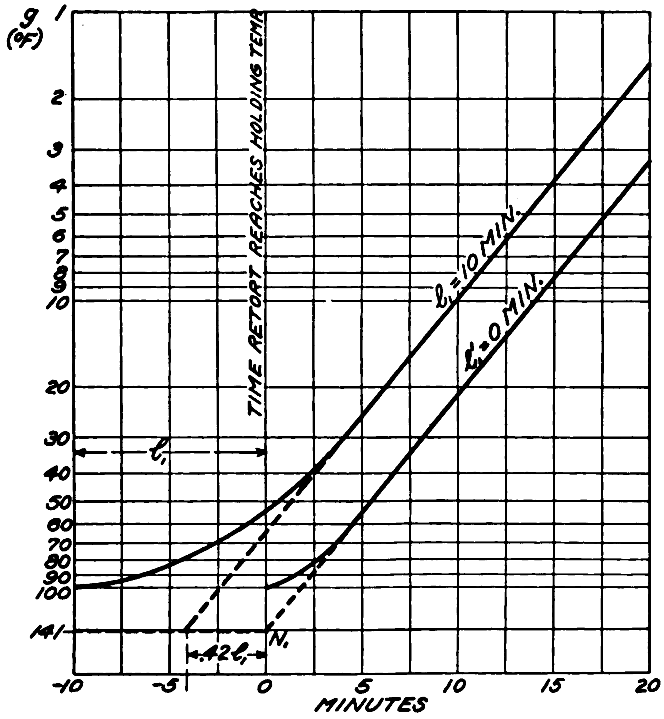

FIG. 13. Heating curves for different cans, for which different lengths of time are taken to bring the retort to processing temperature.

In these experiments, the initial temperatures of the bath and of the contents of the cans were the same. If the retort starts at a temperature below that of the can, the period, l_1, begins at the instant the retort reaches the temperature of the can. If the initial temperature of the retort is higher than that of the can, the value to the process, of the period, l_1, will be increased to a somewhat greater quantity than that given in equation (37).

To determine the equivalent of the period, l_1, in process time the heating curves were plotted on semi-log paper. It was found that increasing the period, l_1, merely moves the log curve to the left with respect to the vertical line representing the time at which the retort reaches processing temperature. This is illustrated in Fig. 13. The number of minutes through which a curve is moved to the left from the position, N_1, is the process time equivalent of the period, l_1.

The principle just set forth obviously cannot be applied when the penetration of heat into the can is so rapid that the temperature at the center of the can follows the retort temperature very closely as the latter rises; so that the temperature of the can is but a few degrees below processing temperature at the time the retort reaches this temperature. For instance, the temperature of a can of peas follows that of the retort so closely that the shape of the heating curves for this substance may be made to vary greatly by varying the time used to bring the retort up to processing temperature.

Peas in No. 2 cans, when no "coming up time" is used in their process, give a heating curve for which the value of f_h is less than 3; while for peas in No. 10 cans, when processed under the same condition, the value of f_h is about 8. If, however, time is taken to bring the retort to processing temperature, the difference between the heating curves for No. 2 and No. 10 cans is not so great; and the tendency of the curves to become similar increases as this period of time increases. When the time required for the rise in temperature of the retort is as great as 15 minutes, it is hardly possible to distinguish between the heating curves for No. 2 and No. 10 cans.

INFLUENCE OF INITIAL TEMPERATURE

Both Bigelow et al (A) and Magoon and Culpepper (F) state that, no matter what the initial temperatures of different cans of the same substance are; if the cans are processed together, theoretically the centers of all cans will reach retort temperature in the same length of time.

This statement is true only when it is referred to infinite time. Theoretically all cans reach retort temperature in infinite time. But when referring to any finite time, and to any fraction of a degree below retort temperature, no matter how infinitesimal is that fraction, the statement is not true. An inspection of Fig. 14 will make this clear. Theoretically, the heating curves of different cans of the same substance, when plotted logarithmically, are parallel. Therefore, they will continue infinitely to be separated by a constant interval, l_2 minutes measured in the direction of abscissæ. Fig. 14 shows the heating curves of three similar cans, the initial temperature of one of which is 50° below retort temperature; of another 100° below; and of the third, 500° below. It is seen that when the curves cross the horizontal line representing .01° below retort temperature they are separated by the same intervals, measured in the direction of abscissæ, as at 30° below retort temperature. (For the present, in Fig. 14, regard all curves as

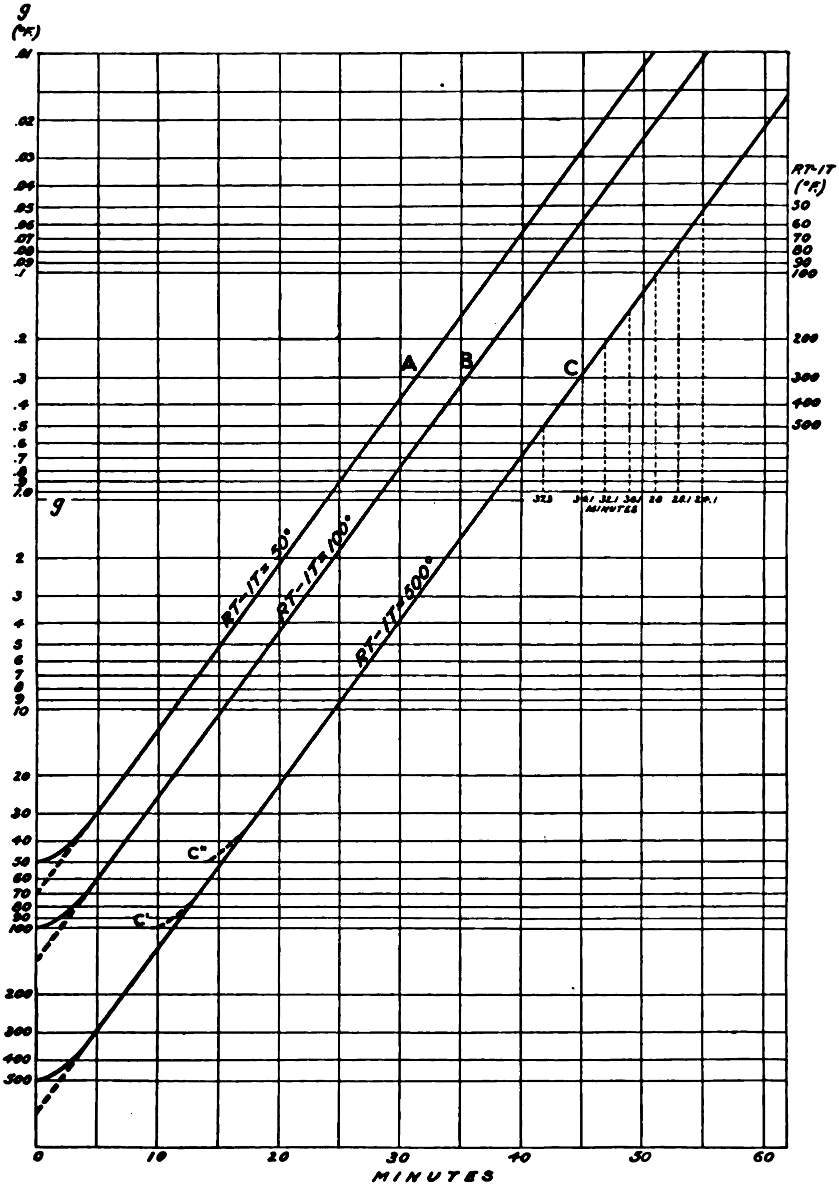

FIG. 14. Heating curves for cans starting at different initial temperatures.

plotted to the scales in the left hand margin and at the bottom of the figure.)

Therefore, approaching the subject of the influence of initial temperature upon process time in this way, an equation expressing the relation desired can be deduced directly from the equation of the heating curve. Such an equation is

$$\frac{B - D}{f_h} = \log \frac{I}{I'} \qquad (38)$$

in which

D = the length of process which has been previously determined for the condition, $RT - IT = I'$ degrees.

B = the length of process which is to be calculated for the condition, $RT - IT = I$ degrees. (I may be given any value, but if $I < 80°$ the accuracy of the result will depend upon the manner in which the contents of the can are brought up to the initial temperature.)

For example: if, by the use of curve B, we have found the required length of process to be 28 minutes, and $g = 1.1°$, the required length of process for cans represented by the curves, A and C (for which all conditions except initial temperature are the same as for the first can) can be found by letting g equal 1.1° for each of them. This gives for can, A, a process of 24 minutes, and for can, C, a process of 37.3 minutes.

Applying equation (38) to the problem,

$I' = 100°$
$f_h = 12.75$ minutes.
$D = 28$ minutes

I = (for A) 50°; (for C), 500°
B = (for A) 24 minutes; (for C), 37.3 minutes.
B is the quantity sought.

The significance of equation (38) may be made clearer by considering curve, B, Fig. 14, to be a section, $C'C$, of curve, C, which has been moved to the left 9.3 minutes; and curve, A, to be a section, $C''C$, of curve, C, which has been moved to the left 13.3 minutes. So the heating curve of a can may, by a mere adjustment of the scale, be made to represent equation (38). In the upper right hand section of Fig. 14, such an adjustment is shown, making the upper portion of curve, C, represent the equation. The ordinate scale represents the quantity, $RT - IT$ (the difference between retort temperature and initial temperature, expressed in degrees F.); and is placed so as to conform to the logarithmic ruling of the paper. The determination of required length of process needs to have been made for only one initial temperature in order for one to be able to establish the scale of minutes. For instance, if the length of process has been determined for the initial temperature 100° below retort temperature, the vertical line passing through the intersection of the curve with the horizontal line marked 100 is given the value of 28 minutes, and the rest of the scale is added to agree with this. Attention is called to the fact that the direction of increase in this scale is from right to left.[1]

If the required length of process of a given can of food has been determined for one initial temperature for each of several different retort temperatures, a very compact chart can be constructed from the data thus obtained which will give the length of process required by the can under any combination of retort temperature and initial temperature. The chart is based primarily upon the considerations which have just been discussed in connection with initial temperature; so the construction of the chart will be merely described here and detailed reasons for each step in the construction will not be given.

A sample chart is shown in Fig. 15. It is constructed as follows:

(1) Choose a vertical line of reference, *AB*.

(2) Draw curve, *CD*, (with the same slope as the heating curve) through the point of intersection, *O*, of the line, *AB*, with the horizontal line separating two logarithmic cycles. Omit the scale of minutes (given in parentheses at the bottom of the figure).

(3) Place the scale of retort temperatures at the bottom.

(4) Draw the system of dot and dash lines, which are given initial temperature values; so that each crosses one unit on the vertical scale shown in the right hand margin for each degree crossed on the horizontal scale. (Omit the scale in the right hand margin from the chart.)

(5) Give the line of this system which passes through the point, *O*, the value 100° lower than the retort temperature represented by the vertical line, *AB*. Give the other lines values as indicated, according to the number of units which separates them on the scale of ordinates, along *AB*.

(6) Place the scale of minutes in the left hand margin.

(7) Find the intersection of the vertical line representing any retort temperature and the dot and dash line representing the initial temperature for which the required length of process has been determined. Move horizontally from this point to the curve, *CD*. Then proceed vertically upwards from this point to the horizontal line representing the length of process, which has been determined. Mark this point. Do likewise for the other retort temperatures. If the length of process determined for the different retort temperatures were all for the condition, $RT - IT = 100^\circ$, the points determined as described in this paragraph will all lie on the line, *AB*.

(8) Draw the system of solid lines; which are given retort temperature values; so that each line crosses one minute on the vertical scale for each minute on the horizontal scale at the bottom of the figure.

---

[1] There are substances for which difference in initial temperature has such an effect upon the penetration of heat to the center of the can that the principle here discussed is not applicable to them. Discussion of this subject cannot be given here.

(9) Give each line of this system a retort temperature value corresponding to the point [located by step (7)], by means of which the position of the line has been determined.

To read the chart: Example. It is desired to find the length of

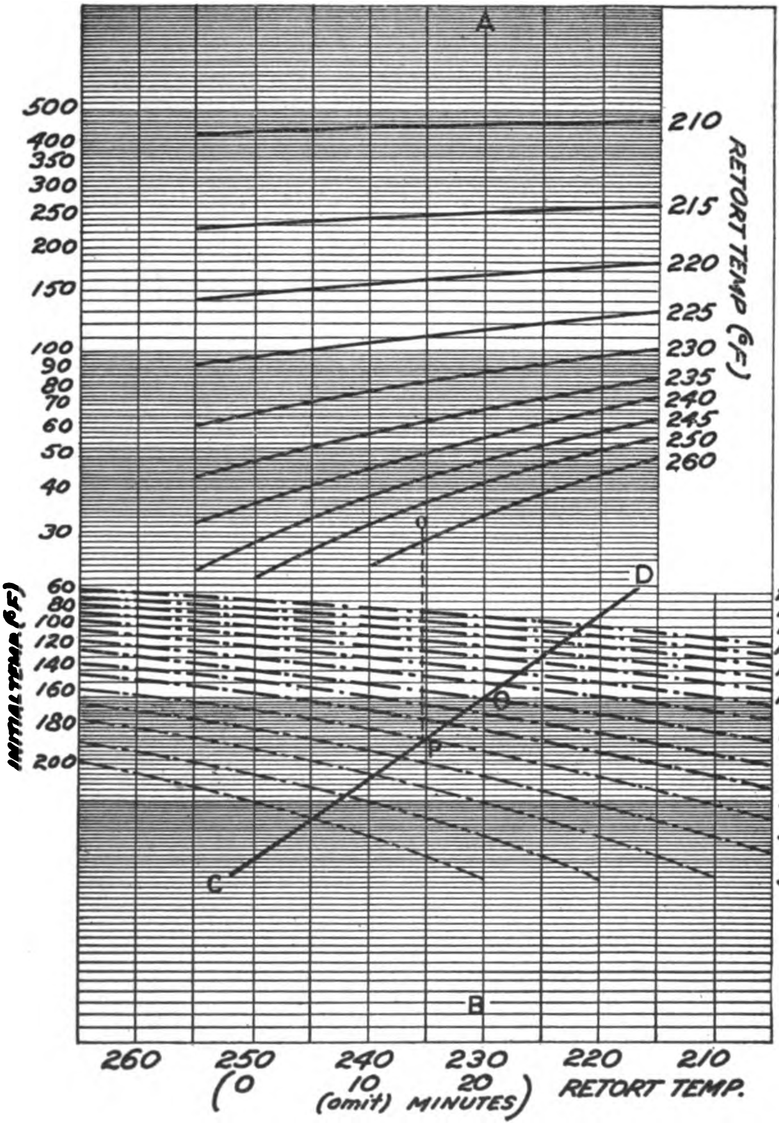

FIG. 15. Chart giving length of process required under any combination of initial and retort temperatures. Scales marked "Omit" have value only during construction of chart.

process required when the can enters the retort with an initial temperature of 180°, and is processed at 255°.

(*a*) Find the intersection of the vertical line representing retort temperature 255° with the dot and dash line representing initial temperature 180°, (point *P*, Fig. 15).

(*b*) From *P*, move horizontally (dash line) to the curve, *CD*, (point *R*).

(*c*) From *R*, move vertically to the solid line representing retort temperature, 255°. Since this line is not given on the chart, it is necessary to interpolate between 250° and 260°. The required process is found to be about 32 minutes.

EQUIVALENT PROCESSES

Occasionally, it is desired to find for one can under any given conditions, a process which is equivalent in lethal value to a given process for a different can under different conditions.

By equation (17), for the known process, $\frac{f_h'C'}{t'} = A'$. Primed symbols will be used in connection with the given process. Since the length of this process is known, the value of g' may be obtained from the heating curve, then C' and t' from their respective curves. Thus, A' is evaluated by equation (17).

Now, for the unknown process

$$\frac{f_hC}{t} = \frac{f_h'C'}{t'} = A';$$

or

$$C = \frac{t}{\frac{f_h}{A'}} \tag{39}$$

Since f_h and A' are both known, equation (39) may be solved to give g by either of the methods given in Sec. III. If the second method is used, $N\left[= \log \frac{f_hT}{U}\right]$ must be replaced by N', which equals $\frac{f_h}{U}\frac{T}{A'}$

RELATIONSHIP BETWEEN HEATING CURVES FOR DIFFERENT SIZES OF CANS

By a comparison between the simple logarithmic formula for the heating curve used in this paper and the formula given by Williamson and Adams (D) for the temperature at the center of a cylinder at any instant during the external application of heat, the following equation has been derived to express the relationship existing between the diameter of a can and the slope of its logarithmic heating curve:

$$d^2 = 4 \frac{K}{f_{\text{II}}} f_h \tag{40}$$

in which d is the diameter of the can in cm.

f_{II} is a factor the value of which is fixed by the ratio, $\frac{\text{length of can}}{\text{diameter of can}}$;

and K = the diffusivity constant = $\frac{\text{thermal conductivity}}{\text{specific heat} \times \text{density}}$.

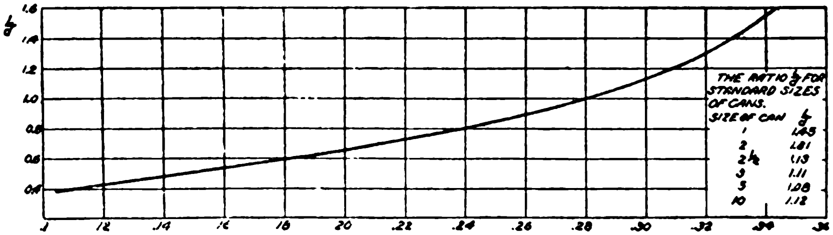

FIG. 16. Curve showing the values of the factor, f_{II}, corresponding to different values of the ratio, $\frac{L}{d}$.

The values of f_{II} corresponding to different values of the ratio, $\frac{L}{d}$, are given in Table VII; and are plotted in Fig. 16. This figure contains also a table giving the values of the ratio, $\frac{L}{d}$, for standard size cans.

TABLE VII

$\frac{L}{d}$	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	∞
f_{II}	0.109	0.148	0.181	0.212	0.239	0.261	0.280	0.296	0.308	0.320	0.328	0.337	0.343	0.397

Equation (40) is the simple equation of a parabola, the shape of which (when we consider f_{II} to remain constant) varies only with the diffusivity constant; i.e., with the character of the contents of the can.

If two parabolas, plotted with common axes, are represented by the equations,

$$d'^2 = 4 \frac{K'}{f_{\text{II}}} f'_h \tag{41}$$

$$\text{and } d''^2 = 4 \frac{K''}{f_{\text{II}}} f''_h \tag{42}$$

and d' is placed equal to d'', the following relation is obtained:

$$f''_h = \frac{K'}{K''} f'_h \tag{43}$$

A curve plotted from equation (41) will give the slope of heating curve corresponding to any diameter of can for the food whose diffusivity constant equals K'; and a curve plotted from equation (43) will, from this value, f''_h, give the slope of heating curve corresponding to the same diameter of can for any other product, the diffusivity constant of which equals K''. Fig. 17 illustrates this method of plotting. The parabola, only a half of which is plotted, represents the food for which $\frac{K'}{f_{\text{II}}} = 1.2$. Any value of this constant may be used, however, in plotting the parabola. The straight line curves to the right of the f'_h-axis represent equation (43) for different values of K''.

Horizontal lines, which may be considered to represent the different sizes of cans are located by the points of intersection to the parabola with vertical lines representing the diameters of the respective cans. These horizontal lines for the parabola, *OP*, are shown to the left of the f'_h-axis in Fig. 17. They are located by considering f_{II} as constant for all sizes of cans and its value is taken as 0.3, which is a little above the average of the values for all sizes except Nos. 1 and 2 (see Fig. 16). It is evident, however, that these same horizontal lines cannot be regarded as representing the different sized cans containing the same product if the correct value is given to f_{II} for each size of can; for K' would have to vary inversely as f_{II} in order to keep $\frac{K'}{f_{\text{II}}}$ constant in the equation of the parabola, *OP*.

If K' be kept constant and f_{II} be allowed to vary with different sizes of cans, the quantity, $\frac{K'}{f_{\text{II}}}$ will vary, and a different parabola will be required to represent each size of can. The different values of f'_h may be calculated by means of equation (41) without the necessity of constructing the different parabolas. The tabulated calculations with their results are given in Table VIII. The values of f_{II} are taken from

TABLE VIII

1	2	3	4	5	6	7	8
Size of can	f_{II}	$\frac{K'}{f_{\text{II}}}$	$4\frac{K'}{f_{\text{II}}}$	d' (*cm.*)	d'^2 (*cm.*^{2})	f'_h ($K' = .36$)	f'_h ($f_{\text{II}} = 0.3$) ($\frac{K'}{f_{\text{II}}} = 1.2$)
1	.3322	1.083	4.332	6.53	42.53	9.82	8.87
2	.3202	1.123	4.492	8.35	69.70	15.52	14.52
2½	.2995	1.202	4.808	9.89	97.50	20.25	20.32
3	.2970	1.212	4.848	10.50	110.10	22.74	22.96
5	.2928	1.230	4.920	12.63	159.70	32.41	33.23
10	.2982	1.208	4.832	15.30	233.80	48.38	48.65

Fig. 16. In column 8 of the table are given the calculated values of f'_h when $f_{II} = 0.3$ and $\frac{K'}{f_{II}} = 1.2$. In column 7 are given the corrected values of f'_h; and horizontal lines, located to correspond to these values, are shown in Fig. 17 to the right of the f'_h-axis. The correction, as is readily seen by a comparison of the portion of the figure to the right

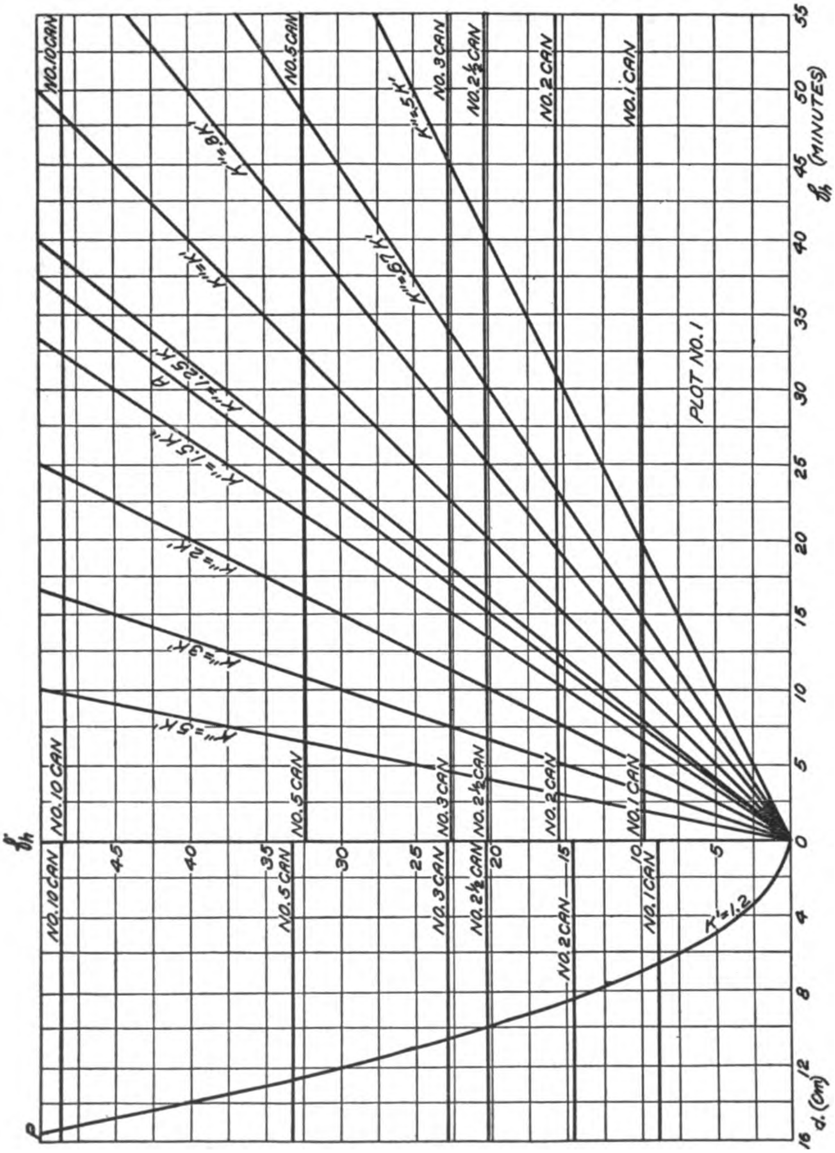

FIG. 17. Curves showing the relation between the diameter of the can and the slope of the heating curve.

of the f'_h-axis with the portion to the left of that axis, somewhat diminishes the range of variation between the different sizes of cans.

To use Fig. 17, the portion to the left of the f'_h axis is not needed, and may therefore be eliminated. The scale on this axis also is unnecessary. The straight line curves radiating from the origin, *O*, are considered to represent different foods and the curve for any particular food may be located by carefully determining the heating curve for one size of can. For example, if it is found by making a heat penetration run with a No. 10 can of food, *A*, that the value of f_h is 36.25 minutes, and the point of intersection of the horizontal line representing No. 10 cans with the vertical line representing 36.25 minutes is joined by a straight line to the origin, *O*, this line is the curve for the food, *A*. This curve gives for the No. 5 can, $f_h = 24.3$ min.; for the No. 3 can, 17 min.; for the No. 2½ can, 15 min.; for the No. 2 can, 11.6 min.; and for the No. 1 can, 7.4 min.

If any one of the radiating straight line curves of Fig. 17 be taken, and for each size of can, the value of f''_h, as indicated by the intersection of this curve with the horizontal line representing that size of can, be listed in a table; the values of f_h for all sizes of cans of any given substance may be obtained by multiplying all the values of f''_h given in the table by a certain factor, the value of which depends upon the substance.

In column 7, Table VIII, the values of f'_h which are given, are also the values of f''_h, which would be obtained from the curve marked "$K'' = K'$" in Fig. 17. Therefore, these values, multiplied by a certain factor, will give f_h for all sizes of cans of a certain substance. The value of the factor, which represents any substance, is obtained by determining experimentally the heating curve for any size of can and dividing the value of f_h for this heating curve by the number in column 7 corresponding to the size of can used.

Example: If a No. 10 can of a substance gives a curve for which $f_h = 36.25$ minutes, the factor for this substance is $\frac{36.25}{48.38}$ or .75. Then, for a No. 3 can, $f_h = .75 \times 22.74 = 17$ minutes; for a No. 2 can, $f_h = .75 \times 15.52 = 11.6$ minutes, etc. These values are the same as those taken directly from curve *A*, Fig 17.

It will be recalled that this relation does not hold strictly for substances having very steep heating curves when a period of time is consumed in bringing the retort to processing temperature. This was discussed on page 48 in connection with peas.

V

THE CONSTRUCTION OF A GENERAL CHART SHOWING RELATIONSHIPS EXISTING BETWEEN ALL VARYING CONDITIONS

Co-relation of Variable Factors

A study of the second method of calculating process times described in Section III reveals the fact that, when considering a single value of z and a single value of $m + g$, any given value of the ratio $\frac{f_h}{U}$ has a value of g corresponding to it. It makes no difference what are the individual values of f_h and of U; as long as their ratio, as well as z and $m + g$ are constant, g also will be constant.

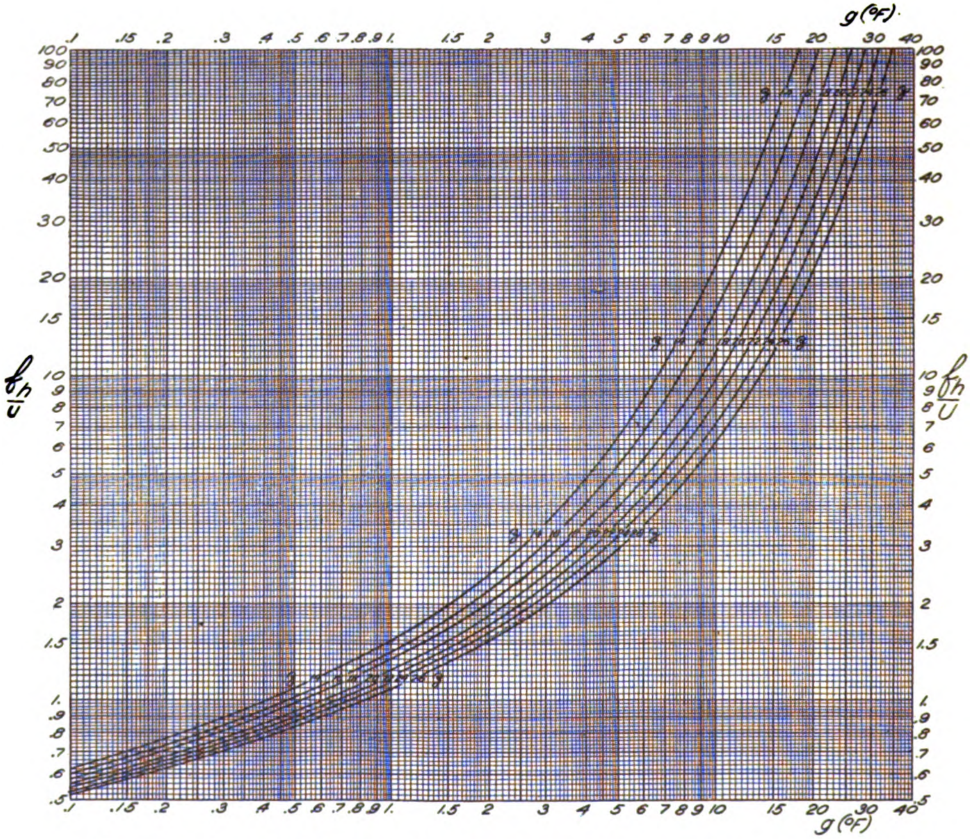

Fig. 18. Curves giving the value of g corresponding to any value of the ratio, $\frac{f_h}{U}$, for the condition that the temperature of the cooling water is 180° below retort temperature.

For instance, when the ratio $\frac{f_h}{U}$ equals 5, z equals 20°, and $m + g =$ 180°, g has the value, 6.18; while, in satisfying this condition, f_h may have any value as long as the value of U is such as to keep the ratio $\frac{f_h}{U}$ equal to 5.

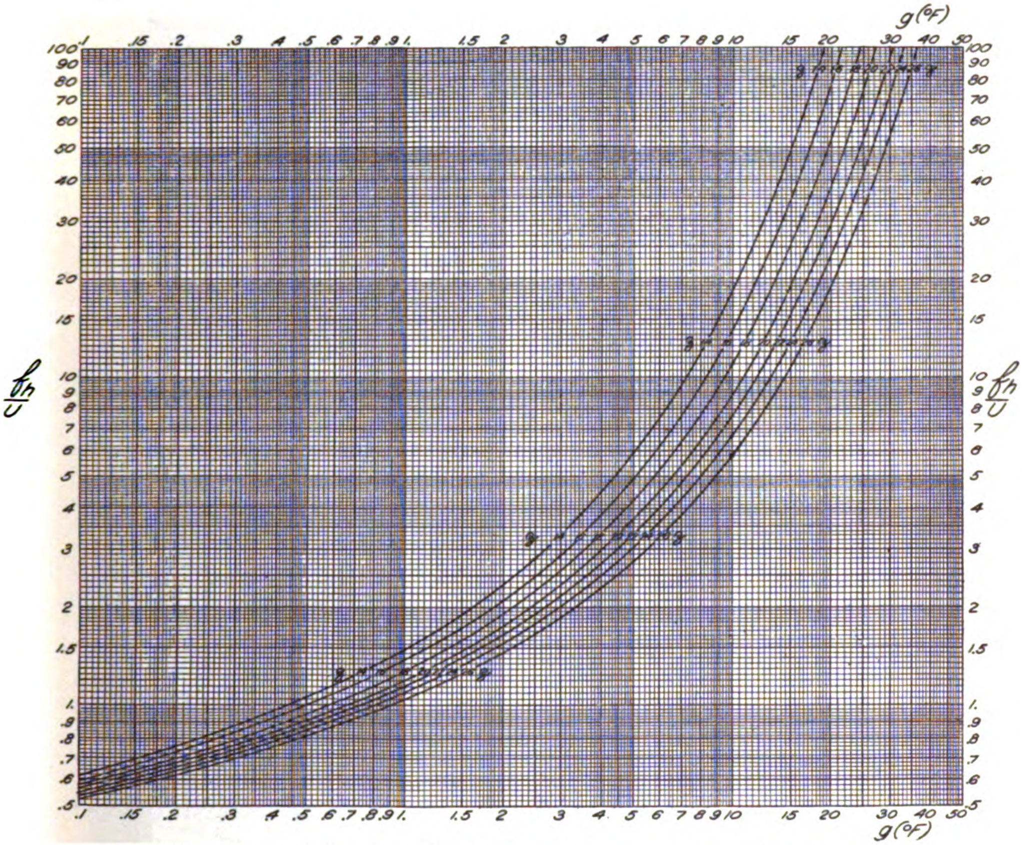

FIG. 19. Curves giving the value of g corresponding to any value of the ratio, $\frac{f_h}{U}$, for the condition that the temperature of the cooling water is 130° below retort temperature.

Figures 18 and 19 contain curves, plotted from calculated results, from which may be obtained the value of g corresponding to any value of the ratio $\frac{f_h}{U}$ up to 100, for all values of z from 14 to 26 inclusive. Each curve represents a constant value of z.

These curves of Figure 18 refer to the condition that the temperature of the cooling water is 180° below the retort temperature, i. e., that $m + g = 180°$. For the curves of Figure 19, $m + g = 130°$. The difference in the values of g between the two conditions, $m + g = 180°$

and $m + g = 130°$ are slight, as can be seen by comparing the curves of Figure 18 with those of Figure 19. Interpolation of g can therefore easily be made for values of $m + g$ intermediate between 180° and 130°.

Now, having obtained from the curves the value of g, this may be converted into process time by transferring it directly to the heating curve; or, if the latter has not been constructed, by the use of substitute curves, which are shown in Fig. 20. The latter curves represent equation (44) which is the equation of the heating curve [equation (1)] after x_h and y_h have been replaced by B and jI respectively.

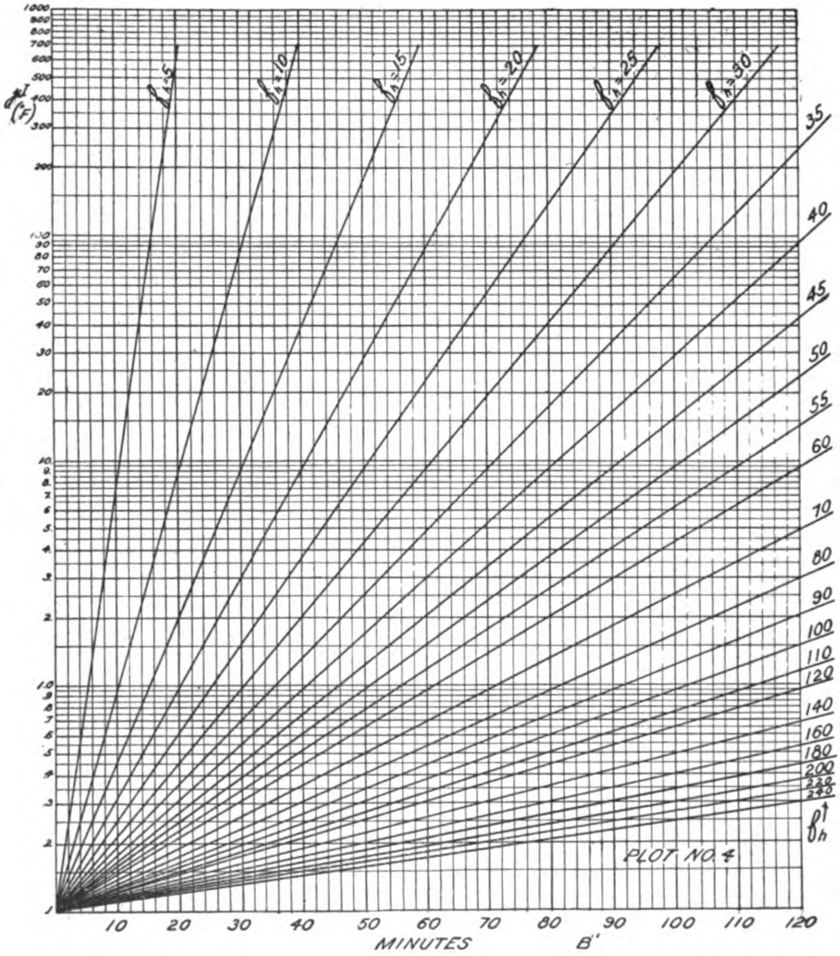

Fig. 20. Curves giving length of process when the value of g and of jI are known.

$$\frac{B}{f_h} = \log \frac{jI}{g} \tag{44}$$

B, in this equation, represents length of process in minutes.

Equation (44) is equivalent to the equation,

$$\frac{B}{f_h} = \log \frac{jI}{.1} - \log \frac{g}{.1} \tag{45}$$

which gives a clear idea of the method of using the curves of Fig. 20. This is as follows:

Knowing, as given conditions, the value of the f_h and of the product jI,

(1) Choose the curve corresponding to the given value of f_h.

(2) From this curve, find the value on the B' scale corresponding to the given value of jI.

(3) From the same curve, find the value on the B' scale corresponding to the given value of g referred to the jI scale.

(4) The difference between the two values obtained in steps (2) and (3) respectively is the length of process required.

When the value of g is shown by the curves of Fig. 18 or of Fig. 19 to be less than .1°, the procedure is indicated by the equation,

$$\frac{B}{f_h} = \log \frac{jI}{.1} + \frac{h}{f_h} \tag{46}$$

in which

B is length of process in minutes, and

h is a quantity whose value is obtained from equation (36).

As explained on page 44, the value of $C_{.1}$ in equation (36) is taken from column 4, Table V; so, the expression may be made clearer by substituting $T_{.1}$ for $C_{.1}$. Also, $t_{.1}$ is practically equal to U; therefore $t_{.1}$ may be replaced by U in equation (36).

This gives, after the indicated substitution is made for h in equation (46), the following equation:

$$\frac{B}{f_h} = \log \frac{jI}{.1} + \frac{U - f_h T_{.1}}{f_h} \tag{47}$$

or, after transposing,

$$\frac{B - U + f_h T_{.1}}{f_h} = \log \frac{jI}{.1} \tag{48}$$

Equation (48) shows that the length of process, B, may be obtained as follows:

(1) Choose the curve (Fig. 20) corresponding to the given value of f_h.

(2) From this curve, find the value on the B' scale corresponding to the given value of jI.

(3) Take the value of $T_{.1}$ from Table V, and obtain the value of the expression, $U - f_h T_{.1}$.

(4) The sum of the two values obtained in steps (2) and (3) respectively is the length of process required.

Plotting the General Chart

From the developments of the formula which have just been presented, a general chart may be constructed, by means of which the length of process may be found without the necessity of solving equations. Only the size of can, the nature of contents, the retort temperature, the initial temperature, and the thermal death time curve need be known. The chart consists of a series of four plots, which are as follows:

Plot No. 1—the right hand portion of Fig. 17, which gives the value of f_h for any food and size of can.

Plot No. 2—the thermal death time curve, which gives the value of U and of z.

Plot No. 3—Figure 18 or Figure 19 which gives the value of g, corresponding to the value of the ratio $\frac{f_h}{U}$ and of z obtained from plots Nos. 1 and 2.

Plot No. 4—Figure 20, which gives the length of process corresponding to the value of g obtained from plot No. 3.

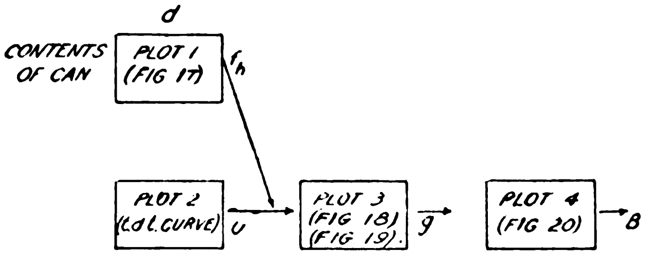

Fig. 21. Diagram of the general chart.

The only mathematical computations that need be made in using this chart are

(1) Finding the value of the ratio $\frac{f_h}{U}$ after obtaining the value of f_h from plot No. 1 and of U from plot No. 2.

(2) Finding the value of the expression, $U - f_h T_{.1}$, which is to be used only in the case, $g < .1°$.

The scheme of the chart is shown diagrammatically in Fig. 21.

Figure 20 is too small to show the curves of plot No. 5 on a scale large enough to be used readily. Since, however, the plot consists merely of a series of straight-line curves radiating from a point, it is very easy to construct, and can be readily copied to a larger scale by anyone desiring to use the chart. The abscissa scale (minutes) should be extended to whatever range is desired.

If it is not desired to use the chart in making calculations, g will be found by one of the methods of Section III, then the length of process may be obtained by solving equation (44) if $g > .1°$, or equation (48) if $g < .1°$. These equations, solved for B, give the following equations, respectively:

$$B = f_h \log \frac{jI}{g} \tag{49}$$

$$B = f_h \log \frac{jI}{.1} + U - f_h T_{.1} \tag{50}$$

A convenient scheme to follow when making calculations by these equations is a table, the columns of which have the following headings in the order given:

$$g; \frac{jI}{g}; \log \frac{jI}{g}; f_h \log \frac{jI}{.1}; \ T_{.1}; \ f_h T_{.1}; \ U - f_h T_{.1}; \text{ and } B.$$

Such a table would be used in connection with Table VI, and may be considered as being supplementary to it.

VI

DEVIATIONS FROM THE SIMPLE THEORETICAL RESULTS

Mention has already been made numerous times of the effect upon length of process of conditions which differ from the theoretical conditions. The variations which introduce special difficulties into the methods of the calculation of the length of process, which have hereinbefore been developed, are those which change the relation existing between the factor, C, and the quantity, g, which relation is represented by the curves of Fig. 7. Such variations are:

(1) Change in the slope[1] of the heating or of the cooling curve during the period of the process.

(2) Difference in the factor, j, applied to the cooling curve, from the value 1.41.

(3) Difference in value between the slope of the heating curve and that of the cooling curve for the same can.

(4) Change in slope of the thermal death time curve.

CHANGE IN SLOPE OF THE HEATING AND COOLING CURVES

It quite often happens that the heating curve has a very marked change in slope during the period of sterilization. This change is, as a rule, quite abrupt; and is evidenced by the presence of a sharp bend in the curve. In cases where the change occurs more or less gradually it is still possible to represent the heating curve quite accurately in a number of straight line sections which describe obtuse angles at their points of intersection with each other.

To calculate the length of process of a can, the heating curve of which contains one sharp bend, it is simply necessary to calculate first the lethal value of the process up to the point of the change, then to find the amount of additional process, under the new slope of heating curve which is required to produce sterility. Curve *A b D*, Fig. 22 represents such a heating curve—this one having an increase in slope at the point *b*. To calculate the lethal value of the process up to the point *b*, the formula of equation (17a) is used. This is

$$\frac{f_{h1}C}{t} = A'' \qquad (51)$$

when applied to the present conditions; f_{h1} being used to represent the slope of the heating curve up to the point *b*; and *A''*, the area beneath the lethality curves which represent a process ending at the point, *b*.

---

[1] The expression, "change in slope" is used in this section to signify a deviation from the straight line form when the curve is plotted on semi-log paper.

Since the area, *A''*, represents the combined lethal value of the heating and the cooling portions of the process, a certain percentage of this area represents that of the heating portion only. This percentage depends upon (1) slope of thermal death time curve; (2) difference between retort temperature and temperature of the cooling water; and (3) length of process. The curves in Fig. 23 give the values of this percentage divided by 100, which correspond to different conditions; the values are represented by the symbol ρ; and ρ is plotted against g.

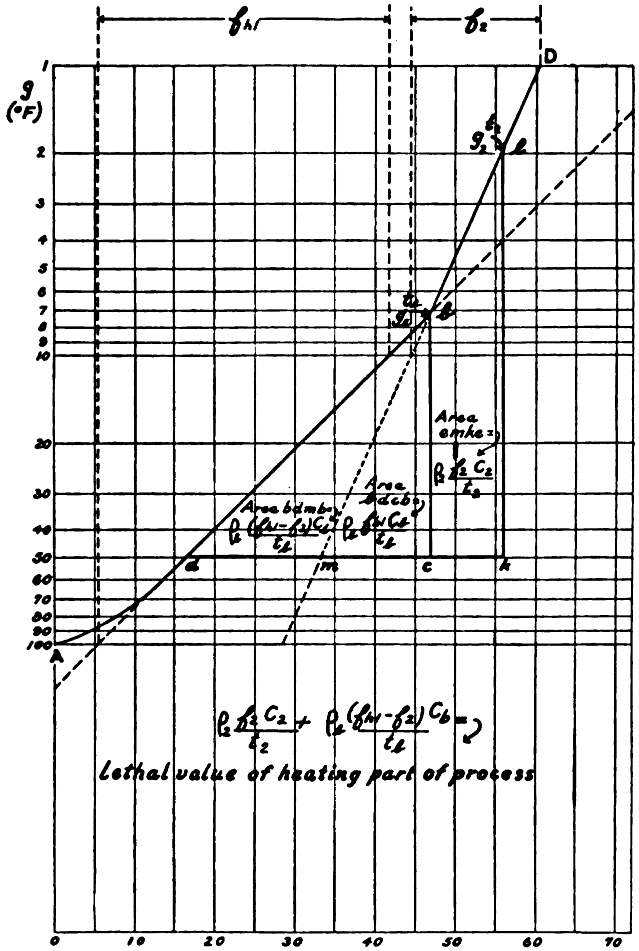

FIG. 22. Diagram illustrating the method of calculating length of process when the heating curve has a change in slope.

Now, the lethal value of the heating portion of the process up to the point *b* may be represented by the expression,

$$\rho_b \frac{f_{h1}\ C_b}{t_b} = A_{1h} \tag{52}$$

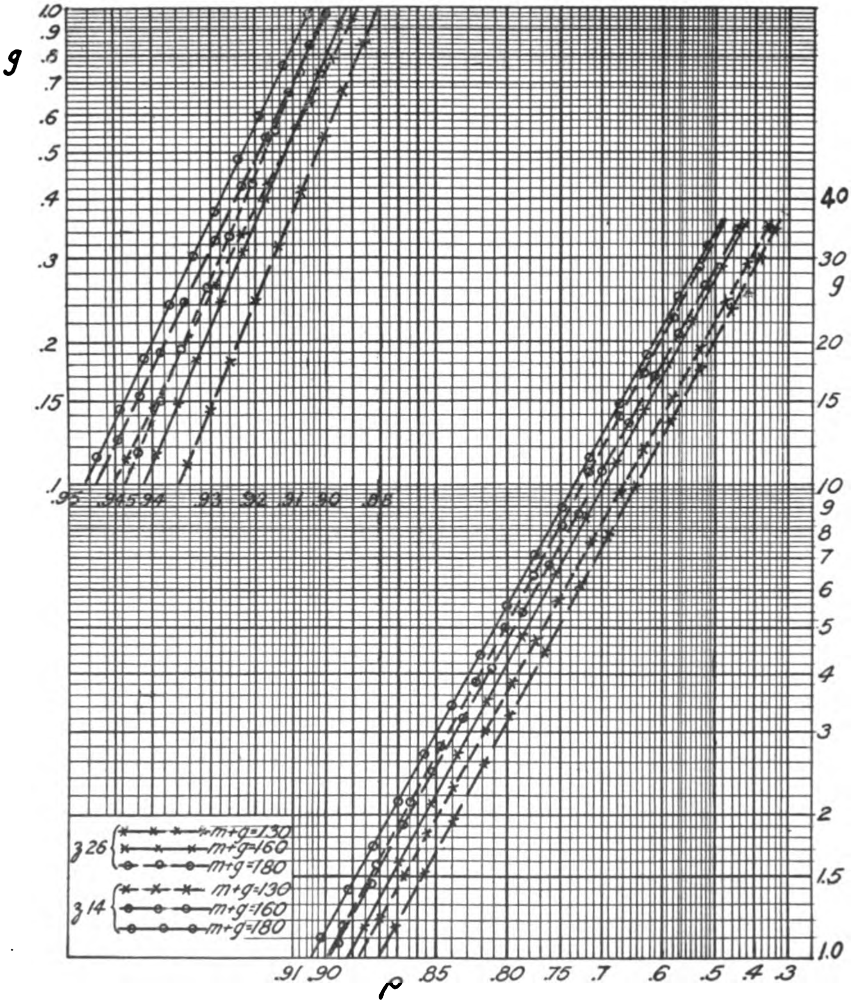

FIG. 23. Curves giving the lethal value of the heating period of a process expressed as a percentage of the total lethal value of the process when this equals unity.

in which

ρ_b = the percentage of lethality due to the heating period when the final limit of the process is marked by the point, *b*.

C_b = the value of the factor, *C*, (Fig. 7) corresponding to the point, *b*.

t_b = the length of time required to destroy the organism at the temperature represented by the point, *b*.

To facilitate the explanation of this calculation, it may be assumed that the area, *b c d b*, Fig. 22, represents A_{1h}, although it is not in reality possible to thus graphically represent absolute areas on semi-logarithmic paper. Then, also, the area, *b c m b*, may be said to represent the lethal value of the heating portion of a process, the termination of which is the point *b*, when the slope of the heating curve is f_2; and the area, *e k m e*, bears a similar relation to a process extending to the point, *e*, when the slope of the heating curve is f_2.

Now, if the point, *e*, represents the end of the process which is necessary to produce sterility when the heating curve is *A b D*, the total lethal value of the heating period of this process is represented by the area, *e b d k e*, which is the sum of the two areas, *e k m e* and *b m d b*; while the area, *b m d b*, is the difference between the areas, *b c d b* and *b c m b*. Therefore the lethal value of the heating period of the process is

$$A_h = \rho_2 \frac{f_2 C_2}{t_2} + \rho_b \frac{(f_{h1} - f_2)C_b}{t_b} \tag{53}$$

This value plus the lethal value of the cooling period equals 1. If the slope of the cooling curve is the same as that of the final portion of the heating curve, the lethal value of the cooling period is

$$(1 - \rho_2) \frac{f_2 C_2}{t_2}$$

Adding this to the second member of equation (53) gives

$$A_h = \frac{f_2 C_2}{t_2} + \rho_b \frac{(f_{h1} - f_2)C_b}{t_b} \tag{54}$$

If the factor, ρ_b, be omitted from equation (54), it implies that the cooling curve also changes its slope from f_2 to f_{h1} upon reaching the temperature represented by the point, *b*. Fig. 24 shows the cooling curve (*A′ b′ n′*) corresponding to the heating curve of Fig. 22 when calculated in this manner. The area, *A′ b′ D′ k′ A′*, may be said to represent, in lethal value, the expression,

$$(1 - \rho_2) \frac{f_2 C_2}{t_2}$$

the area, *b′ m′ c′ b′*, the expression

$$(1 - \rho_b) \frac{f_2 C_b}{t_b};$$

and the area, *b′ d′ c′ b′*, the expression

$$(1 - \rho_b) \frac{f_{h1} C_b}{t_b}.$$

OVERNIGHT

Now area $A'\,b'\,D'\,k'\,A'$ + (area $b'\,d'\,c'\,b'$ − area $b'\,m'\,c'\,b$

$$= (1-\rho_2)\frac{f_2C_2}{l_2} + (1-\rho_b)\frac{(f_{h1}-f_2)C_b}{l_b}$$

Adding the second member of equation (55) to the second
equation (53) gives equation (54) with the factor, ρ_b, omitte

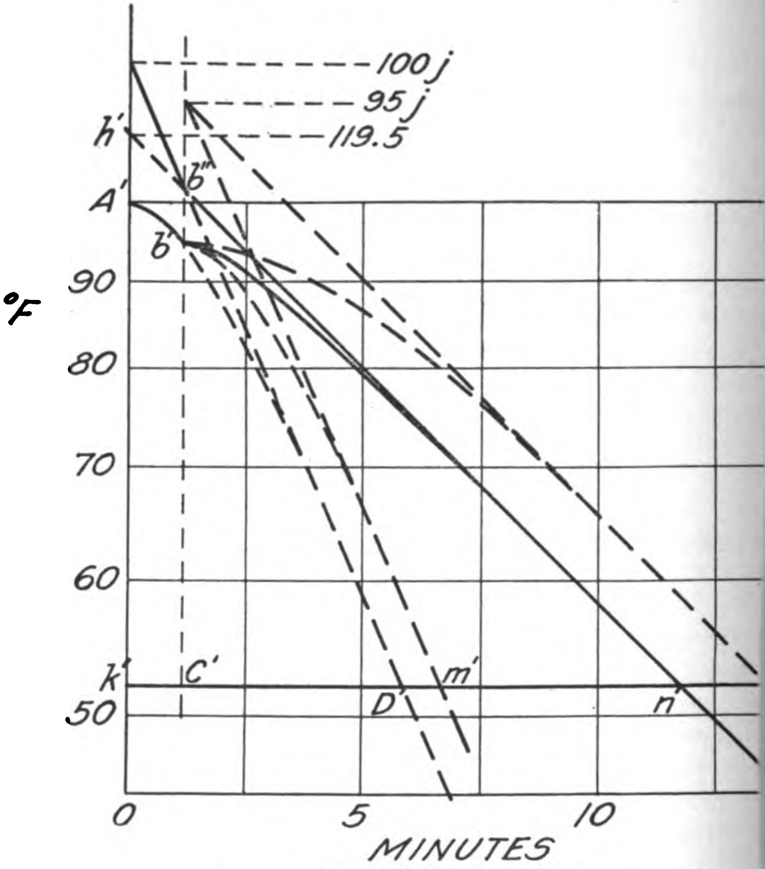

Fig. 24. Cooling curve having a change in slope.

fore, when this expression is used to make the calculation,
Fig. 22, which is allowed to represent the lethal value of
period is the sum of the areas, $A'\,b'\,D'\,k'\,A'$ and $b'\,d'\,m'\,l$
nearly, although not quite, the same as the area $A'\,b'\,n'\,k$
latter area would represent the lethal value of the cooling pe
were a simple change in slope in the curve at the point, b.

For convenience, in constructing Fig. 24, it is assume
temperature of the cooling water is 132° below retort temp

that the maximum temperature of the can is 100° above the temperature of the cooling water; and the lowest temperature which is considered to have a practical sterilizing value is 52° above cooling water.

The last term of equation (54) represents a known quantity since the exact location of the point, b, is known, For sterility, $A_k = 1$. Then, for the quantity,

$$1 - \rho_b \frac{(f_{k1} - f_2)C_b}{t_b}$$

may be substituted a symbol, V, giving the equation

$$C_2 = \frac{t_2}{\dfrac{f_2}{V}} \tag{56}$$

which may be solved by either of the methods given in Section III for solving equation (18); by merely substituting for f_k in the latter equation, the quantity, $\frac{f_2}{V}$. This equation, of course, will apply only if the process extends beyond the point, b. It is solved in a similar manner if the factor, ρ_b, is omitted.

It may be found convenient at times to represent either the heating or the cooling curve—or both—as containing more than one sharp bend. The equations given below represent five conditions different from the two which have already been discussed in connection with equation (54). In these equations, f_{k1} represents the slope of the heating curve before the first change; f_{k2}, the slope immediately before the second change (if there is a second change); f_2, the slope of the last part of the heating curve and of the first part of the cooling curve; f_{c2}, the slope of the cooling curve after the first change; and f_{c3}, the slope of the cooling curve after the second change. C_b represents the value of the factor, C, corresponding to the point of the first change in slope of the heating curve; $C_{b'}$ corresponds to the point of the second change in the heating curve; C_2 corresponds to the maximum point of the process; $C_{c'}$ corresponds to the point of first change in the cooling curve; and C_c corresponds to the point of the second change in the cooling curve. t and ρ have subscripts corresponding to those of C.

When there is one change in the cooling curve but no change in the heating curve, the equation is

$$\frac{f_2C_2}{t_2} + (1 - \rho_{c'}) \frac{(f_{c2} - f_2)C_{c'}}{t_{c'}} = 1. \tag{57}$$

When there are two changes in the heating curve and two corresponding changes[1] in the cooling curve, the equation is

$$\frac{(f_{h1} - f_{h2})C_b}{t_b} + \frac{(f_{h2} - f_2)C_{b'}}{t_{b'}} + \frac{f_2C_2}{t_2} = 1. \tag{58}$$

When there are two changes in the heating curve and no changes in the cooling curve, the equation is

$$p_b\,\frac{(f_{h1} - f_{h2})C_b}{t_b} + p_{b'}\,\frac{(f_{h2} - f_2)C_{b'}}{t_{b'}} + \frac{f_2C_2}{t_2} = 1. \tag{59}$$

When there are two changes in the cooling curve and no changes in the heating curve, the equation is

$$\frac{f_2C_2}{t_2} + (1 - p_{c'})\,\frac{(f_{c2} - f_2)C_{c'}}{t_{c'}} + (1 - p_c)\,\frac{(f_{c3} - f_{c2})C_c}{t_c} = 1. \tag{60}$$

When there are two changes in the heating curve; and also two changes in the cooling curve, which do not correspond to those in the heating curve, the equation is

$$\begin{aligned} &p_b\frac{(f_{h1} - f_{h2})C_b}{t_b} + p_{b'}\frac{(f_{h2} - f_2)C_b'}{t_b'} + \frac{f_2C_2}{t_2} \\ &+ (1 - p_{c'})\,\frac{(f_{c2} - f_2)C_{c'}}{t_{c'}} + (1 - p_c)\,\frac{(f_{c3} - f_{c2})C_c}{t_c} = 1. \end{aligned} \tag{61}$$

WHEN THE FACTOR, j, FOR THE COOLING CURVE, IS NOT 1.41

By introducing abrupt changes of slope into the cooling curve at the proper points, it will be possible to produce a curve which differs but little from almost any experimental curve. The accuracy of calculations based upon curves corrected in this manner will depend, of course, upon the accuracy of the representation of the curve by the equation. It would be possible to better represent the curve if a correction were made for variation in the value of the coefficient, j, by computing new tables for the factor, C. As indicated in the foot-note on p. 22, for cooling curves it appears that the coefficient, j, generally has a value different from 1.41. It is as a rule less than this value. How such a condition might appear from merely a change in the slope of the curve is evident from Fig. 24; for if $A'b'n'$ is the cooling curve calculated it is seen that when the log curve, $n'b''$, is extended to the vertical axis it intersects the latter at the point, h', which does not represent 1.41 x 100 degrees above cooling water temperature but about 1.195 x 100 degrees, instead. This illustrates how correction may be made for variations in the value of j without the necessity of computing new values for the factor, C. It appears at present that the advantage to be gained by a retabulation of C for this purpose does not warrant the use of the time necessary to perform the work.

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[1] By "corresponding change" is meant an equal and opposite change occurring at a point representing the same temperature at which the change, to which it is referred, occurred in the heating curve. See p. 64.

Difference in Value Between the Slope of the Heating Curve and That of the Cooling Curve

As is evident from Fig. 24, if there is an early change in slope in the cooling curve, it is difficult to tell by inspection, just what slope the first part of the curve actually assumes. If it is decided, however that it is different from the final slope of the heating curve, f_2, account of this may be taken in the above equations by letting $C_{c'} = C_2$ and $t_{c'} = t_2$, meaning that the first change in slope of the cooling curve occurs at the beginning of the cooling period. Referring this to equation (61); if we represent the expression,

$$1 - \left(\rho_b \frac{(f_{h1} - f_{h2})\, C_b}{t_b} + \rho_{b'} \frac{(f_{h2} - f_2)\, C_{b'}}{t_{b'}} + (1 - \rho_c) \frac{(f_{c3} - f_{c2})\, C_c}{t_c} \right),$$

by the symbol Q, the equation may be reduced to

$$C_2 = \frac{t_2}{\dfrac{f_{c2} + \rho_2 (f_2 - f_{c2})}{Q}}. \tag{62}$$

Since the values of the quantities, C_2, t_2, and ρ_2, all depend upon the value of g for the process, equation (62) cannot be solved by any of the methods given for the solution of equation (18) unless a value for ρ_2 be assumed. When an accurate assumption of the value of ρ_2 is impossible, it will probably be better to consider the first change in the cooling curve as occurring at some point which is known to be below the maximum temperature which is attained by the can during the process, and the problem solved by one of the equations, (54) to (62) inclusive, as it stands.

The consistency of some canned foods seems to be controlled by the temperature, so that whatever changes occur in the slopes of their heating curves, always occur at about the same temperature for any one substance. Thus, with a substance of this kind, the temperature at which any specific change in slope of the heating curve will occur will be separated from retort temperature by an interval which varies with the retort temperature. This must be borne in mind when calculating lengths of process. The rate of heat penetration may either increase or decrease with increase in temperature.

Besides the retort temperature and the inherent nature of the substance itself, it is thought that other factors have an influence upon the characteristics of the heating and cooling curves. Two of these factors are the treatment to which the food is subjected before processing and the initial temperature.

A sufficient study of heating and cooling curves from this point of view has not been made to make possible a presentation of the characteristics of the curves for different substances at this time. A thorough study of heating and cooling curves must include a study of the influence of air cooling upon the cooling curve and upon the length of process There is room for a great deal of work here.

CHANGE IN SLOPE OF THE THERMAL DEATH TIME CURVE

The fourth and last variation which introduces difficulty into the calculations is change in the slope of the thermal death time curve. It has been found since the derivation of equation (17) was completed that the thermal death time curves for the spores of micro-organisms are not, as a rule, absolutely straight semi-log curves. However, by proper choice of scales, they can be plotted as practically straight lines upon paper, which has the logarithmic scale both horizontally and vertically. The equation of the curve is considered to be the following:

$$\frac{\log \frac{x_o + G}{G}}{\log \frac{J + G}{G}} = \log \frac{y_o}{t} \tag{63}$$

in which x_o, y_o, and t are the same as in equation (4),

G = the value on the horizontal log scale of the intersection of the curve with the horizontal line representing the value, t minutes. Although G actually represents retort temperature, its absolute value depends upon the organism and upon the properties possessed by the substance which acts as the bacterial medium. An important influencing factor in this connection is the hydrogen-ion concentration.

J = the difference measured in units along the horizontal scale between the two points on the curve, $y_o = t$ and $y_o = 10\,t$.

The factors expressing lethal rate (see p. 19) when obtained from equation (63) are of course not the same as those which were derived from equation (4).

$$\text{From equation (63) } y_o = t\left(\frac{x_o + G}{G}\right)^{\log \frac{J+G}{G}} \tag{64}$$

Now, when the same substitutions for x_o are made as were made in equation (8), the factors obtained are as follows:

(*a*) For the heating curve, the reciprocal of

$$t\left(\frac{y_h - g + G}{G}\right)^{\log\frac{J+G}{G}} \tag{64a}$$

(*b*) For the hyperbola, the reciprocal of

$$t\left(\frac{y_{ch} - a + G}{G}\right)^{\log\frac{J+G}{G}} \tag{64b}$$

(*c*) For the log cooling curve, the reciprocal of

$$t\left(\frac{m - y_{cl} + G}{G}\right)^{\log\frac{J+G}{G}} \tag{64c}$$

If these factors are substituted for the factors (8a), (8b), and (8c) (see p. 19) in the expressions (8d), (8e), and (8f) respectively, and the integrations are performed, a result different from that given in equation (17) will be obtained, but one which can no doubt be handled in the same way. Time will not be taken before the publication of this paper to carry out these calculations, because it is felt that the present value of the results would not warrant the delay. The departure of the thermal death time curves from the straight line semi-log curve is slight throughout the range within which the lethal rate value is of much practical consequence. The deviation increases as the thermal death time increases. Moreover, the slight error, which results from the use of the simple straight line semi-log equation is on the side of safety; that is, the calculated process is lengthened slightly.

SUMMARY AND CONCLUSION

The lengths of time necessary to destroy bacterial spores by subjecting them to heat at various temperatures in media under given conditions are called the thermal death times for these spores corresponding to the respective temperatures used.

The rate of heat penetration of a can of food is the rate of rise in temperature at the center of the contents during the heating of the can at constant temperature and the rate of drop in temperature at the center of the contents during the cooling of the can at constant temperature.

It is possible, by a method of correlating thermal death times with the rate of heat penetration to obtain the length of time, corresponding to any given temperature of process, which is theoretically necessary to destroy all bacterial spores of the kind being considered, if they are present in the food within the can. These theoretical lengths of process will be the actual lengths of process necessary provided all conditions are given absolutely accurate control.

A formula, which is expressed in the form $C = \frac{t}{f_h}$ has been developed for obtaining the necessary length of process of a can of food under certain conditions, which may be called the ideal conditions. These include the following:

(1) The curve which represents the death times of the spores corresponding to different temperatures must apply to conditions as to bacterial media which exist within the can during the process, and must be a straight line when plotted on semi-logarithmic paper.

(2) The curves which represent the rate of heating and the rate of cooling of the can of food, except the first part of each, must be straight lines when plotted on semi-logarithmic paper and both must have the same slope.

The formula is applicable to any retort temperature, any initial temperature, any temperature of the cooling water, and any slopes of thermal death time and heating curves.

In the application of the formula the length of process is found by first obtaining the difference in degrees which will exist between the retort temperature and the maximum temperature which must be attained by the center of the can during the minimum process which will destroy the spores. This difference in temperature is symbolized by g. By simply referring this value of g to the heating curve, the length of process is read directly from this curve.

Two methods are developed for obtaining the value of g under ideal conditions, and a sample calculation is made under each method.

The reader, if he so desires, may overlook these methods and still obtain the results given by them by simply referring to the curves of Figures 18 and 19, which give values of g corresponding to different values of the ratio $\frac{f_h}{U}$; f_h representing the slope of the heating curve, and U representing the time, taken from the thermal death time curve, that is necessary to destroy the organism at retort temperature. Therefore, if one has simply a sufficient understanding of the construction of the heating curve and of the thermal death time curve, which are described in the introduction, to enable him to obtain the values of these two quantities; he can, from the curves of Figures 18 and 19, obtain the value of g desired without having to resort to the methods of calculation described in Section III.

Figures 18 and 19, however, can be used only in cases in which g is equal to, or greater than .1°. When g is less than .1° the formula must be applied as described in the latter part of Section III.

Figures 18 and 19 constitute a portion of a general chart of four plots that may be constructed, by means of which one can quickly obtain the theoretical length of process when he is given merely the size and the contents of the can. In the construction of this chart various relationships are involved which are discussed in Sections IV and V. In these sections are discussed the effect upon the process of the time taken to bring the retort to processing temperature, the influence of initial temperature, the calculation of equivalent processes, and the relationships existing between different sizes of cans.

The formula is extended in section VI to apply to certain deviations from the ideal conditions relating to the heating and cooling curves In this discussion is included a comparison of the value, from the standpoint of sterilization, of the heating portion of the process to that of the cooling portion. Depending upon the conditions, the value of the heating portion may vary from less than 50 per cent to more than 95 per cent of the total value of the process.

Mention of influences which introduce error into the calculations described in this paper has been so frequent that the reader no doubt feels inclined now to ask, "What is the use? Of what value is it? Can any work which contains so many inherent sources of error have any practical value?"

The true value of it, of course, remains to be determined. While a single error is generally insignificant, it is true that if several should be so combined that their effects become additive, this combined effect may become quite appreciable. As has been stated previously, until the methods which have been described herein shall have been considerably refined their principal use will not be the direct calculation of the absolute length of a process which should exactly satisfy any given set of conditions. It will be rather the comparison of the relative merits of different combinations of factors which have an influence upon the process of sterilizing canned foods. It is hoped that the facts that have been set forth will be of material assistance to the canner in the search for fundamental causes of many of his troubles, and in the effort to solve many of his problems of processing; or, that they may at least give to others a few pointers which will enable them to make more rapid progress in future research.

The writer is greatly indebted, for valuable suggestions leading to the accomplishment of the integration of the expressions on page 10, to the National Research Council, to Dr. H. L. Hodgkins, President of George Washington University, and especially to Dr. L. B. Tuckerman, U. S. Bureau of Standards. He wishes also to express his appreciation of the kindness of Dr. H. L. Curtis, U. S. Bureau of Standards, in granting him permission to use the instrument known as the integraph, and to Miss J. Busse for assistance rendered in the use of this instrument.

Valuable suggestions on the improvement and correction of the text were given by Dr. W. D. Bigelow and Mr. R. J. Thompson, of this laboratory, for which thanks are hereby extended.

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PREFACE

For the benefit of those who desire to engage in research or graduate study in any branch of science or technology, this Bulletin sets forth in a concise way the fellowships and scholarships offered for that purpose by American educational institutions, associations, or industrial concerns. They are listed under the names of such institutions with brief indications as to the subject in which each is awarded, if restricted to any particular science or sciences, the persons who are eligible, the conditions of tenure and the stipend when specified. A few teaching fellowships and assistantships in which the holder must give part of his time to teaching or otherwise assisting in class or laboratory work have been included, but no attempt has been made to furnish a complete list of such opportunities.

Lists of fellowships and scholarships known by special names, showing the institution under which each is described and of industrial corporations and trade associations supporting fellowships, together with a subject index, by which the fellowships available in each science may be located, are appended.

The word "unrestricted" wherever used in this compilation means that the award may be made in any branch of learning and is not limited to science and technology. It may be assumed that both men and women, who hold the bachelor's degree from any institution of recognized standing, are eligible unless some limitation or additional requirement is expressly stated in the text.

Applications for fellowships should, in most cases, reach the respective institutions by March 1 preceding the academic year for which the award is to be made. For full particulars as to regulations and requirements, the institutions themselves should be directly consulted.

So far as possible the information herein given has been obtained by direct correspondence with the institutions which hold and administer the funds. When information could not be thus directly obtained the latest available reports, yearbooks, or catalogues, were consulted and the institutions requested to verify and supplement the data.

The Council wishes to make appreciative acknowledgment of the generous and ready assistance given by the various institutions which have cooperated in making this compilation possible.

DESCRIPTIVE LIST OF FELLOWSHIPS BY INSTITUTION

Agnes Scott College, Decatur, Ga.

GENERAL COLLEGIATE FELLOWSHIP. Unrestricted. Annual appropriation by the College. For women who are graduates of Agnes Scott College. Stipend, $500.

Akron, Municipal University of, Akron, Ohio.

FIRESTONE TIRE & RUBBER CO. FELLOWSHIP. Chemistry of India Rubber. Established by Firestone Tire & Rubber Co. All university fees remitted. Stipend, $500.

GOODYEAR TIRE & RUBBER CO. FELLOWSHIP. Chemistry of India Rubber. Established by Goodyear Tire & Rubber Co. All university fees remitted. Stipend, $500.

Alabama, University of, Tuscaloosa, Ala.

BUREAU OF MINES FELLOWSHIP. Research in Mining, Metallurgy and Chemical Technology. Established by University, research directed by Bureau of Mines. Exemption from laboratory and non-resident fees. Five; each $540.

Alabama Federation of Women's Clubs.

See under VIRGINIA, UNIVERSITY OF.

Alfred University, Alfred, N. Y.

WAR MEMORIAL GRADUATE SCHOLARSHIP. Unrestricted. For graduates of Alfred University. Work may be done elsewhere. Capital fund $5000. Stipend, $300.

American Association of University Women, 1634 I St., Washington, D. C.

AMERICAN ASSOCIATION OF UNIVERSITY WOMEN EUROPEAN FELLOWSHIP. Unrestricted. Open to American women who have met all the requirements for the Ph. D. degree with the possible exception of the completion of the dissertation, for study and research in Europe. Supported by A. A. U. W. funds. Stipend, $1000.

AMERICAN ASSOCIATION OF UNIVERSITY WOMEN INTERNATIONAL FELLOWSHIP. Unrestricted. Open to women holding a Doctor's degree or the equivalent, for research anywhere except in the country in which the Fellow has received her previous education or habitually resides.

Applicant should be a member of some federation of university women included in the International Federation of University Women. Stipend, $1000.

SARAH BERLINER RESEARCH FELLOWSHIP. Physics, Chemistry, or Biology. Open to American women who are holders of a Doctor's degree or its equivalent, for research. Supported by endowment fund. Stipend, $1000-$1200.

BOSTON ALUMNAE FELLOWSHIP. Unrestricted. Open to American women, for graduate work in Europe or in America. Supported by the Boston Branch of the A. A. U. W. aided by the Radcliffe Alumnae Association and the Boston Alumnae clubs of various colleges. Stipend, $500.

LATIN-AMERICAN FELLOWSHIP. Open to women born in the Latin-American republics, who have the equivalent of a college education in the universities or the best normal schools of their countries and a sufficient knowledge of English, for preparation for public service to their countries in (a) education, including scientific investigation, teaching, library work, etc.; (b) social service; or (c) public health and sanitation. Supported by A. A. U. W. and awarded by its Committee on Fellowships. Stipend, $1000.

ALICE FREEMAN PALMER MEMORIAL FELLOWSHIP. Unrestricted. Open to American women who are holders of a Doctor's degree or its equivalent, for research. Supported by endowment fund. Stipend, $1000.

PHI MU FELLOWSHIP. Unrestricted. Open to American women having a degree from any university or college in which Phi Mu has a chapter, for graduate work. Supported by Phi Mu Fraternity. Stipend, $1000.

JULIA C. G. PIATT MEMORIAL FELLOWSHIP. Unrestricted. Open to American women intending to enter the teaching profession, preference being given to those who have had successful experience in teaching and at least two years of graduate study. Offered in alternate years. Supported by memorial fund given by Julia C. G. Piatt Association. Stipend, $750.

SPANISH FELLOWSHIP. Unrestricted. Open to American women who can speak Spanish fluently, for graduate study at the University of Madrid, which offers the fellowship to be awarded by the A. A. U. W. Free tuition, and in addition, board and lodging at the Residencia.

Applications and recommendations for any of the above fellowships should be made by letter to the chairman of the Committee on Fellowships, Professor

Agnes L. Rogers, Smith College, Northampton, Mass., not later than February 1st.

ROSE SIDGWICK MEMORIAL FELLOWSHIP. Unrestricted. Open to British women of graduate standing, for graduate study in American colleges or universities. Established by A. A. U. W.; awarded by a committee of the British Federation of University Women. Stipend, $1000.

Further information regarding this fellowship may be obtained from Miss Theodora Bosanquet, 92 Victoria Street, London, S. W. 1, England.

See also under WOODS HOLE, MASS., MARINE BIOLOGICAL LABORATORY.

American Institute of Baking, 1135 Fullerton Ave., Chicago, Ill.

FELLOWSHIP IN BAKING RESEARCH. Chemistry, Biochemistry, Physics, Biology. Open to men who, if they have not completed the Master's degree, may spend part of the time in a university graduate school to do so. Supported by funds of the Institute. Two; each $500.

American Physiological Society, C. W. Greene, Secretary, Columbia, Mo.

PORTER FELLOWSHIP FOR PHYSIOLOGICAL RESEARCH. Physiology, research. Degree of M. D. or Ph. D. required. Work may be done anywhere at approved choice of candidate. Bequest of Dr. Wm. T. Porter. Two; each $1200.

American-Scandinavian Foundation, 25 W. 45th St., New York.

AMERICAN-SCANDINAVIAN FOUNDATION EXCHANGE FELLOWSHIPS. Unrestricted except for 1 in oceanography and weather forecasting at the Bergen Geophysical Institute in Norway, 1 for metallurgy in Sweden. Usually 1 in forestry, 1 or more in chemistry, 1 in hydroelectric engineering in Norway and 1 or 2 in cooperative agriculture in Denmark. Half of the fellowships (20) are for Scandinavian students for study in the United States; the other 20 are for students of American birth for study in Scandinavian countries: Sweden 10; Denmark 5; Norway 5. Supported by the Foundation through donations of individuals and firms. Forty; minimum stipend, each $1000.

PRINCETON-LEACH FELLOWSHIP. Unrestricted. Awarded in alternate years to a Princeton student and for study in a Norwegian or Danish university, and to a Scandinavian student for study at Princeton. Supported by the Foundation through donation by Dr. Henry Goddard Leach. Stipend, $1500.

Amherst College, Amherst, Mass.

RUFUS B. KELLOGG UNIVERSITY FELLOWSHIP. Natural Science, Philology, Mathematics, or humanistic subjects. For an alumnus of Amherst College. An especially good knowledge of Latin and German required. Awarded for a term of seven years, the first three to be spent at a German university or elsewhere, the other four as a lecturer at Amherst. About $30,000 given for purpose by the late Rufus B. Kellogg, Class of '58. Stipend, income.

JOHN WOODRUFF SIMPSON FELLOWSHIPS. (1) Medicine. Open to a graduate of Amherst College for study at any school approved by the Board of Trustees. (2) Unrestricted. Open to a graduate of Amherst College for study in England, at Oxford or Cambridge. (3) Unrestricted. Open to a graduate of Amherst College for study at the Sorbonne in Paris. Capital fund $150,000 for support of these three fellowships among others. Memorial gift by Mrs. John Woodruff Simpson and daughter. Stipend, part of income.

Arizona, University of, Tucson, Ariz.

BUREAU OF MINES FELLOWSHIPS. Research in Mining, Metallurgy and Chemical Technology. Established by University, research directed by Bureau of Mines. Exemption from laboratory fee when work is done in Bureau of Mines laboratories. Two; each $750.

Bernice Pauahi Bishop Museum, Honolulu, H. T.

See under YALE UNIVERSITY.

Boston Society of Architects, C. H. Blackall, Secretary, 20 Beacon St., Boston, Mass.

ROTCH TRAVELING SCHOLARSHIP. Architecture. Open to men under 30 years of age who have been engaged in professional work during two years in Massachusetts in the employ of a practising architect resident in Massachusetts. Holders of a degree in architecture from Massachusetts Institute of Technology, Columbia University, University of Pennsylvania, Cornell University, Harvard University or University of Illinois are admitted to competition without preliminary examination. For two years of foreign travel and studies. Founded by the children of the late Benjamin S. Rotch, of Boston. Stipend, $2800.

Boston University, Boston, Mass.

AUGUSTUS HOWE BUCK FELLOWSHIPS. Unrestricted. Open to men who are graduates of Boston University and who have been Augustus Howe Buck scholars during their undergraduate work. Work may be done abroad, if desirable. Several; stipend, all expenses.

Ada Draper Scholarships. Unrestricted. Open to women who are graduates of Boston University. Work to be done abroad. Financial need a consideration. Founded through bequest of Miss Ada Augustus Draper of Boston. Income (about $900) divided among several.

Jacob Sleeper Fellowships. Unrestricted. Open to men who are graduates of Boston University. Available for traveling. Supported by general University funds. Given now only twice in three years. Two; each $750.

Bowdoin College, Brunswick, Me.

Charles Carroll Everett Scholarship. Unrestricted. Open to men who are graduates of Bowdoin College. Available for traveling and for work in some other institution. Stipend, $500.

Brooklyn Institute of Arts and Sciences, Brooklyn, N. Y.

Temple Prime Scholarship. Biological research. For work at Biological Laboratory, Cold Spring Harbor, L. I. Stipend, $100.

Brown University, Providence, R. I.

Oliver Cromwell Gorton Arnold Biological Fellowship. Biology. Capital fund $10,000 given by Dr. Oliver Henry Arnold. Stipend, income or part of it.

duPont Fellowship. Chemistry. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

Morgan Edwards Fellowship. Unrestricted. For original research. Open to men who are graduates of Brown University of not more than 10 years standing. Work may be done wherever facilities are best. Capital fund $10,000 given by the Philadelphia alumni of Brown. Stipend, $1000.

Grand Army of the Republic Fellowship. Unrestricted. Applicants should be candidates for Ph. D. or have obtained that degree. Open to men who are graduates of Brown. Must reside at the University. Descendants of Union Veterans of the Civil War given preference. Capital fund $10,000 given by Grand Army of the Republic, Department of Rhode Island. Stipend, income.

Bryn Mawr College, Bryn Mawr, Pa.

Bryn Mawr European Fellowship. Unrestricted. Open to a member of graduating class of Bryn Mawr for a year's study in an English or Continental university. Founded. Stipend, $500.

Mary Elizabeth Garrett European Fellowship. Unrestricted. Open to candidates for Ph. D. at Bryn Mawr, who have had two years of graduate work there, for completion of studies at an English or Continental university. Founded by Miss Mary Elizabeth Garrett of Baltimore. Stipend, $500.

Helen Schaeffer Huff Memorial Research Fellowship. Physics or Chemistry. Open to women who have already done advanced work either at Bryn Mawr or elsewhere and have shown capacity for research. The work may be done where circumstances make it most desirable. Founded by an anonymous donor. Stipend, $1200.

Resident Fellowships. Biology, Chemistry, Geology, Physics or Mathematics. Open to women. Four; each $810.

Resident Scholarships. Unrestricted. Open to women. Twenty; each $350.

Helene and Cecil Rubel Foundation Fellowship. Unrestricted. Open to women who are graduates of any other college of recognized standing but who have also studied at Bryn Mawr. If holder has sufficient income to waive the stipend it may be used by another student in the same year. The work may be done wherever circumstances make it desirable and may sometimes be used as a traveling fellowship. Stipend, $1500.

Scholarship for European Women Students. Unrestricted. Open to European women holding the equivalent of a Bachelor's degree. Three for British, three for French, three for other European countries, usually. Holders may do a small amount of teaching if they desire. Stipend to cover board, residence and tuition at Bryn Mawr for one academic year. Stipend, $720.

President M. Cary Thomas European Fellowship. Unrestricted. Open to women who are candidates for Ph. D. at Bryn Mawr and who are in first year of graduate work. To be applied toward expenses of one year's study and residence at some English or Continental university. Founded by Miss Mary Elizabeth Garrett of Baltimore. Stipend, $500.

C. R. B. Educational Foundation, Inc., 42 Broadway, New York City.

Graduate Fellowships for Belgian Universities. Bacteriology, Botany, Civil Engineering, Electricity, Geology, Mathematics, Medicine, Mining Engineering, Morphology, Organic Chemistry, Physiology, Psychology, Surgery, and Zoology, as well as certain humanistic studies. A candidate for a fellowship, to be eligible, must be an American citizen, must have a thorough speaking and reading knowledge of French, and if a member of the faculty of a college or university, must hold a grade below that of associate professor. Preference given to applicants between the ages of 25 and 33 who are unmarried and who intend to take up teaching or research as a profession. For definite program of work in Belgian universities or technical schools. Established to commemorate the work of the Commission for Relief in Belgium during the Great War and to promote closer relations and the exchange of intellectual ideas between Belgium and America. First class traveling expenses and tuition fees paid in addition to stipend. Applications to be filed by February 15. In 1924-25 not more than six; stipend, each 15,000 francs.

California, University of, Berkeley, Cal.

ASSOCIATED RADIOGRAPH LABORATORIES FELLOWSHIP. Dentistry. For research in radiography relating to dentistry. Established by Associated Radiograph Laboratories of San Francisco. Stipend, $1200.

BASE HOSPITAL NO. 30 SCHOLARSHIP IN MEDICINE. Medicine. Supported by balance of fund collected for the University of California Medical School Hospital Unit No. 30. Stipend, $400.

EDITH CLAYPOLE MEMORIAL RESEARCH FELLOWSHIP IN PATHOLOGY. Pathology. Supported by friends of the late Edith Claypole. Stipend, $1200.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

FELLOWSHIP FOR RESEARCH IN ANTHROPOLOGY OF ABORIGINAL CALIFORNIA. Anthropology. Supported by the Regents of the University. Stipend, $600 minimum.

MARJORIE GREEN FOSTER MEMORIAL SCHOLARSHIP IN RESEARCH MEDICINE. Medicine, research. Offered as a prize for most meritorious work during the year. Supported by the gift of Edythe Foster. Stipend, $150.

JAMES M. GOEWEY FELLOWSHIP IN NATURAL SCIENCE. Natural Science. Supported by the family of the late James M. Goewey. Stipend, $750-$900.

HATTIE HELLER GRADUATE SCHOLARSHIP. Unrestricted. Supported by an anonymous trust fund. One or more; each $500.

HONORARY TRAVELING FELLOWSHIPS. Unrestricted. May be awarded at any time to distinguished graduate students in any department.

HOOPER FELLOWSHIPS IN RESEARCH MEDICINE. Medicine, research. Candidates should have at least one and a half years medical training in the University of California Medical School or equivalent preparation. Supported by the Regents of the University. One or more; each $500.

GEORGE WILLIAMS HOOPER FOUNDATION FELLOWSHIP. Neurosurgery. Open to members of the house staff of the University of California Hospital or allied hospitals or to persons having equivalent training. Stipend, $1000-$1200.

HORGAN ESTATE ASSISTANTSHIPS IN AGRICULTURE. Agriculture. For definite research problems in this subject. Supported by estate of the late James C. Horgan. Stipend, $600.

MARTIN KELLOGG FELLOWSHIP IN ASTRONOMY. Astronomy. Under certain conditions work may be done elsewhere. Supported by estate of the late Mrs. Martin Kellogg. One or more; each $1200.

WILLIAM WATT KERR SCHOLARSHIP IN MEDICINE. Medicine. Supported by Faculty and Alumni of the Medical School. Stipend, $400.

LECONTE MEMORIAL FELLOWSHIP. Unrestricted. Open to graduates of the University of California. Under certain conditions the work may be done elsewhere. Supported by the Alumni Association. One or more; each $600.

LICK OBSERVATORY FELLOWSHIPS IN ASTRONOMY. Astronomy. Supported by Regents of the University. Three; each $600.

JAMES MONROE MCDONALD SCHOLARSHIPS. Chemistry, Sanitary and Municipal Engineering, Mining, Physics. Open to graduates of the University of California. One in each subject, which may be awarded for one year of post-graduate work at discretion of departments concerned. Supported by the gift of Mrs. McDonald. Four; each $400-$600.

JOHN W. MACKAY, JR., FELLOWSHIPS IN ELECTRICAL ENGINEERING. Electrical Engineering. Supported by the gift of Clarence W. Mackay and his mother, Mrs. John W. Mackay. Two; each $600.

PACIFIC COAST GAS ASSOCIATION FELLOWSHIP. Engineering. For research in engineering or allied sciences of physics and chemistry relating directly to gas engineering. Maintained by the Pacific Coast Gas Association. Stipend (may be divided between two fellowships), $1500.

JAMES ROSENBURG MEMORIAL SCHOLARSHIP IN AGRICULTURE. Agriculture. Supported by the Estate of James Rosenburg. Stipend, $600.

SHEFFIELD SANBORN SCHOLARSHIPS. Medicine. Open to graduates of the University of California who have not received a degree in medicine and who otherwise would not have the opportunity to acquire a university training. Supported by the gift of Mrs. James Horsburg (formerly Mrs. F. B. Sanborn). Two; each $150.

BERTHA HENICKE TAUSSIG MEMORIAL SCHOLARSHIP. Architecture, among other subjects not scientific. Open to women. Available for traveling. Established under the will of the late Hugo A. Taussig. Stipend, $600-$900.

TEACHING FELLOWSHIPS OR ASSISTANTSHIPS. Unrestricted. Appointments made in many departments. Appointees give one-half their time to teaching and one-half to graduate work. Maintained by the University. Variable number; each $500-$750.

WILLARD D. THOMPSON MEMORIAL SCHOLARSHIPS. Unrestricted. Open to students from Utah. Established under the will of Jennie D. Thompson. Five; each $600.

UNIVERSITY FELLOWSHIPS. Unrestricted. Maintained by the University. Eight or ten; each $500-$750.

MARY J. WATSON SCHOLARSHIP IN MEDICINE. Medicine. Homeopathy. Supported by Dr. Mary J. Watson. Stipend, $150.

WHITING FELLOWSHIPS. Physics. Under certain circumstances may be used for work elsewhere. Supported by income from the Whiting Fund. One or more; each $750.

California Institute of Technology, Pasadena, Cal.

ASSISTANTSHIPS. Physics. Open to men. Supported by funds of the Institute. Nine in all; three each at stipends of $600, $750 and $900.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry or Chemical Engineering. Open to men. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

RESEARCH FELLOWSHIP. Physics. Open to men. Supported by funds of the Institute. Exemption from tuition fee. Stipend, $1000.

TEACHING FELLOWSHIPS.

Chemistry. Open to men, preferably those holding Master's degree. Six in all; three each at stipends of $900 and $1000.

Mechanical Engineering. Open to men, preferably those holding Master's degree. Stipend, $900.

Physics. Open to men preferably those holding Master's degree. Six; each $900.

These teaching fellowships are supported by funds of the Institute. Holders devote not more than 15 hours a week to instruction duties.

Carnegie Institute of Technology, Pittsburgh, Pa.

CARNEGIE FELLOWSHIPS IN MINING ENGINEERING. Mining Engineering. Open to men. These include four Bureau of Mines Fellowships (see U. S. Bureau of Mines) and two fellowships granted this year by the Hillman Coal and Coke Co. and the Mine Safety Appliances Co. Exemption from tuition fee. Six; each $750.

CARNEGIE FELLOWSHIP IN COMMERCIAL ENGINEERING. Commercial Engineering. Open to men. Maintained by regular funds of the Carnegie Institute of Technology. Exemption from tuition fee. Stipend, $750.

CARNEGIE FELLOWSHIP IN ENGINEERING. Engineering (unassigned). Open to men. Maintained by regular funds of the Carnegie Institute of Technology. Exemption from tuition fee. Stipend, $750.

FELLOWSHIPS OF BUREAU OF PERSONNEL RESEARCH. Psychology. Candidates must have good grounding in psychology and economics or education. Incumbents act as assistants in research work. Two to five; each $500-$750.

Case School of Applied Science, Cleveland, Ohio.

CASE FELLOWSHIPS. Unrestricted. Open to graduates of Case School for further work there. Several; stipend, individually determined.

DRURY FELLOWSHIP. Civil, Mechanical, Electrical, Mining, Metallurgical, Chemical Engineering, or Physics. Open to men who are graduates of other colleges. Established by Mr. E. F. Drury of Cleveland. Five; each $600.

GRASSELLI FELLOWSHIPS. Chemical Engineering. Established by the Grasselli Chemical Co. of Cleveland. May be held for more than one year if necessary. Two; one each at $500 and $750.

TEACHING FELLOWSHIP IN CHEMISTRY. Chemical Engineering. Fellow to devote about a quarter of his time to laboratory and instruction duties. Award usually continued for two years. Remission of tuition and fees. Stipend, $300.

Charles A. Coffin Foundation, Schenectady, N. Y.

CHARLES A. COFFIN FELLOWSHIPS. Electricity, Physics and Physical Chemistry. Open to men. Applications received from seniors about to graduate as well as from graduates of universities, colleges and technical schools. For research work in educational institutions here or abroad. Financial need a consideration. The Foundation was established by the General Electric Co. to make possible the award of $5000 annually for this purpose. Seven; minimum stipend of each, $500.

Chicago, University of, Chicago, Ill.

In general the candidate should have spent at least one year in resident study after receiving his Bachelor's degree and have demonstrated ability to conduct original investigation. Fellows will be asked to give a modicum of service.

EDITH BARNARD MEMORIAL FELLOWSHIP IN CHEMISTRY. Chemistry. Established by family and friends of Miss Barnard. Stipend, $150.

SEYMOUR COMAN RESEARCH FUND FELLOWSHIPS. Medicine. For "scientific research with special reference to cause, prevention and cure of disease." Work to be done under direction of Dr. Joseph A. Capps, Chicago, Ill., or Dr. Wilber E. Post, Chicago, Ill. Capital fund about $200,000, residue of estate of Seymour Coman. Net income used for the purpose. Several; stipend, income.

DUPONT FELLOWSHIP. Chemistry. For research in any field of chemistry. Granted by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

FLEISCHMANN FELLOWSHIP. Physiological Chemistry. Appropriated by Fleischmann Mfg. Co. Stipend, $750-$800.

STANTON A. FRIEDBERG FELLOWSHIP. Otolaryngology. Awarded by and work done under the direction of Dr. E. Irons and Dr. Geo. E. Shambaugh of Rush Medical College, Chicago, Ill. Capital fund $8000. Stipend, $450.

GRADUATE HONOR SCHOLARSHIPS. Unrestricted. For honor graduates of any department of the Senior College to continue work in the Graduate Schools for three quarters. Several; exemption from tuition.

GYPSUM FELLOWSHIPS. Research relative to use of Gypsum in Agriculture. Open to men who have scientific training dealing with problems of soil and plant nutrition. Maintained by The Gypsum Industries, Chicago. Stipend, $1200.

HOME ECONOMICS FELLOWSHIPS. Research in Home Economics. Open to women who present evidence of graduate work in an institution of high standing. Provided by friends of the Department of Home Economics. Two; each $600.

JOSEPH B. LOEWENTHAL FELLOWSHIP IN CHEMISTRY. Chemistry. Endowed by Mr. Berthold Loewenthal of Chicago. Stipend, $400.

MR. AND MRS. FRANK G. LOGAN RESEARCH FELLOWSHIPS. Research in Bacteriology and Pathology. Awarded in alternate years. On the organization of the Medical School, research fellowships will also be

Applicant should be a member of some federation of university women included in the International Federation of University Women. Stipend, $1000.

SARAH BERLINER RESEARCH FELLOWSHIP. Physics, Chemistry, or Biology. Open to American women who are holders of a Doctor's degree or its equivalent, for research. Supported by endowment fund. Stipend, $1000-$1200.

BOSTON ALUMNAE FELLOWSHIP. Unrestricted. Open to American women, for graduate work in Europe or in America. Supported by the Boston Branch of the A. A. U. W. aided by the Radcliffe Alumnae Association and the Boston Alumnae clubs of various colleges. Stipend, $500.

LATIN-AMERICAN FELLOWSHIP. Open to women born in the Latin-American republics, who have the equivalent of a college education in the universities or the best normal schools of their countries and a sufficient knowledge of English, for preparation for public service to their countries in (a) education, including scientific investigation, teaching, library work, etc.; (b) social service; or (c) public health and sanitation. Supported by A. A. U. W. and awarded by its Committee on Fellowships. Stipend, $1000.

ALICE FREEMAN PALMER MEMORIAL FELLOWSHIP. Unrestricted. Open to American women who are holders of a Doctor's degree or its equivalent, for research. Supported by endowment fund. Stipend, $1000.

PHI MU FELLOWSHIP. Unrestricted. Open to American women having a degree from any university or college in which Phi Mu has a chapter, for graduate work. Supported by Phi Mu Fraternity. Stipend, $1000.

JULIA C. G. PIATT MEMORIAL FELLOWSHIP. Unrestricted. Open to American women intending to enter the teaching profession, preference being given to those who have had successful experience in teaching and at least two years of graduate study. Offered in alternate years. Supported by memorial fund given by Julia C. G. Piatt Association. Stipend, $750.

SPANISH FELLOWSHIP. Unrestricted. Open to American women who can speak Spanish fluently, for graduate study at the University of Madrid, which offers the fellowship to be awarded by the A. A. U. W. Free tuition, and in addition, board and lodging at the Residencia.

Applications and recommendations for any of the above fellowships should be made by letter to the chairman of the Committee on Fellowships, Professor

Agnes L. Rogers, Smith College, Northampton, Mass., not later than February 1st.

ROSE SIDGWICK MEMORIAL FELLOWSHIP. Unrestricted. Open to British women of graduate standing, for graduate study in American colleges or universities. Established by A. A. U. W.; awarded by a committee of the British Federation of University Women. Stipend, $1000.

Further information regarding this fellowship may be obtained from Miss Theodora Bosanquet, 92 Victoria Street, London, S. W. 1, England.

See also under WOODS HOLE, MASS., MARINE BIOLOGICAL LABORATORY.

American Institute of Baking, 1135 Fullerton Ave., Chicago, Ill.

FELLOWSHIP IN BAKING RESEARCH. Chemistry, Biochemistry, Physics, Biology. Open to men who, if they have not completed the Master's degree, may spend part of the time in a university graduate school to do so. Supported by funds of the Institute. Two; each $500.

American Physiological Society, C. W. Greene, Secretary, Columbia, Mo.

PORTER FELLOWSHIP FOR PHYSIOLOGICAL RESEARCH. Physiology, research. Degree of M. D. or Ph. D. required. Work may be done anywhere at approved choice of candidate. Bequest of Dr. Wm. T. Porter. Two; each $1200.

American-Scandinavian Foundation, 25 W. 45th St., New York.

AMERICAN-SCANDINAVIAN FOUNDATION EXCHANGE FELLOWSHIPS. Unrestricted except for 1 in oceanography and weather forecasting at the Bergen Geophysical Institute in Norway, 1 for metallurgy in Sweden. Usually 1 in forestry, 1 or more in chemistry, 1 in hydro-electric engineering in Norway and 1 or 2 in cooperative agriculture in Denmark. Half of the fellowships (20) are for Scandinavian students for study in the United States; the other 20 are for students of American birth for study in Scandinavian countries: Sweden 10; Denmark 5; Norway 5. Supported by the Foundation through donations of individuals and firms. Forty; minimum stipend, each $1000.

PRINCETON-LEACH FELLOWSHIP. Unrestricted. Awarded in alternate years to a Princeton student and for study in a Norwegian or Danish university, and to a Scandinavian student for study at Princeton. Supported by the Foundation through donation by Dr. Henry Goddard Leach. Stipend, $1500.

Amherst College, Amherst, Mass.

RUFUS B. KELLOGG UNIVERSITY FELLOWSHIP. Natural Science, Philology, Mathematics, or humanistic subjects. For an alumnus of Amherst College. An especially good knowledge of Latin and German required. Awarded for a term of seven years, the first three to be spent at a German university or elsewhere, the other four as a lecturer at Amherst. About $30,000 given for purpose by the late Rufus B. Kellogg, Class of '58. Stipend, income.

JOHN WOODRUFF SIMPSON FELLOWSHIPS. (1) Medicine. Open to a graduate of Amherst College for study at any school approved by the Board of Trustees. (2) Unrestricted. Open to a graduate of Amherst College for study in England, at Oxford or Cambridge. (3) Unrestricted. Open to a graduate of Amherst College for study at the Sorbonne in Paris. Capital fund $150,000 for support of these three fellowships among others. Memorial gift by Mrs. John Woodruff Simpson and daughter. Stipend, part of income.

Arizona, University of, Tucson, Ariz.

BUREAU OF MINES FELLOWSHIPS. Research in Mining, Metallurgy and Chemical Technology. Established by University, research directed by Bureau of Mines. Exemption from laboratory fee when work is done in Bureau of Mines laboratories. Two; each $750.

Bernice Pauahi Bishop Museum, Honolulu, H. T.

See under YALE UNIVERSITY.

Boston Society of Architects, C. H. Blackall, Secretary, 20 Beacon St., Boston, Mass.

ROTCH TRAVELING SCHOLARSHIP. Architecture. Open to men under 30 years of age who have been engaged in professional work during two years in Massachusetts in the employ of a practising architect resident in Massachusetts. Holders of a degree in architecture from Massachusetts Institute of Technology, Columbia University, University of Pennsylvania, Cornell University, Harvard University or University of Illinois are admitted to competition without preliminary examination. For two years of foreign travel and studies. Founded by the children of the late Benjamin S. Rotch, of Boston. Stipend, $2800.

Boston University, Boston, Mass.

AUGUSTUS HOWE BUCK FELLOWSHIPS. Unrestricted. Open to men who are graduates of Boston University and who have been Augustus Howe Buck scholars during their undergraduate work. Work may be done abroad, if desirable. Several; stipend, all expenses.

ADA DRAPER SCHOLARSHIPS. Unrestricted. Open to women who are graduates of Boston University. Work to be done abroad. Financial need a consideration. Founded through bequest of Miss Ada Augustus Draper of Boston. Income (about $900) divided among several.

JACOB SLEEPER FELLOWSHIPS. Unrestricted. Open to men who are graduates of Boston University. Available for traveling. Supported by general University funds. Given now only twice in three years. Two; each $750.

Bowdoin College, Brunswick, Me.

CHARLES CARROLL EVERETT SCHOLARSHIP. Unrestricted. Open to men who are graduates of Bowdoin College. Available for traveling and for work in some other institution. Stipend, $500.

Brooklyn Institute of Arts and Sciences, Brooklyn, N. Y.

TEMPLE PRIME SCHOLARSHIP. Biological research. For work at Biological Laboratory, Cold Spring Harbor, L. I. Stipend, $100.

Brown University, Providence, R. I.

OLIVER CROMWELL GORTON ARNOLD BIOLOGICAL FELLOWSHIP. Biology. Capital fund $10,000 given by Dr. Oliver Henry Arnold. Stipend, income or part of it.

DUPONT FELLOWSHIP. Chemistry. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

MORGAN EDWARDS FELLOWSHIP. Unrestricted. For original research. Open to men who are graduates of Brown University of not more than 10 years standing. Work may be done wherever facilities are best. Capital fund $10,000 given by the Philadelphia alumni of Brown. Stipend, $1000.

GRAND ARMY OF THE REPUBLIC FELLOWSHIP. Unrestricted. Applicants should be candidates for Ph. D. or have obtained that degree. Open to men who are graduates of Brown. Must reside at the University. Descendants of Union Veterans of the Civil War given preference. Capital fund $10,000 given by Grand Army of the Republic, Department of Rhode Island. Stipend, income.

Bryn Mawr College, Bryn Mawr, Pa.

BRYN MAWR EUROPEAN FELLOWSHIP. Unrestricted. Open to a member of graduating class of Bryn Mawr for a year's study in an English or Continental university. Founded. Stipend, $500.

MARY ELIZABETH GARRETT EUROPEAN FELLOWSHIP. Unrestricted. Open to candidates for Ph. D. at Bryn Mawr, who have had two years of graduate work there, for completion of studies at an English or Continental university. Founded by Miss Mary Elizabeth Garrett of Baltimore. Stipend, $500.

HELEN SCHAEFFER HUFF MEMORIAL RESEARCH FELLOWSHIP. Physics or Chemistry. Open to women who have already done advanced work either at Bryn Mawr or elsewhere and have shown capacity for research. The work may be done where circumstances make it most desirable. Founded by an anonymous donor. Stipend, $1200.

RESIDENT FELLOWSHIPS. Biology, Chemistry, Geology, Physics or Mathematics. Open to women. Four; each $810.

RESIDENT SCHOLARSHIPS. Unrestricted. Open to women. Twenty; each $350.

HELENE AND CECIL RUBEL FOUNDATION FELLOWSHIP. Unrestricted. Open to women who are graduates of any other college of recognized standing but who have also studied at Bryn Mawr. If holder has sufficient income to waive the stipend it may be used by another student in the same year. The work may be done wherever circumstances make it desirable and may sometimes be used as a traveling fellowship. Stipend, $1500.

SCHOLARSHIP FOR EUROPEAN WOMEN STUDENTS. Unrestricted. Open to European women holding the equivalent of a Bachelor's degree. Three for British, three for French, three for other European countries, usually. Holders may do a small amount of teaching if they desire. Stipend to cover board, residence and tuition at Bryn Mawr for one academic year. Stipend, $720.

PRESIDENT M. CARY THOMAS EUROPEAN FELLOWSHIP. Unrestricted. Open to women who are candidates for Ph. D. at Bryn Mawr and who are in first year of graduate work. To be applied toward expenses of one year's study and residence at some English or Continental university. Founded by Miss Mary Elizabeth Garrett of Baltimore. Stipend, $500.

C. R. B. Educational Foundation, Inc., 42 Broadway, New York City.

GRADUATE FELLOWSHIPS FOR BELGIAN UNIVERSITIES. Bacteriology, Botany, Civil Engineering, Electricity, Geology, Mathematics, Medicine, Mining Engineering, Morphology, Organic Chemistry, Physiology, Psychology, Surgery, and Zoology, as well as certain humanistic studies. A candidate for a fellowship, to be eligible, must be an American citizen, must have a thorough speaking and reading knowledge of French, and if a member of the faculty of a college or university, must hold a grade below that of associate professor. Preference given to applicants between the ages of 25 and 33 who are unmarried and who intend to take up teaching or research as a profession. For definite program of work in Belgian universities or technical schools. Established to commemorate the work of the Commission for Relief in Belgium during the Great War and to promote closer relations and the exchange of intellectual ideas between Belgium and America. First class traveling expenses and tuition fees paid in addition to stipend. Applications to be filed by February 15. In 1924-25 not more than six; stipend, each 15,000 francs.

California, University of, Berkeley, Cal.

ASSOCIATED RADIOGRAPH LABORATORIES FELLOWSHIP. Dentistry. For research in radiography relating to dentistry. Established by Associated Radiograph Laboratories of San Francisco. Stipend, $1200.

BASE HOSPITAL NO. 30 SCHOLARSHIP IN MEDICINE. Medicine. Supported by balance of fund collected for the University of California Medical School Hospital Unit No. 30. Stipend, $400.

EDITH CLAYPOLE MEMORIAL RESEARCH FELLOWSHIP IN PATHOLOGY. Pathology. Supported by friends of the late Edith Claypole. Stipend, $1200.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

FELLOWSHIP FOR RESEARCH IN ANTHROPOLOGY OF ABORIGINAL CALIFORNIA. Anthropology. Supported by the Regents of the University. Stipend, $600 minimum.

MARJORIE GREEN FOSTER MEMORIAL SCHOLARSHIP IN RESEARCH MEDICINE. Medicine, research. Offered as a prize for most meritorious work during the year. Supported by the gift of Edythe Foster. Stipend, $150.

JAMES M. GOEWEY FELLOWSHIP IN NATURAL SCIENCE. Natural Science. Supported by the family of the late James M. Goewey. Stipend, $750-$900.

HATTIE HELLER GRADUATE SCHOLARSHIP. Unrestricted. Supported by an anonymous trust fund. One or more; each $500.

HONORARY TRAVELING FELLOWSHIPS. Unrestricted. May be awarded at any time to distinguished graduate students in any department.

HOOPER FELLOWSHIPS IN RESEARCH MEDICINE. Medicine, research. Candidates should have at least one and a half years medical training in the University of California Medical School or equivalent preparation. Supported by the Regents of the University. One or more; each $500.

GEORGE WILLIAMS HOOPER FOUNDATION FELLOWSHIP. Neurosurgery. Open to members of the house staff of the University of California Hospital or allied hospitals or to persons having equivalent training. Stipend, $1000-$1200.

HORGAN ESTATE ASSISTANTSHIPS IN AGRICULTURE. Agriculture. For definite research problems in this subject. Supported by estate of the late James C. Horgan. Stipend, $600.

MARTIN KELLOGG FELLOWSHIP IN ASTRONOMY. Astronomy. Under certain conditions work may be done elsewhere. Supported by estate of the late Mrs. Martin Kellogg. One or more; each $1200.

WILLIAM WATT KERR SCHOLARSHIP IN MEDICINE. Medicine. Supported by Faculty and Alumni of the Medical School. Stipend, $400.

LECONTE MEMORIAL FELLOWSHIP. Unrestricted. Open to graduates of the University of California. Under certain conditions the work may be done elsewhere. Supported by the Alumni Association. One or more; each $600.

LICK OBSERVATORY FELLOWSHIPS IN ASTRONOMY. Astronomy. Supported by Regents of the University. Three; each $600.

JAMES MONROE MCDONALD SCHOLARSHIPS. Chemistry, Sanitary and Municipal Engineering, Mining, Physics. Open to graduates of the University of California. One in each subject, which may be awarded for one year of post-graduate work at discretion of departments concerned. Supported by the gift of Mrs. McDonald. Four; each $400-$600.

JOHN W. MACKAY, JR., FELLOWSHIPS IN ELECTRICAL ENGINEERING. Electrical Engineering. Supported by the gift of Clarence W. Mackay and his mother, Mrs. John W. Mackay. Two; each $600.

PACIFIC COAST GAS ASSOCIATION FELLOWSHIP. Engineering. For research in engineering or allied sciences of physics and chemistry relating directly to gas engineering. Maintained by the Pacific Coast Gas Association. Stipend (may be divided between two fellowships), $1500.

JAMES ROSENBURG MEMORIAL SCHOLARSHIP IN AGRICULTURE. Agriculture. Supported by the Estate of James Rosenburg. Stipend, $600.

SHEFFIELD SANBORN SCHOLARSHIPS. Medicine. Open to graduates of the University of California who have not received a degree in medicine and who otherwise would not have the opportunity to acquire a university training. Supported by the gift of Mrs. James Horsburg (formerly Mrs. F. B. Sanborn). Two; each $150.

BERTHA HENICKE TAUSSIG MEMORIAL SCHOLARSHIP. Architecture, among other subjects not scientific. Open to women. Available for traveling. Established under the will of the late Hugo A. Taussig. Stipend, $600-$900.

TEACHING FELLOWSHIPS OR ASSISTANTSHIPS. Unrestricted. Appointments made in many departments. Appointees give one-half their time to teaching and one-half to graduate work. Maintained by the University. Variable number; each $500-$750.

WILLARD D. THOMPSON MEMORIAL SCHOLARSHIPS. Unrestricted. Open to students from Utah. Established under the will of Jennie D. Thompson. Five; each $600.

UNIVERSITY FELLOWSHIPS. Unrestricted. Maintained by the University. Eight or ten; each $500-$750.

MARY J. WATSON SCHOLARSHIP IN MEDICINE. Medicine. Homeopathy. Supported by Dr. Mary J. Watson. Stipend, $150.

WHITING FELLOWSHIPS. Physics. Under certain circumstances may be used for work elsewhere. Supported by income from the Whiting Fund. One or more; each $750.

California Institute of Technology, Pasadena, Cal.

ASSISTANTSHIPS. Physics. Open to men. Supported by funds of the Institute. Nine in all; three each at stipends of $600, $750 and $900.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry or Chemical Engineering. Open to men. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

RESEARCH FELLOWSHIP. Physics. Open to men. Supported by funds of the Institute. Exemption from tuition fee. Stipend, $1000.

TEACHING FELLOWSHIPS.

Chemistry. Open to men, preferably those holding Master's degree. Six in all; three each at stipends of $900 and $1000.

Mechanical Engineering. Open to men, preferably those holding Master's degree. Stipend, $900.

Physics. Open to men preferably those holding Master's degree. Six; each $900.

These teaching fellowships are supported by funds of the Institute. Holders devote not more than 15 hours a week to instruction duties.

Carnegie Institute of Technology, Pittsburgh, Pa.

CARNEGIE FELLOWSHIPS IN MINING ENGINEERING. Mining Engineering. Open to men. These include four Bureau of Mines Fellowships (see U. S. Bureau of Mines) and two fellowships granted this year by the Hillman Coal and Coke Co. and the Mine Safety Appliances Co. Exemption from tuition fee. Six; each $750.

CARNEGIE FELLOWSHIP IN COMMERCIAL ENGINEERING. Commercial Engineering. Open to men. Maintained by regular funds of the Carnegie Institute of Technology. Exemption from tuition fee. Stipend, $750.

CARNEGIE FELLOWSHIP IN ENGINEERING. Engineering (unassigned). Open to men. Maintained by regular funds of the Carnegie Institute of Technology. Exemption from tuition fee. Stipend, $750.

FELLOWSHIPS OF BUREAU OF PERSONNEL RESEARCH. Psychology. Candidates must have good grounding in psychology and economics or education. Incumbents act as assistants in research work. Two to five; each $500-$750.

Case School of Applied Science, Cleveland, Ohio.

CASE FELLOWSHIPS. Unrestricted. Open to graduates of Case School for further work there. Several; stipend, individually determined.

DRURY FELLOWSHIP. Civil, Mechanical, Electrical, Mining, Metallurgical, Chemical Engineering, or Physics. Open to men who are graduates of other colleges. Established by Mr. E. F. Drury of Cleveland. Five; each $600.

GRASSELLI FELLOWSHIPS. Chemical Engineering. Established by the Grasselli Chemical Co. of Cleveland. May be held for more than one year if necessary. Two; one each at $500 and $750.

TEACHING FELLOWSHIP IN CHEMISTRY. Chemical Engineering. Fellow to devote about a quarter of his time to laboratory and instruction duties. Award usually continued for two years. Remission of tuition and fees. Stipend, $300.

Charles A. Coffin Foundation, Schenectady, N. Y.

CHARLES A. COFFIN FELLOWSHIPS. Electricity, Physics and Physical Chemistry. Open to men. Applications received from seniors about to graduate as well as from graduates of universities, colleges and technical schools. For research work in educational institutions here or abroad. Financial need a consideration. The Foundation was established by the General Electric Co. to make possible the award of $5000 annually for this purpose. Seven; minimum stipend of each, $500.

Chicago, University of, Chicago, Ill.

In general the candidate should have spent at least one year in resident study after receiving his Bachelor's degree and have demonstrated ability to conduct original investigation. Fellows will be asked to give a modicum of service.

Edith Barnard Memorial Fellowship in Chemistry. Chemistry. Established by family and friends of Miss Barnard. Stipend, $150.

Seymour Coman Research Fund Fellowships. Medicine. For "scientific research with special reference to cause, prevention and cure of disease." Work to be done under direction of Dr. Joseph A. Capps, Chicago, Ill., or Dr. Wilber E. Post, Chicago, Ill. Capital fund about $200,000, residue of estate of Seymour Coman. Net income used for the purpose. Several; stipend, income.

duPont Fellowship. Chemistry. For research in any field of chemistry. Granted by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

Fleischmann Fellowship. Physiological Chemistry. Appropriated by Fleischmann Mfg. Co. Stipend, $750-$800.

Stanton A. Friedberg Fellowship. Otolaryngology. Awarded by and work done under the direction of Dr. E. Irons and Dr. Geo. E. Shambaugh of Rush Medical College, Chicago, Ill. Capital fund $8000. Stipend, $450.

Graduate Honor Scholarships. Unrestricted. For honor graduates of any department of the Senior College to continue work in the Graduate Schools for three quarters. Several; exemption from tuition.

Gypsum Fellowships. Research relative to use of Gypsum in Agriculture. Open to men who have scientific training dealing with problems of soil and plant nutrition. Maintained by The Gypsum Industries, Chicago. Stipend, $1200.

Home Economics Fellowships. Research in Home Economics. Open to women who present evidence of graduate work in an institution of high standing. Provided by friends of the Department of Home Economics. Two; each $600.

Joseph B. Loewenthal Fellowship in Chemistry. Chemistry. Endowed by Mr. Berthold Loewenthal of Chicago. Stipend, $400.

Mr. and Mrs. Frank G. Logan Research Fellowships. Research in Bacteriology and Pathology. Awarded in alternate years. On the organization of the Medical School, research fellowships will also be awarded in medicine and surgery. Endowed by Mr. Frank. G. Logan. Stipend of each, $1000.

PRIZE SCHOLARSHIPS IN MEDICAL COURSE. Medicine. Open to seniors who are about to graduate from colleges recognized by the University of Chicago, to encourage research. The stipends are equal to tuition for from one to three quarters. Offered by University. Three or four; each $75, $150 or $225.

PRIZE SCHOLARSHIP IN MEDICAL COURSE. Medicine. Open to graduate students of colleges recognized by the University of Chicago. Sum equal to tuition for three quarters. Offered by University. Stipend, $225.

GUSTAVUS F. SWIFT FELLOWSHIP IN CHEMISTRY. Chemistry. Awarded on nomination of the Department of Chemistry and recommendation of President of the University for unusual ability in research. Endowed by Mrs. Gustavus F. Swift of Chicago. Stipend, $920.

UNIVERSITY FELLOWSHIPS. Unrestricted. $21,000 appropriated from general funds of the University annually. About seventy; each $180-$580.

SIDNEY WALKER III SCHOLARSHIP. Physiology. Holder nominated by head of Department of Physiology, University of Chicago. Annual payment until endowed. Stipend, $200.

Cincinnati, University of, Cincinnati, Ohio.

BALDWIN FELLOWSHIPS. Bacteriology, Biochemistry, Engineering, Mathematics, Physiology, Physics, among other subjects not scientific. Preference given to candidates for Doctor's degree. Founded by bequest of Francis Howard Baldwin. Tuition free. Ten; each $500-$750.

BURCHENAL FELLOWSHIP. Biochemistry. For research in problems relating to biochemistry of potable waters. Founded by John J. Burchenal. Tuition free. Stipend, $1500.

DAVIS TEACHING FELLOWSHIP IN PREVENTIVE MEDICINE. Medicine. Capital fund $15,000, established by Union Central Life Insurance Co. Tuition free. Stipend, income.

GRADUATE SCHOLARSHIPS. Unrestricted. Maintained by the University. Exemption from tuition.

HANNA FELLOWSHIP. Physics. Established by Mrs. Henry Hanna and Miss Mary Hanna. Tuition free. Stipend, $500.

MERRELL FELLOWSHIP. Biochemistry. For investigation of the nature of the natural cure of disease. Established by Mr. Charles G. Merrell of the William S. Merrell Co. Tuition free. Stipend, $1500.

RANSOHOFF FELLOWSHIP IN SURGERY. Surgery or Surgical Anatomy. Capital fund $25,000, presented by Mrs. Joseph Ransohoff. Tuition free. Stipend, $1000.

SEELY TEACHING FELLOWSHIP IN INTERNAL MEDICINE. Medicine. $20,000 given to endowment fund of college for this purpose by Mrs. W. W. Seely. Tuition free. Stipend, income.

TAFT FELLOWSHIPS. Unrestricted. Preference given to candidates for Ph. D. Annual gift. Tuition free. Four; each $500-$750.

Clark University, Worcester, Mass.

"JUNIOR" FELLOWSHIPS. Chemistry, Geography, Physics, Psychology, among other subjects. Available for persons holding Master's degree or equivalent. Tuition remitted. Supported by Geo. F. Hoar Fund of $100,000, the gift of Andrew Carnegie. Variable number; each $100.

"SENIOR" FELLOWSHIPS. Chemistry, Geography, Physics, Psychology, among other subjects. Available for persons within one year of the Doctorate. Tuition remitted. Supported by Geo. F. Hoar Fund of $100,000, the gift of Andrew Carnegie. Variable number; each $200.

UNIVERSITY SCHOLARSHIPS. Chemistry, Geography, Physics, Psychology, among other subjects. Variable number; exemption from tuition.

Cold Spring Harbor, L. I., Biological Laboratory.

See under BROOKLYN INSTITUTE OF ARTS AND SCIENCES (Temple Prime Scholarship); COLUMBIA UNIVERSITY (John D. Jones Scholarship.)

Colorado School of Mines, Golden, Colo.

POST-GRADUATE FELLOWSHIPS. Mining, Metallurgical, Geological and Chemical Research. Open to men who are qualified to undertake research. Purpose of fellowships is to solve problems in these subjects of importance to State of Colorado. Fellows must give 18 hours a week as laboratory assistants, etc. Several; each $675.

Columbia University, New York City.

ERNEST KEMPTON ADAMS RESEARCH FELLOWSHIP. Physical sciences or their practical applications. Appointment may be made from among the teaching staff, alumni or students of the University or from among the distinguished physicists of the United States or any foreign country. Studies may be carried on at Columbia or elsewhere. Founded by Edward Dean Adams. Stipend, $1250.

BARNARD FELLOWSHIP FOR ENCOURAGING SCIENTIFIC RESEARCH. Physical Science. Open to graduates of Columbia. Endowed by bequest of late President Barnard. Fellow receives net income which shall accrue for three years, available every third year beginning with 1918. Stipend (given as last awarded), $1362.50.

GEORGE BLUMENTHAL, JR., FELLOWSHIPS. Medicine and Surgery. For advanced study and research. One or more at maximum stipend of $1000.

GEORGE BLUMENTHAL, JR., SCHOLARSHIP. Medicine. Supported by George Blumenthal Scholarship Fund. Stipend, $500 maximum.

BORING FELLOWSHIP IN ARCHITECTURE. Architecture. Awarded every third year beginning with 1925 conjointly with the Perkins Fellowship (q. v.). Fellow receives net income accruing during three years of Boring Fellowship Fund. Stipend, income.

SAMUEL WILLARD BRIDGHAM FELLOWSHIP. Unrestricted. Incumbent should reside in New York City or vicinity during academic year. Supported by Samuel Willard Bridgham Fellowship Fund of $20,000. Stipend, income.

RICHARD BUTLER SCHOLARSHIP. Unrestricted (except as not available in Teachers College or College of Pharmacy). Open to men born in Ohio. For one year but renewable for two more. Capital fund $5000. Stipend, income.

CLARK SCHOLARSHIP. Medicine. For research. Established by will of late Alonzo Clark, M. D., LL. D., formerly President of College of Physicians and Surgeons. Stipend, $762.50.

CLASS OF '70 FELLOWSHIP. Unrestricted. Filled by University Council, subject to such regulations as may be prescribed. Stipend, $500.

CURTIS UNIVERSITY SCHOLARSHIPS. Unrestricted. Open to women who are candidates for higher degree. Under same regulations as University Scholarships (*q. v.*). Four; each $240.

WILLIAM BAYARD CUTTING TRAVELING FELLOWSHIPS. Science, Medicine (among other subjects not scientific). Open to men who have been in residence as graduate students and candidates for higher degree under Faculty of Pure Science for at least one academic year; to graduates of Columbia University in Medicine or Applied Science who have had at least one year of additional study as graduate students in the University; and to graduating seniors (not more than two in one year). May be used for study either in the United States or in a foreign country. Indeterminate number at $1000 minimum.

ABRAM DUBOIS FELLOWSHIP. Medicine. Open to a graduate of the School of Medicine who has completed one year as interne in a New York hospital or elsewhere, which time has been largely devoted to diseases of the eye. For clinical and laboratory study of these diseases in American or foreign universities. Capital fund $18,000. Stipend, income.

DUPONT FELLOWSHIP. Chemistry. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

EMMONS GEOLOGICAL FELLOWSHIP. Economic Geology. For research to be carried on at any place selected by the holder, who must give whole time to problem selected unless otherwise authorized by Committee of Award. Stipend, prescribed by Committee in each case.

FERGUSON FELLOWSHIP IN CHEMISTRY. Chemistry. For research in synthetic drugs and medicine under direction of Department of Chemistry. Subject to same regulations as University Fellowships (*q. v.*). Capital fund $10,000. Stipend, income.

SAMUEL ANTHONY GOLDSCHMIDT FELLOWSHIP IN APPLIED CHEMISTRY. Chemistry. Subject to same regulations as University Fellowships (*q. v.*). Capital fund $16,250, gift of George B. Goldschmidt. Stipend, income.

GOTTSBERGER FELLOWSHIP. Unrestricted. Open to a graduate of Columbia College who has been for at least two years a resident graduate student at the University, taking also Master's degree. Available for study abroad with consent of department in charge of his major subject. Awarded in alternate years. Net income for two years of Cornelius Heeney Gottsberger Fund of $9500. Stipend, income.

HARSEN SCHOLARSHIPS. Medicine and Surgery. Open to students in College of Physicians and Surgeons, two each in second and third years of the medical course, and one in fourth year. Fund established by late Jacob Harsen, M. D. Five; each $250.

JOHN D. JONES SCHOLARSHIP. Zoology. Includes use of a table at biological laboratory at Cold Spring Harbor, L. I. Annual gift of Wawepex Society. Stipend, $200.

MCKIM FELLOWSHIP IN ARCHITECTURE. Architecture. Open to graduates of the School of Architecture within three years following graduation and to all matriculated students who have completed the requirements for the thesis. For foreign study and travel. Available every third year beginning with 1917-18, carrying net income accruing for two years of fund of $20,000, gift of Charles F. McKim. Stipend, income.

William Mitchell Fellowship. Science (or Letters). Open to graduates of the College, preference being given to members of graduating class and graduates of not more than three years standing. Capital fund $10,000 bequeathed by Samuel D. Silliman. Stipend, income.

Perkins Fellowship in Architecture. Architecture. Open to graduates of the School of Architecture within three years following graduation and to all matriculated students who have completed requirements for thesis. For foreign travel and study. Available every third year beginning with 1902. Fellow receives net income accruing during three years from fund of $5700, established by bequest of Willard P. Perkins. Stipend, income.

Isaac Plaut Fellowship. Pharmacy. Open to the Bachelor of Science in College of Pharmacy who has shown greatest aptitude for original investigation in his class. For study in an approved European institution selected by himself. Established by Albert Plaut. Stipend, $500.

Maria McLean Proudfit Fellowship. Internal Medicine. Open to any son of native-born American parents, a graduate in medicine and unmarried. Work to be done under direction of Medical Faculty at Columbia or elsewhere. Appointment made every four years upon terms designated by Faculty. Endowed by late Alexander Moncrief Proudfit, A. B. '92. Capital fund $15,000. Stipend, income.

Lydia C. Roberts (Chamberlain) Fellowships. Unrestricted. Open to graduates of Iowa colleges who were born in Iowa and will return to live in Iowa. For travel and study. Capital fund $419,000, gift of Mrs. Lydia C. Chamberlain. About twenty; each $850 maximum.

Schermerhorn Fellowship in Architecture. Architecture. Open to graduates of School of Architecture within three years following graduation and to all matriculated students who have completed requirements for thesis. For foreign travel and study. Available every third year beginning with 1921-22, with net income accruing during three years from fund of $13,000, gift of F. Augustus Schermerhorn, E. M. '68. Stipend, income.

William Petit Trowbridge Fellowship in Engineering. Engineering. Fellow recommended by Committee consisting of President of the University and heads of Departments of Mining, Civil and Mechanical Engineering. Capital fund $10,000. Stipend, income.

Tyndall Fellowship for Encouragement of Research in Physics. Physics. Open to a graduate of Columbia or an unusually promising student in the University. Awarded every second and third year in a cycle of three years. Fellow receives net income of capital sum constituting endowment, for one year and a half. Endowed by Professor John Tyndall. Stipend, $972 minimum.

UNIVERSITY FELLOWSHIPS. Unrestricted. Open to candidates for Ph. D. with decided fitness for special line of study. For sufficient reasons Fellows may be reappointed for one year. Pecuniary emolument may be waived for benefit of additional Fellows, as the fellowships are awarded as honors. Fellows not ordinarily allowed to accept remuneration for outside work. Twelve; each $750.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Open to all qualified to be enrolled as candidates for higher degree at Columbia, not over thirty years of age, preference being given to candidates for University Fellowships who have failed of appointment after having been recommended by any faculty or department. Stipend may be relinquished without losing honor of designation. For sufficient reasons holder may be reappointed for one year. Twenty-eight; each $240.

Commonwealth Fund, New York City.

See under NATIONAL COMMITTEE FOR MENTAL HYGIENE.

Cornell University, Ithaca, N. Y.

ERASTUS BROOKS FELLOWSHIP IN MATHEMATICS. Mathematics. Tuition free. Maintained by the University. Stipend, $400.

CHARLES BULL EARLE MEMORIAL FELLOWSHIP IN MECHANICAL AND ELECTRICAL ENGINEERING. Mechanical and Electrical Engineering. Tuition free. Founded by Charles Earle. Stipend, $400.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

HERMAN FRASCH FELLOWSHIPS. Agriculture. Given by Union Sulphur Co. Two; each $1000.

GRADUATE SCHOLARSHIPS. Mathematics; Chemistry; Physics; Civil Engineering; Archaeology and Comparative Philology; Animal Biology; Botany, Geology or Physical Geography; Architecture; Veterinary Medicine; among other subjects not scientific. One in each. Tuition free. Maintained by the University. Nine; each $200.

GRASSELLI FELLOWSHIP IN CHEMISTRY. Chemistry. Given by the Grasselli Chemical Co. Stipend, $750.

GYPSUM INDUSTRIES FELLOWSHIP. Research relative to use of Gypsum in Agriculture. Open to men who have had scientific training dealing with problems of soil and plant nutrition. Maintained by the Gypsum Industries. Stipend, $1500.

HONORARY FELLOWSHIPS. Unrestricted. Open to persons having Doctor's degree or equivalent preparation. Residence at the University and registration in Graduate School required. Exemption from all fees except laboratory charges. Awarded at discretion of Faculty of Graduate School. One or more; tuition.

INTERNATIONAL MILK DEALERS ASSOCIATION FELLOWSHIP. Agriculture. Given by the International Milk Dealers Association. Stipend, $600.

MCGRAW FELLOWSHIP IN CIVIL ENGINEERING. Civil Engineering. Tuition free. Maintained by the University. Stipend, $400.

EDGAR J. MEYER MEMORIAL FELLOWSHIP IN ENGINEERING RESEARCH. Engineering Research. Founded by Eugene Meyer. Stipend, $400.

NATIONAL CANNERS ASSOCIATION FELLOWSHIP IN CHEMISTRY. Chemistry. Given for 1923-24 by National Canners Association. Stipend, $800.

SAGE FELLOWSHIP IN CHEMISTRY. Chemistry. Tuition free. Maintained by the University. Stipend, $400.

SCHUYLER FELLOWSHIP IN ANIMAL BIOLOGY. Animal Biology. Tuition free. Maintained by the University. Stipend, $500.

SIBLEY FELLOWSHIP IN MECHANICAL AND ELECTRICAL ENGINEERING. Mechanical and Electrical Engineering. Tuition free. Maintained by the University. Stipend, $400.

GOLDWIN SMITH FELLOWSHIP IN BOTANY, GEOLOGY, OR PHYSICAL GEOGRAPHY. Botany, Geology, or Physical Geography. Tuition free. Maintained by the University. Stipend, $400.

UNIVERSITY FELLOWSHIPS. Architecture, and Agriculture or Veterinary Science, among other subjects not scientific. Tuition free. No stipend in Architecture. Maintained by the University. One of each; stipend, each $400.

WESTERN NEW YORK FRUIT GROWERS PACKING ASSOCIATION FELLOWSHIP. Agriculture. Given for this year by Western New York Fruit Growers Packing Association. Stipend, $750.

PRESIDENT WHITE FELLOWSHIP IN PHYSICS. Physics. Tuition free. Maintained by the University. Stipend, $400.

WILLIAMSON CO-OPERATIVE VEGETABLE ASSOCIATION FELLOWSHIP. Agriculture. Given for this year by Williamson Co-operative Vegetable Association. Stipend, $1000.

Crop Protection Institute, Paul Moore, Secretary, c/o National Research Council, Washington, D. C.

SULPHUR FELLOWSHIPS. Agriculture—applications of sulphur. For research. Capital fund $7500 a year for two years beginning 1922. Given jointly by Union Sulphur Co., Freeport Sulphur Co. and Texas Gulf Sulphur Co. Two; each $2500.

SCALECIDE FELLOWSHIPS. Agriculture—scalecide studies. For research. Capital fund $4500 for three years beginning 1923. Given by B. G. Pratt Co., New York. Stipend, $2500.

FURFURAL FELLOWSHIP. Agriculture—applications of furfural. For research. Given by the Quaker Oats Co., Chicago, Ill. Stipend, $2500.

The above fellowships are awarded to individuals who are appointed by the Board of Governors of the Crop Protection Institute and do the work at institutions designated by the Board. Extra funds are provided for traveling expenses, supplies, cooperative work, etc.

Dartmouth College, Hanover, N. H.

GEORGE E. CHAMBERLIN FELLOWSHIPS. Unrestricted—advanced studies, not professional. Open to men who are graduates of Dartmouth College. Available for traveling and work elsewhere. Fund supplemented by College. Fifteen; each $1000.

BETSEY R. LANG SCHOLARSHIP. Unrestricted—advanced studies, not professional. Open to men who are graduates of Dartmouth College. Capital fund $5000. Stipend, $200.

ANN L. PAIGE SCHOLARSHIP. Unrestricted—advanced studies, not professional. Open to men who are graduates of Dartmouth College. Capital fund $4248. Stipend, $200.

HENRY ELIJAH PARKER FELLOWSHIPS. Unrestricted—advanced studies, not professional. Open to men who are graduates of Dartmouth College. Available for traveling and work elsewhere. Fund supplemented by College. Twenty-three; each $1000.

SMYTH FUND SCHOLARSHIP. Unrestricted—advanced studies, not professional. Open to men who are graduates of Dartmouth College. Capital fund $5000. Stipend, $200.

WILLIAM JEWETT TUCKER FELLOWSHIPS. Unrestricted—advanced studies, not professional. Open to men who are graduates of Dartmouth College. Available for traveling and work elsewhere. Maintained by Alumni Fund. Six; stipend, variable.

Delaware, University of, Newark, Del.

DUPONT SCHOLARSHIP. Chemistry. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $300.

GRADUATE SCHOLARSHIPS. Chemistry, Bacteriology, among other subjects in which graduate work is offered. Open to men who are graduates of the University. Supported by general funds of the University. Three; each $300.

E. I. duPont de Nemours & Company, Wilmington, Del.

DUPONT FELLOWSHIPS IN CHEMISTRY. Chemistry. Granted at present in the following institutions: Brown University, California Institute of Technology, University of California, University of Chicago, Columbia University, Cornell University, University of Delaware (scholarship), Harvard University, University of Illinois. Johns Hopkins University, Lafayette College, Massachusetts Institute of Technology, University of Michigan, University of Minnesota, Ohio State University, Princeton University, University of Virginia, University of Washington, University of Wisconsin, Yale University. Twenty-one, stipends ranging from $300 to $750 (see under each institution named).

Florida, University of, Gainsville, Fla.

GRADUATE STUDENT ASSISTANTSHIP. Farm Management. Open to men who are graduates of the University in the College of Agriculture. Not available for work outside of Florida. Supported by legislative appropriation. Stipend, $600.

Food Research Institute, Stanford University, Cal.

See under STANFORD UNIVERSITY.

Forest Products Laboratory, Madison, Wis.

See under WISCONSIN, UNIVERSITY OF.

General Education Board, New York City.

See under NATIONAL RESEARCH COUNCIL (National Research Fellowships—Medicine).

Goucher College, Baltimore, Md.

JOHNS HOPKINS UNIVERSITY RESIDENT FELLOWSHIPS. Unrestricted. Open to women who are graduates of Goucher College for study at Johns Hopkins University. Tuition paid direct to University. Two; no stipend, free tuition.

DEAN VAN METER ALUMNAE FELLOWSHIP. Unrestricted. Open to women who are graduates of Goucher College for graduate study at some approved university elsewhere. Available for traveling. Capital fund $60,000. Stipend, $500.

Grasselli Chemical Co., Cleveland, O.

See under CASE SCHOOL OF APPLIED SCIENCE; CORNELL UNIVERSITY; INDIANA UNIVERSITY; MASSACHUSETTS INSTITUTE OF TECHNOLOGY; OBERLIN COLLEGE; OHIO STATE UNIVERSITY.

Gypsum Industries, America Fore Bldg., Chicago, Ill.

GYPSUM INDUSTRIES FELLOWSHIPS.

Research relative to use of Gypsum in Agriculture. Open to men, graduates of recognized colleges who have had scientific training dealing with problems of soil and plant nutrition. The Gypsum Industries now maintains these fellowships in the University of Chicago, Cornell University, Iowa State College, University of Kentucky and North Carolina State College. Five; stipends ranging from $1000 to $1500 (see under each institution named).

Research relative to use of Gypsum in Building Construction. Open to men, graduates of recognized colleges, who have had scientific training dealing with problems of soil and plant nutrition. Maintained at the Bureau of Standards, Washington, D. C., where fellow works under direction of the Head of the Plastering Section. Stipend, $1800.

Hamilton College, Clinton, N. Y.

ROOT FELLOWSHIP IN SCIENCE. Science. Open to men who are graduates of Hamilton College for work at some other institution of approved standing. Endowed. Stipend, $500.

Harvard University, Cambridge, Mass.

In general the following conditions apply to the fellowships listed below:

In making resident appointments preference is usually, though not always, given to those who have given evidence of qualifications at the University. Non-resident appointments, however, are only for graduates of some department of the University or for students who have been there for some time.

Appointments are made for one year, renewable for a second year but never for more than the third year.

Tuition is paid out of stipends except by non-resident holders of fellowships.

Women are not eligible for appointment.

The following abbreviations are used to show in which schools of the University the fellowships may be assigned, according to conditions. (*) used

in connection with medical scholarships indicates that they have that stipend only when the student pays full tuition.

Arch.—Faculty of Architecture.	**Gr.—Graduate School of Arts and Sciences.**
Bus.—Bussey Institution.	**Med.—Medical School.**
Den.—Dental School.	**Pub. H.—School of Public Health.**
Eng.—Engineering School.	

ANNA C. AMES SCHOLARSHIP. Applied Biology, Research. (Bus.) Founded by Professor Oakes Ames. Stipend, $450.

JULIA AMORY APPLETON TRAVELLING FELLOWSHIP IN ARCHITECTURE. Architecture. (Arch.) Open to Masters in Architecture of Harvard University not over thirty years of age on day of appointment. This and the Nelson Robinson Jr. Travelling Fellowship are awarded in alternate years and are renewable for second year when fellow has done creditable work. Founded by Charles F. McKim of New York. Capital fund $20,000. Stipend, $2200.

EDWARD AUSTIN FELLOWSHIPS. Unrestricted. (Gr.) Open to resident graduate students showing fitness to pursue advanced studies in the departments in which the degree of Doctor of Philosophy is given. From income of Edward Austin fund. Four; each $600.

AUSTIN FELLOWSHIPS IN MEDICINE. Medicine. (Med.) Open to resident graduate students. From income of Edward Austin Fund. Four; each $750.

AUSTIN SCHOLARSHIPS IN ARCHITECTURE. Architecture. (Arch.) From income of Edward Austin Fund. Two; each $400.

AUSTIN SCHOLARSHIP IN LANDSCAPE ARCHITECTURE. Landscape Architecture. (Arch.) From income of Edward Austin Fund. Stipend, $400.

AUSTIN TEACHING FELLOWSHIPS. Unrestricted. (Gr.) Open to needy, meritorious students recommended by a division or department of the Faculty of Arts and Sciences. From income of Edward Austin Fund. Thirty; each $750.

EDWARD M. BARRINGER SCHOLARSHIP NO. 1. Medicine. (Med.) Preference given to students in the fourth class. Edward M. Barringer Fund. Stipend, $300 minimum.

EDWARD M. BARRINGER SCHOLARSHIP NO. 2. Medicine. (Med.) Preference given to students in the fourth class. Edward M. Barringer Fund. Stipend, $200 minimum.

GORDON BARTLETT SCHOLARSHIP. Medicine. (Med.) Preference given to graduates of Dartmouth. Usually divided between two men. Established by Mr. and Mrs. Donald Gordon. Stipend, $350.

Matthew and Mary E. Bartlett Scholarship. Medicine. (Med.) Financial need a consideration. Founded by Miss Fannie Bartlett of Boston. Stipend, $550.

Lucius F. Billings Scholarship. Medicine. (Med.) Founded from bequest of Lucius F. Billings. May be divided between two or more. Stipend, $250.

Bliss Scholarships. Applied Biology, Research. (Bus.) To be used in the Division of Forestry. Usually divided among from three to five holders. Founded by Mrs. William H. Bliss of New York. Stipend, in all $1250.

Edward Hickling Bradford Fellowship. Medical Research or Instruction. (Med.) May be awarded separately or in connection with any other foundation. Capital fund $25,000. Anonymous donor. Stipend, $1250.

Bright Scholarship. Architecture. (Arch.) Preference given to descendants of Henry Bright, Jr., who bear the name of Bright. Established under will of Jonathan Brown Bright of Waltham, Mass. Five; each $350.

John White Browne Scholarship. Medical Research. (Med.) Investigations may be pursued elsewhere if desirable. Established under will of Mrs. Francis B. Greene. Stipend, $950.

Daniel A. Buckley Scholarships. Dentistry. (Den.) Candidates should be graduates of the public schools of Cambridge. Established under will of Daniel A. Buckley, of Cambridge, Mass. Several; each $230.

Unrestricted. (Gr., Arch., Bus., Med.*) Candidates should be graduates of the public schools of Cambridge. Established under will of Daniel A. Buckley, of Cambridge, Mass. Several; each $250.

Bullard Fellowships. Medicine. See George Cheyne Shattuck Memorial Fellowship; John Ware Memorial Fellowship; Charles Eliot Ware Memorial Fellowship. May be awarded singly or in combination. Donor, William Story Bullard.

Caspar Henry Burton, Jr., Scholarship. Unrestricted. (Gr., Arch., Bus., Med.) Open to students nominated by Rev. Spence Burton, A. B. 1903, and Wm. Greenough Wendell, A. B. 1909. Established by Mr. and Mrs. Caspar Henry Burton. Stipend, $350.

Arthur Tracy Cabot Fellowship. Surgery. (Med.) Incumbent may not ordinarily engage in private practice, but is to devote himself to the advancement of surgery in the United States or elsewhere. Capital fund $25,000, gift of Dr. and Mrs. Frederick C. Shattuck. Stipend, $1250.

JAMES JACKSON CABOT FELLOWSHIP. Medicine, preferably. (Med.) Capital fund $10,000. Established by Arthur T. Cabot, Samuel Cabot and Guy C. Cabot. Stipend, $500.

VICTOR EMMANUEL CHAPMAN MEMORIAL FELLOWSHIP. Unrestricted. (Gr., Bus., Arch., Med.) Open to French youths nominated by committee of French scholars. Application should be made to Chairman of the Committee, Professor Emile Legouis, University of Paris. Founded by several persons. Stipend, $1650.

JOSEPH HODGES CHOATE MEMORIAL FELLOWSHIP. Unrestricted. (Gr., Arch., Bus., Med.*) Open to a British subject from the University of Cambridge, Eng. Established by the Harvard Club of New York City. Stipend, $2000.

FRANCIS HATHAWAY CUMMINGS SCHOLARSHIP. Landscape Architecture. (Arch.) Established by Charles A. Cummings and Margaret K. Cummings. Capital fund $5000. Stipend, $400.

GEORGE AND MARTHA DERBY. Unrestricted. (Gr.) Capital fund $5000. Bequest of Miss Martha C. Derby. Stipend, $350.

ORLANDO W. DOE SCHOLARSHIP. Medicine. (Med.) Founded by bequest of Orlando Witherspoon Doe. One-half of income to be used for this purpose. Capital fund $5000. Stipend, $150.

duPONT FELLOWSHIP. Chemistry. (Gr.) Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

CHARLES ELIOT TRAVELLING FELLOWSHIP IN LANDSCAPE ARCHITECTURE. Landscape Architecture. (Arch.) Open to men not over thirty years of age who have been recommended for, or have taken the Master's degree in Landscape Architecture at Harvard. For one year of travel and study. Capital fund $25,000. Stipend, $1200.

GEORGE H. EMERSON SCHOLARSHIPS.

Applied Biology, Research. (Bus.)

Geology, Mineralogy, and Chemistry. (Gr.) Open to graduates of, though not paid assistants in, one of these departments.

Capital fund about $25,000, residuary bequest of George H. Emerson. Four, one in each department, stipend, $450.

JOSEPH EVELETH SCHOLARSHIPS.

Architecture. (Arch.) Stipend, $300.
Dentistry. (Den.) Stipend, $250.
Engineering. (Eng.) Two; each $300.

Medicine. (Med.) Three; each $250.

Capital fund $37,897.14, residuary bequest of Joseph Eveleth of Boston and Watertown.

HORACE PUTNAM FARNHAM SCHOLARSHIP. Medicine. (Med.) Financial need considered. Founded from bequest of Mrs. Eliza Cary Farnham. Two; each $225.

CHARLES FOLLEN FOLSOM FELLOWSHIP. Hygiene or Mental and Nervous Diseases. (Med.) A teaching fellowship. At present assigned to Department of Hygiene. Founded by more than sixty persons. Capital fund over $11,000. Stipend, $625.

VIRGINIA BARRETT GIBBS SCHOLARSHIP. Zoology, preferably Marine Zoology. (Gr.) Applicant need not necessarily be candidate for degree. Work may be done in any place approved. Stipend, one or more at $375.

HARRIS FELLOWSHIP. Unrestricted (some subject other than Law, Theology or Medicine). (Gr.) Holder must live in Cambridge and be unmarried. Established by William Minot, Jr., as executor of will of Henry Harris of Boston. Stipend, $600.

JOHN HARVARD FELLOWSHIPS. Unrestricted. (Gr.) Selected on nominations of several departments or divisions. Holder may be resident or non-resident. Established by President and Fellows of the University. Indeterminate number. No stipend.

HARVARD CLUB OF BUFFALO SCHOLARSHIP. Unrestricted. (Gr.) Preference given to men from Erie Co., N. Y. Founded by Harvard Club of Buffalo. Stipend, $250.

HARVARD CLUB OF CHICAGO SCHOLARSHIP. Unrestricted. (Gr.) Open to a graduate of an Illinois university or college. Founded by Harvard Club of Chicago. Stipend, $350.

HARVARD CLUB OF LOUISIANA SCHOLARSHIP. Unrestricted—advanced non-professional studies. (Gr.) Open to graduates of Tulane University. Founded by Harvard Club of Louisiana. Stipend, $250.

HARVARD CLUB OF OKLAHOMA SCHOLARSHIP. Unrestricted. (Gr., Bus., Arch., Med.*) Founded by Harvard Club of Oklahoma. Stipend, $250.

HARVARD CLUB OF PHILADELPHIA SCHOLARSHIP. Unrestricted. (Gr.) Preference given to men from Philadelphia or some part of Pennsylvania from which membership of Club is constituted. Founded by Harvard Club of Philadelphia. Stipend, one or more at $350.

Harvard Club of St. Louis Scholarship. Unrestricted. (Gr.) Preference given to men whose preparation and home have been in or about St. Louis or elsewhere in Missouri. Founded by Harvard Club of St. Louis. Stipend, $300.

Harvard Engineering Society of New York City Scholarship. Engineering. (Eng.) Preference given to men from New York City or vicinity. Established by the Society. Stipend, $200.

Lewis and Harriet Hayden Scholarship. Medicine. (Med.) Open to colored students. Founded from bequest of Mrs. Harriet Hayden. Stipend (may be divided between two or more), $275.

Hemenway Fellowship. Anthropology. (Gr.) Capital fund $10,000 given by Mrs. Mary Hemenway. Stipend, $675.

William Hilton Scholarship. Engineering. (Eng.) Founded from the bequest of William Hilton. Two; each $325.

Priscilla Clark Hodges Scholarship. Applied Biology, Research. (Bus.) Capital fund $4000, bequest of Miss Priscilla Clark Hodges, of Boston. Stipend, $275.

Edward William Hooper Fellowship. Unrestricted. (Gr.) Capital fund $25,000, gift of Henry Lee Higginson and others. Stipend, $1400.

Hennen Jennings Scholarship. Mining or Metallurgy. (Eng.) Founded by Hennen Jennings. Stipend, $500.

William Otis Johnson Scholarship. Medicine. (Med.) Founded from bequest of Mrs. William Otis Johnson. Stipend, $200.

Claudius M. Jones Scholarship. Medicine. (Med.) Capital fund $6000, bequest of Marcellus Jones. Stipend, $325.

John Thornton Kirkland Fellowship. Unrestricted. (Gr.) Incumbent must, as a rule, have resided three years at the University and he has "leave to repair to a foreign country for instruction." Capital fund $11,000. Endowed by George Bancroft. Stipend, $675.

Lincoln Scholarship. Unrestricted. (Gr.) Preference given to men from Lincoln, Mass. Founded by Miss Levina Hoar, of Lincoln. Stipend, $400.

Alfred Hosmer Linder Scholarship. Medicine. (Med.) Financial need considered. Founded by Mrs. George Linder. Stipend, $275.

J. Ewing Mears Scholarship. Medicine. (Med.) For the four years' course. Founded from bequest of Dr. J. Ewing Mears, of Philadelphia. Stipend, $200.

WILLIAM O. MOSELEY, JR., TRAVELLING FELLOWSHIPS. Medicine. (Med.) Open to men who have attended the Medical School for three or four years. For study in Europe. Founded from bequest of Mrs. William O. Moseley. Capital fund $74,000. Two; each $2325.

LADY MOWLSON SCHOLARSHIP. Unrestricted. (Gr.) First Scholarship in Harvard (1643). Preference given to kinsmen of Lady Ann Mowlson, of London, the founder. Stipend, $375.

WILLIAM STANISLAUS MURPHY SCHOLARSHIP. Unrestricted. (Gr., Bus., Med.,* Eng.) Open to men of the name of Murphy. From whole number usually not more than three awarded for graduate work. Founded by William Stanislaus Murphy. Three; up to $310 each.

JOSEPH PEARSON OLIVER SCHOLARSHIP. Medicine. (Med.) Financial need considered. Founded by patients of Dr. Joseph Pearson Oliver. Stipend, $425.

PARKER FELLOWSHIP. Unrestricted. (Gr.) Open to men with talent for some science. Founded from bequest of John Parker, Jr. Four; each $950.

FRANCIS PARKMAN FELLOWSHIP. Unrestricted. (Gr.) Capital fund $10,000, from estate of Miss Eliza W. S. Parkman. Stipend, $575.

CHARLES ELLIOT PERKINS SCHOLARSHIP. Unrestricted. (Gr., Arch., Bus., Med.*) Preference given to a graduate of an Iowa college or university. Founded by Mrs. Charles Elliot Perkins of Burlington, Vt. Stipend, $450.

CHARLES B. PORTER SCHOLARSHIP. Medicine. (Med.) Capital fund $5000, bequest of William L. Chase. Stipend, $275.

PRINCETON FELLOWSHIP. Unrestricted. (Gr., Arch., Bus., Med.*) Preference given to graduates of Princeton. Capital fund $10,000, gift of an alumnus of Princeton. Stipend, $600.

PUBLIC HEALTH FELLOWSHIPS. Public Health. (Pub. H.) Limited number at stipend of $1200.

NELSON ROBINSON, JR., TRAVELLING FELLOWSHIPS IN ARCHITECTURE. Architecture. (Arch.) Open to Masters in Architecture of Harvard University, not over thirty years of age on day of appointment. This and the Julia Amory Appleton Travelling Fellowship are awarded in alternate years and are renewable for second year when fellow has done creditable work. Established from income of the Nelson Robinson, Jr., Memorial Fund. Stipend, $2200.

Rocky Mountain Harvard Club Scholarship. Unrestricted. (Gr., Arch.) Open to graduates of Colorado colleges or schools. Maintained by the Rocky Mountain Harvard Club, of Denver. Stipend, $250.

James A. Rumrill Scholarship. Unrestricted. (Gr., Arch., Bus., Med.*) Open to a properly qualified graduate of a college in Virginia, South Carolina, Florida, Georgia, Tennessee or Kentucky. Founded by Mrs. James A. Rumrill. Stipend, $325.

Edward Russell Scholarship. Unrestricted. (Gr.) Financial need considered. Founded by Edward Russell. Stipend, $375.

Leverett Saltonstall Scholarship. Unrestricted. (Gr.) For resident or non-resident study. Preference given to relatives or descendants of Leverett Saltonstall, A. B. 1802, or donors of the scholarship, Leverett Saltonstall, A. B. 1844, and Henry Saltonstall, A. B. 1848. Stipend, $500.

James Savage Scholarship. Unrestricted. (Gr.) Founded from part of bequest of $40,000 made by James Savage. Stipend, $400.

Flavius Searle Scholarships. Engineering. (Eng.) Medicine. Founded by Miss Mary L. Searle. One in each department at stipend of $200.

Philip H. Sears Scholarship. Psychology. (Gr.) To be assigned on recommendation of the Department of Philosophy and Psychology. Founded by Mrs. Philip H. Sears. Stipend, $900.

George Cheyne Shattuck Memorial Fellowship. Medicine. (Med.) For original investigation. See Bullard Fellowships. Stipend, $275.

Frederick Sheldon Fund for Travelling Fellowships. Unrestricted. (Gr., Arch., Bus., Med.*) Incumbents must be at time of appointment members of Harvard University. For investigation and study "in this country—outside Harvard University—or abroad." Not assigned in fixed amounts. Founded from residuary bequest of Mrs. Amey Richmond Sheldon, about $400,000. Variable number; stipend, variable.

Eugene Hanes Smith Scholarship. Dentistry. (Den.) Awarded in third or fourth year to a student in the Dental School who has been in good standing during first and second years. Founded by Harvard Dental Alumni Association. May be divided between two or given to one. Stipend, $250.

PETER E. STRAUSS SCHOLARSHIP. Dentistry. (Den.) Established by Mrs. Emily R. Strauss. Stipend, $500.

CHARLES PRATT STRONG SCHOLARSHIP. Medicine. (Med.) Founded by friends and patients of Dr. Charles Pratt Strong. Stipend, $250.

ISAAC SWEETSER SCHOLARSHIP. Medicine. (Med.) Founded by Mrs. Anne M. Sweetser. Stipend, $325.

JOHN THOMSON TAYLOR SCHOLARSHIP. Medicine. (Med.) Founded by Mrs. Frederick D. Phillips. Stipend, $250.

THAYER FELLOWSHIPS AND SCHOLARSHIPS. Unrestricted. (Gr.) Financial need considered. From income of $50,000 bequeathed in trust by John Eliot Thayer of Boston. Fellowships: five; each $550. Scholarships: five; each $400.

GORHAM THOMAS SCHOLARSHIP. Unrestricted. (Gr.) Capital fund $3000, gift of Alexander Thomas. Stipend, $300.

TOWNSEND SCHOLARSHIP. Unrestricted. (Gr.) Capital fund $20,000, bequest of Miss Mary P. Townsend. Five; each $350.

JOHN TYNDALL FELLOWSHIP. Physics. (Gr.) Open to American students, for original investigation. Founded by Professor John Tyndall, of London, with sum of $10,800. One or more at stipend of $700.

UNIVERSITY SCHOLARSHIPS.

Architecture and Landscape Architecture. (Arch.) Six; each $250.

Applied Biology, Research. (Bus.) Stipend, $250.

Unrestricted. (Gr.) Twenty-five annually assigned to seniors of high standing graduating from Harvard or other colleges. Regard had to geographical distribution. Maintained by Harvard University. Forty; each $250.

HENRY PICKERING WALCOTT FELLOWSHIP. Medicine. (Med.) For teaching and clinical research. Incumbent may not engage in private practice. Founded by Dr. Frederick Cheever Shattuck. Stipend, $1650.

CHARLES ELIOT WARE MEMORIAL FELLOWSHIP. Medicine. (Med.) For original investigation. See Bullard Fellowships. Stipend, $300.

JOHN WARE MEMORIAL FELLOWSHIP. Medicine. (Med.) For original investigation. See Bullard Fellowships. Stipend, $275.

MAJOR HARRISON BRIGGS WEBSTER, U. S. A., SCHOLARSHIP. Medicine. (Med.) Preference given to sons of members of Class of 1905. Founded by Mrs. L. Florence Webster. Stipend, $250.

Christopher M. Weld Scholarship. Unrestricted. (Gr.) Financial need considered. Capital fund $10,000, bequest of Mrs. Ann P. Weld. Stipend, $600.

Whiting Fellowships. Physics. (Gr.) Capital fund $20,000, bequest of Harold Whiting. Four; each $400.

Edward Wigglesworth Scholarship. Medicine. (Med.) Founded by family of Edward Wigglesworth. Stipend, $275.

Robert C. Winthrop Scholarship. Archaeology and Ethnology. (Gr.) Assigned to Peabody Museum of Archaeology and Ethnology. Capital fund $5000, bequest of Robert Charles Winthrop. Stipend, $375.

Idaho, University of, Moscow, Idaho.

Bureau of Mines Fellowships. Research in Mining, Metallurgy and Chemical Technology. Supported by the University, research done under direction of Bureau of Mines. Exemption from fees. Two; each $750.

Illinois, University of, Urbana, Ill.

Carr Fellowship in Chemistry. Chemistry or Chemical Engineering. Capital fund $10,000, established by Hon. Robert F. Carr. Stipend, income.

duPont Fellowship. Chemistry. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

Research Graduate Assistantships in the Engineering Experiment Station. Engineering. Open to men who are graduates of any approved institution, American or foreign. Awarded for a term of two years. Exemption from dues. Supported by University appropriation. Fourteen; each $600.

Social Hygiene Board Fellowships. Chemistry. Capital fund $3000, part of gift of U. S. Interdepartmental Social Hygiene Board. Three at $800 each; one at $600.

University Fellowships. Unrestricted. Open to candidates who have had one or two years of graduate work and are not over thirty years old. Exemption from tuition and other fees. Several, at $350-$500 each.

University Scholarships. Unrestricted. Open to candidates in first year of graduate work who are not over thirty years old. Exemption from tuition and other fees. Several, at $300 each.

Indiana University, Bloomington, Ind.

GRASSELLI CHEMICAL COMPANY FELLOWSHIP. Chemistry. Given by Grasselli Chemical Company of Cleveland. One or two at $750 each.

LAWRENCE FELLOWSHIP IN ASTRONOMY. Astronomy. Open to candidates for Master's degree. Work is done at Observatory at Flagstaff, Ariz., where holder gives general assistance. Travelling expenses to and from the Observatory, also furnished room, are provided. Exempt from contingent fees. Established by the late Percival Lowell. Stipend, $600.

NATIONAL LIME ASSOCIATION FELLOWSHIP. Chemistry. Given by National Lime Association, of Washington, D. C. Stipend paid at rate of $50 a month for twelve months. Stipend, $600.

SPECIAL FELLOWSHIPS. Unrestricted. For persons of exceptional ability and merit who may or may not have received the Ph. D. degree. Three; each $1000.

UNIVERSITY FELLOWSHIPS. Unrestricted. Open to graduate students who have had at least one year of graduate work. Fellows may be required to spend some time in service of their department. Several, at $300-$500.

Iowa, State University of, Iowa City, Ia.

GRADUATE ASSISTANTSHIPS. Unrestricted. Open to those who have graduated with a high record from a standard college and have taken advanced courses in the subjects in which they may serve as assistants. They must give one-half their time to undergraduate instruction and have the privilege of carrying from one-half to two-thirds of the graduate schedule. About sixty, at $700-$800.

JUNIOR FELLOWSHIPS. Unrestricted. Open to those who have had one or more years of graduate study or equivalent preparation. Preference given to graduates of standard colleges of the State. Free tuition. Eighteen; each $300-$500.

RESEARCH ASSISTANTSHIPS. Unrestricted. Open to those who have qualified for research in some specific field. They must give one-half their time to assistance in research, with the privilege of carrying a maximum of two-thirds graduate schedule. Fifteen; each $600.

RESEARCH ASSOCIATES. Unrestricted. A sort of post-doctorate fellowship for those who have proved their ability in graduate work. Entire time to be devoted to research. Offered by Graduate College. Five; each $1000 minimum.

SCHOLARSHIPS. Unrestricted. Preference given to graduates of standard colleges of the State. Free tuition. Offered by Graduate College. Eighteen; each $200-$400.

SENIOR FELLOWSHIPS. Unrestricted. Open to students who have Doctor's degree or equivalent preparation. Preference given to graduates of standard colleges of the State. Free tuition. Offered by Graduate College. Three; each $600-$800.

Iowa State College of Agriculture and Mechanic Arts, Ames, Ia.

FELLOWSHIP OF NATIONAL ASSOCIATION OF MANUFACTURERS OF CARBONATED BEVERAGES. Chemistry, food and sanitary. Given by the National Association of Manufacturers of Carbonated Beverages for two years, 1923-24, 1924-25. Stipend, $2000-$2500.

GYPSUM INDUSTRIES FELLOWSHIP. Research relative to use of Gypsum in Agriculture. Open to men who have had scientific training dealing with problems of soil and plant nutrition. Maintained by The Gypsum Industries. Stipend, $1250.

RESEARCH FELLOWSHIPS. Agricultural Economics, Agricultural Engineering, Animal Husbandry, Botany, Dairying, Farm Crops and Soils, Forestry, Horticulture. Open to holders of Bachelor's or Master's degree. Supported by general funds of the College. Exemption from tuition. One or more awards in each subject. Fourteen; each $540.

RESEARCH SCHOLARSHIPS. Agricultural Economics. Open to holders of Bachelor's or Master's degree. Supported by general funds of the College. Exemption from tuition. Four; each $270.

TEACHING FELLOWSHIPS. Agricultural Engineering, Animal Husbandry, Bacteriology, Botany, Dairying, Farm Crops and Soils, Horticulture, Forestry, Home Economics, Comparative Physiology. Open to holders of Bachelor's or Master's degree. Supported by general funds of the College. Exemption from tuition. One or more awards in each subject. Nineteen; each $540.

TEACHING SCHOLARSHIPS. Animal Husbandry, Dairy Husbandry. Open to holders of Bachelor's or Master's degree. Supported by general funds of the College. Exemption from tuition. Four awards in Animal Husbandry, one in Dairy Husbandry. Five; each $270.

Johns Hopkins University, Baltimore, Md.

ADAM T. BRUCE FELLOWSHIP. Biological Science, especially Animal Morphology. Preference given to those who have been fellows at Johns Hopkins and are not more than thirty years old. For original research at a place approved by the President of the University. Capital fund $10,000, given by the mother of Adam T. Bruce. Tuition free. Stipend, income.

DUPONT FELLOWSHIPS. Chemistry. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Two; each $750.

FELLOWSHIPS IN SCHOOL OF HYGIENE AND PUBLIC HEALTH. Hygiene and Public Health. Preference given to applicants already students in the school, who are candidates for degree of Doctor of Science. Granted for one year with privilege of renewal for another. Maintained by the University. Tuition free. Six; each $500.

WILLIAM H. GRAFFLIN SCHOLARSHIP. Industrial Chemistry, Research. Established by bequest of the late William H. Grafflin of Baltimore. Stipend, $1000.

HOPKINS SCHOLARSHIPS.

Unrestricted. Open to graduates of recognized colleges in, and legal residents of, (a) Maryland, Virginia and North Carolina and (b) other states of the South and Southwest. Awarded as an honor. (a) thirty, (b) ten; exemption from tuition but not from laboratory fees.

Medicine. Open to second, third and fourth year students in the Medical Department from Maryland, North Carolina and Virginia. In all six; exemption from tuition.

JOHNSTON SCHOLARSHIPS (THE HENRY E. JOHNSTON, THE JAMES BUCHANAN JOHNSTON, AND THE HENRY E. JOHNSTON, JR.). Unrestricted. For independent research in the chosen subjects. Some teaching may be required. Founded by Mrs. Harriet Lane Johnston, of Washington. Three; each $1500.

WALTER COTTRELL QUINCY FUND GRADUATE SCHOLARSHIP. Physics. Capital fund $20,000, bequest of Mrs. Martha R. Quincy. Income divided among four. Stipend, income.

RESEARCH FELLOWSHIPS. Hygiene and Public Health. Open to persons who hold a Doctor's degree in Medicine, Science or Public Health, or who have conducted and published a meritorious investigation. Awarded by Trustees of the University for one year with privilege of renewal for another. Three; each $1000.

SCHOLARSHIPS IN SCHOOL OF HYGIENE AND PUBLIC HEALTH. Hygiene and Public Health. Granted for one year with privilege of renewal for another. Awarded by Trustees of University. Twelve; exemption from tuition.

SCHOLARSHIPS FOR FOREIGN STUDENTS. Unrestricted. Open to graduate students from following countries, distributed as indicated: Latin-American republics 5; Belgium 5; France 3; Mexico 3; Russia 5; Persia (graduate or undergraduate) 3. Twenty-four; exemption from tuition.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Awarded as honors to graduate students who have been studying at the University during previous part of the session. Open about January 1. Work may be done elsewhere if pecuniary emolument is waived. Exemption from tuition.

GEORGE HUNTINGTON WILLIAMS MEMORIAL FELLOWSHIP. Geology. It is expected that the George Huntington Williams Memorial Fund will provide a fellowship for geological research. Stipend, half of income.

Kalamazoo College, Kalamazoo, Mich.

UPJOHN RESEARCH SCHOLARSHIP IN CHEMISTRY. Chemistry. Open to graduates of Kalamazoo College. Given by the Upjohn Company. Five; each $400.

Kansas, University of, Lawrence, Kans.

KANSAS COLLEGE FELLOWSHIPS. Unrestricted. Open to graduates of any accredited Kansas college, one in each. Maintained by the State. Ten; each $350.

UNIVERSITY FELLOWSHIPS. Unrestricted. Maintained by the State. Fifteen; each $350.

Kansas State Agricultural College, Manhattan, Kans.

GRADUATE ASSISTANTSHIPS. Animal Husbandry, Bacteriology, Botany, Crops, Dairy Husbandry, Food Economy and Nutrition, Household Economics, Mechanical Engineering, Soils, Zoology. Open to persons pursuing work leading to Master's degree. Half of their time is given to laboratory or research assistance. Salary fixed by Board of Administration. Several; stipend, variable.

MILLING INDUSTRY SCHOLARSHIPS. Milling Industry work in departments of Agriculture, General Science, and Engineering. Preference given to residents of the State of Kansas. Given by Kansas Flour Mills Co. Three; each $100.

Kentucky, University of, Lexington, Ky.

ASSISTANTSHIPS. Unrestricted. Supported by general University funds. Eight; each $400.

FELLOWSHIPS. Unrestricted. Supported by general University funds. Two; each $500.

GYPSUM INDUSTRIES FELLOWSHIP. Research relative to use of Gypsum in Agriculture. Open to men who have had scientific training dealing with problems of soil and plant nutrition. Maintained by Gypsum Industries. Stipend, $1200.

SCHOLARSHIPS. Supported by general University funds. Four; each $200.

Lafayette College, Easton, Pa.

DUPONT FELLOWSHIP. Chemistry. Open to men. Given by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $500.

HART FELLOWSHIP. Chemistry. Open to men. Capital fund $10,000. Stipend, $500.

INTERNATIONAL PAPER CO. FELLOWSHIP. Chemistry. Open to men. Given by the International Paper Co. Stipend, $750.

Lake Erie College, Painesville, Ohio.

ELIZA H. WILCOX FELLOWSHIP FOR GRADUATE STUDY. Unrestricted. Open to women graduates of Lake Erie College. Available for traveling and work elsewhere. Founded. Stipend, $300.

Lehigh University, South Bethlehem, Pa.

CALLENDER-CORNELL FELLOWSHIP. Chemistry. Open to men, for research. Annual gift of anonymous donor. Stipend, $1500.

Louisiana State University and Agricultural and Mechanical College, Baton Rouge, La.

FELLOWSHIPS. Chemistry, Agronomy, Horticulture, Physics. Fellows are required to devote some time to work for the University. Five; each $40 per month.

SCHOLARSHIPS. Unrestricted. Open mainly to graduates of the University. Some laboratory or other assistance required. Forty-two; each $15 per month.

Louisville, University of, Louisville, Ky.

FELLOWSHIPS IN PATHOLOGY AND BACTERIOLOGY. Pathology and Bacteriology. Open to graduates of approved medical schools. Maintenance is added. Two, each $300.

J. B. SPEED GRADUATE SCHOLARSHIP. Unrestricted. Open to graduates of the University of Louisville, residents of Louisville. Available for traveling or work elsewhere. Captial fund $10,000. Two; each $250 maximum.

Macalester College, St. Paul, Minn.

FELLOWSHIPS. Unrestricted. Open to graduates of Macalester College for study at University of Minnesota. Fellows required to devote part of their time to instruction in Macalester College. Awarded by Board of Trustees. Several. Stipend, $800.

Maryland, University of, College Park, Md.

FELLOWSHIPS. Agriculture, Biology and Chemistry. Exemption from all fees except diploma. Supported from university funds. Fourteen; each $500.

GRADUATE ASSISTANTSHIPS. Agriculture, Biology and Chemistry. Both research and teaching assistants. Exemption from all fees except diploma. Supported from university funds.

Massachusetts Institute of Technology, Cambridge, Mass.

The Committee on Graduate Courses and Scholarships appropriates money from incomes of various funds as it sees fit, dividing the sums and fixing the stipends at its discretion according to circumstances. Stipends usually range from $300 to $600.

AUSTIN FUND. Unrestricted. Open to (a) candidates for degree of Master of Science and (b) candidates for degree of Doctor of Science and Doctor of Philosophy. Founded by bequest of Edward Austin. Limited number in each. Stipend, (a) $300 maximum; (b) $500 maximum.

AUSTIN RESEARCH FELLOWSHIPS. Unrestricted. Open to candidates for degree of Doctor of Science or Doctor of Philosophy. Tuition free. Two; each $500.

MALCOLM COTTON BROWN FUND. Physics. Open to a senior in high standing in course in physics, for advanced study and research. Usually available for only one year. Established by Charles A. Brown and Caroline C. Brown. Stipend, variable.

COLLAMORE FUND. Unrestricted. Primarily for women. Founded by bequest of Helen Collamore. Stipend, variable.

DALTON FUND. Chemistry. Open to American men who are graduates of the Institute, for advanced study and research, especially applicable to textile industries. Stipend, variable.

DUPONT FELLOWSHIP. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

FOREIGN TRAVELLING SCHOLARSHIPS. Unrestricted. May be awarded to Institute graduates with exceptional qualifications or to applicants who have served on instructing staff of the Institute. Stipend, $500.

GRASSELLI FELLOWSHIP IN CHEMISTRY. Given annually by The Grasselli Chemical Co. Stipend, $500.

MOORE SCHOLARSHIP FUND. Organic Chemistry, especially. Open to Institute graduates who have distinguished themselves in this subject, for study in Europe. Founded by Mrs. F. Jewett Moore. Stipend, variable.

NATIONAL LIME ASSOCIATION FELLOWSHIP. Research in utilization of lime. Given annually by the National Lime Association. Stipend, $1000.

WILLARD B. PERKINS FUND. Architecture. Income available for a traveling fellowship in 1923-24 and every fourth year. Founded by bequest of Willard B. Perkins. Stipend, $1250.

ELLEN H. RICHARDS MEMORIAL RESEARCH FUND. Sanitary Chemistry. Fellowships and assistantships for research. Capital fund $16,000. Stipend, variable.

HENRY BROMFIELD ROGERS FUND. Unrestricted. Fellowships and scholarships for women. Capital fund $21,000. Stipend, variable.

RICHARD LEE RUSSEL FUND. Civil Engineering. Open to a student of high standing in Department of Civil Engineering. Founded by Theodore E. Russel. Stipend, variable.

HENRY SALTONSTALL SCHOLARSHIP FUND. Unrestricted. Capital fund $10,000. Bequest of Henry Saltonstall. Stipend, variable.

JAMES SAVAGE FUND. Unrestricted. Founded by the late James Savage. Stipend, variable.

SUSAN H. SWETT FUND. Unrestricted. Capital fund $10,000. Stipend, variable.

TRAVELING FELLOWSHIP IN ARCHITECTURE. Architecture. Open to regular and special students who have passed at least two consecutive years in the school within the last three years, one of which must have been in graduate class. For travel and study abroad under direction of Department of Architecture. Maintained by the Austin Fund. Stipend, $1250.

JONATHAN WHITNEY FUND. Unrestricted. For students needing financial assistance. Founded by Francis B. Greene. Stipend, variable.

Mayo Clinic, Rochester, Minn.
See under MINNESOTA, UNIVERSITY OF.

Mellon Institute of Industrial Research, University of Pittsburgh, Pittsburgh, Pa.

INDIVIDUAL INDUSTRIAL FELLOWSHIPS. Industrial Research. The following names of this class of Fellowships appear in the Tenth Annual Report of the Institute: Asbestos, Carbon dioxide, Chrysotile, Corrosion, Dental alloy, Edible gelatin, Ester, Emulsion flavors, Enameling, Fiber, Glue, Heavy chemicals, Inks, Insecticides, Insulation, Magnesia products, Medicinal, Metal ware, Metallic oxides, Pharmaceuticals, Roofing, Salt, Silicate, Smoke, Steel, Stove, Synthetic acids, Textile, Varnish, Wood chemicals. Usually awarded to holders of Master's degree or preferably Doctors of Philosophy. An Individual Industrial Fellowship utilizes the services of one research chemist or engineer (with assistants when necessary), who is responsible directly to the executive staff of the Institute. The award is available for traveling if advisable and the work may be done wherever necessary. Supported by industrial associations and individuals. Bonuses and royalties provided in some cases. Thirty fellowships; the foundation sums covering all expenses of the specific researches for which they are donated, including salaries, range from $3000 to $8000.

MULTIPLE INDUSTRIAL FELLOWSHIPS. Industrial Research. The following names of this class of Fellowships appear in the Tenth Annual Report of the Institute: Bread, By-products, Cleaning, Coke, Fertilizer, Food container, Gas, Laundry, Nickel, Oil, Organic synthesis, Perfumes, Pratt, Protected metals, Refractories, Slag, Synthetic resins, Varieties, Vitrified tile. Usually awarded to holders of the Master's degree or preferably Doctors of Philosophy. A Multiple Industrial Fellowship has the services of one or more research men (Junior Fellows) under the direction of a Senior Fellow, who, in turn, is responsible to the executive staff. The award is available for traveling and the work may be done wherever necessary. Supported by industrial associations and individuals. Bonuses and royalties provided in some cases. Twenty fellowships; the foundation sums covering all expenses of the specific researches for which they are donated, including salaries, range from $7500 to $18,000.

Among the trade associations which have established fellowships at this Institute are: Clay Products Association, Compressed Gas Manufacturers Association, Container Club, Eastern Clay Products Association, Edible Gelatin Manufacturers of America, Laundryowners National Association, Magnesia Association of America, Mundatechnical Society of America (formerly International Technical Society of Cleaners and Dyers), National Wood Chemical Association, Refractories Manufacturers Association, Stove Founders' Research Association, and Vitrified Tile Floor Association.

Michigan, University of, Ann Arbor, Mich.

Fellows and Scholars do not pay the tuition fee and are not called upon for instructional duties. With the exception of the University Fellowships and Scholarships, none of the scholarships listed are open to students in their first year of residence.

ACME WHITE LEAD AND COLOR WORKS FELLOWSHIP. Chemistry. For the study of problems connected with the manufacture of paints and varnishes. Given by the Acme White Lead and Color Works, of Detroit. Stipend, $750.

CARL BRAUN FELLOWSHIP. Unrestricted. Capital fund $10,000, given by Mrs. Elsie L. Braun, of Toledo. Stipend, $500.

ROY D. CHAPIN FELLOWSHIP IN HIGHWAY ENGINEERING. Highway Engineering. For investigation in an approved subject relating to hard surfaced roads and pavements. Annual gift. $50 additional for expenses. Stipend, $250.

ROY D. CHAPIN FELLOWSHIP IN HIGHWAY TRANSPORT. Highway Transport. For investigation in an approved subject relating to highway transport. Annual gift. $50 additional for expenses. Stipend, $250.

EMMA J. COLE FELLOWSHIP IN BOTANY. Botany. Established from residue of estate of Miss Emma J. Cole. Stipend, $600.

DETROIT EDISON FELLOWSHIPS IN HIGHWAY ENGINEERING. Highway Engineering. For investigation in an approved subject relating to moderate cost country roads. Offered by the Detroit Edison Company. $50 additional for expenses. Several; each $250.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

EDWIN C. HINSDALE FELLOWSHIP. Zoology. Open especially to students in the Museum of Zoology. Supported by bequest of the late Genevieve S. Hinsdale. One or more; each $500.

LAWTON FELLOWSHIP IN ASTRONOMY AND MATHEMATICS. Astronomy (preferably), or Mathematics. Capital fund $6000, bequest of Francis A. Lawton. Stipend, $180.

NATIONAL ANILINE AND CHEMICAL COMPANY FELLOWSHIP. Organic Chemistry. Gift of National Aniline and Chemical Company. Stipend, $750.

STATE COLLEGE FELLOWSHIPS OR SCHOLARSHIPS. Unrestricted. Open to graduates of not more than four years standing of some college in Michigan. Nominated by faculties of the different colleges. Six; each $300-$500.

FREDERICK STEARNS AND COMPANY FELLOWSHIP IN PHARMACY. Pharmacy. Gift of Frederick Stearns and Company, of Detroit. Stipend, $500.

UNITED STATES RADIATOR COMPANY FELLOWSHIP. Engineering. For investigation of problems connected with the application of different forms of radiators and other heating devices in heating installations. Gift of the United States Radiator Co., of Detroit. Stipend, $500.

UNITED STATES RUBBER PLANTATIONS FELLOWSHIP IN BOTANY. Botany. For the bio-chemical study of the hevea plant. Gift of the United States Rubber Plantations, Inc., for three years. Stipend, $1200.

UNIVERSITY FELLOWSHIPS. Unrestricted. Open to properly qualified students from any college or university or from any part of the world. Limited number; each $500-$1000.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Open to properly qualified students from any college or university or from any part of the world. Limited number; each $150-$500.

ANGELINE BRADFORD WHITTIER FELLOWSHIPS IN BOTANY. Botany. Capital fund $4000, gift of Joseph Bradford Whittier, of Saginaw. Stipend, $400.

Middlebury College, Middlebury, Vt.

TEACHING FELLOWSHIP IN CHEMISTRY. Chemistry. Supported by general fund of the College. One or two; each $750.

Minnesota, University of, Minneapolis, Minn.

CLASS OF 1890 FELLOWSHIP. Science, Engineering, or Architecture. Open to a graduate of the College of Science, Literature and the Arts or the College of Engineering and Architecture. Capital fund $2500, gift of the Class of 1890. Stipend, income.

CALEB DORR FELLOWSHIPS. Agriculture, research. Capital fund $20,000, bequest of the late Caleb Dorr. Two or more; each $500.

DUPONT FELLOWSHIP. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

ALBERT HOWARD SCHOLARSHIP. Unrestricted. Open to a graduate of the College of Science, Literature and the Arts. Endowed. Stipend, $120.

MAYO FOUNDATION FELLOWSHIP. Medicine and Surgery. Open to students having degree of B. S., who are graduates of a high class medical school and who have had one year interneship. Maintained by the Mayo Clinic, Rochester, Minn. One hundred seven; stipend variable.

SHEVLIN FELLOWSHIPS. Agriculture, Chemistry, Medicine, Science. Open to graduates of the Colleges of Agriculture, Chemistry, Medicine, or Science, Literature and the Arts. Established by the late Thomas Shevlin. Four; each $500.

The University assistantships, fellowships and scholarships carry stipends ranging from **$225** to **$1200** with remission of tuition in the Graduate School, being graded according to duties required. Stipend, **$225-$1200**.

UNIVERSITY ASSISTANTSHIPS. Agriculture (16); Animal Biology (8); Anthropology (1); Chemistry (24); Electrical Engineering (1); Engineering, Experiment Station (1); Mathematics (2); Physics (10); Sociology and Anthropology (1). The services of some of these assistants may be almost wholly of a research character; in other cases the duties are more instructional. Maintained by University funds. Sixty-four; stipend, variable.

UNIVERSITY FELLOWSHIPS. Animal Biology (2 teaching); Electrical Engineering (2 teaching); Engineering, Experiment Station (2 research); Mathematics (1 teaching); Physics (4 teaching); Psychology (6 teaching); Botany (6 teaching). Maintained by University funds. Twenty-three; stipend, variable.

UNIVERSITY FELLOWSHIPS IN THE MEDICAL SCHOOL. Medicine and Surgery. Open to students having degree of B. S. or its equivalent who are graduates of a high class medical school and have had one year interneship. Maintained by University funds. Sixteen; stipend, variable.

UNIVERSITY SCHOLARSHIPS. Animal Biology (2); Astronomy (1); Geology and Mineralogy (2); Sociology and Anthropology (1); Psychology (1). Maintained by University funds. Seven; stipend, variable.

Missouri, University of, Columbia, Mo.

AGRICULTURAL RESEARCH FELLOWSHIPS. Agricultural Science, research. Open to students who have completed one year of graduate work and shown ability in research work. Investigation carried on at the Agricultural Experiment Station. Supported by the University. Limited number; stipend, $600.

AGRICULTURAL RESEARCH SCHOLARSHIPS. Agricultural Science, research. Supported by the University. Limited number; stipend, $300.

BUREAU OF MINES FELLOWSHIPS. Research in Mining, Metallurgical and Chemical Technology. Open to candidates for, or holders of, degree of Master of Science in Mining or Metallurgy. Maintained by the University, School of Mines and Metallurgy. Research directed by Bureau of Mines. Four; each $800.

GREGORY FELLOWSHIPS. Unrestricted. Open to students who have had at least one year of graduate work and have shown ability in research. Supported by William Alexander Gregory Fund, not more than $3000 from income of which is used annually for all scholarships and fellowships. Several; each $600.

GREGORY SCHOLARSHIPS. Unrestricted. Scholars may be called upon to do limited amount of work for the University. Supported by William Alexander Gregory Fund, not more than $3000 from income of which is used annually for all scholarships and fellowships. Several; each $300.

UNIVERSITY FELLOWSHIPS. Unrestricted. Open to students who have had at least one year of graduate study and have shown ability in research. Limited number; stipend, $600.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Scholars may be called upon to do limited amount of work for university. Limited number; stipend, $300.

Missouri Botanical Garden, St. Louis, Mo.

See under WASHINGTON UNIVERSITY, St. Louis (Rufus J. Lackland Research Fellowship in Botany).

Mount Holyoke College, South Hadley, Mass.

BARDWELL MEMORIAL FELLOWSHIP. Unrestricted. Open to women who are graduates of Mount Holyoke College of not more than five years standing. Work may be done wherever desirable. Capital fund $10,000, established by Alumnae Association. Stipend, $500.

CLASS OF 1905 FELLOWSHIP. Unrestricted. Open to women who are graduates of Mount Holyoke College. Work may be done wherever desirable. Capital fund $11,000. Stipend, $500.

CLASS OF 1886 FELLOWSHIP. Unrestricted. Open to women who are graduates of Mount Holyoke College for work in some other approved college. Given annually by a member of the Class of 1886. Stipend, $500.

Mary E. Woolley Fellowship. Unrestricted. Open to women who are graduates of Mount Holyoke College for work in some other approved college. Given annually by the General Alumnae Association. Stipend, $500.

Mount Sinai Hospital, 5th Ave. and 100th St. New York, N. Y.

Leo S. Bing Fellowship. Medicine. Provided for one year. Stipend, $1000.

George Blumenthal, Jr., Fellowships. Medicine. Supported by capital fund. Two; each $600.

Theodor Escherich Fellowship. Medicine. Supported by capital fund. Stipend, $600.

Moses Heineman Fellowship. Medicine. Supported by capital fund. Stipend, $600.

Eugene Meyer, Jr., Fellowships. Medicine. Supported by capital fund. Two; each $600.

Harry Sachs Fellowship (Neuro-Anatomy). Provided for one year. Stipend, $1000.

Harry Sachs Fellowship (Neuro-Pathology). Provided for one year. Stipend, $2500.

Harry and Samuel Sachs Fellowship. Medicine. Provided for one year. Stipend, $600.

National Committee for Mental Hygiene, 370 7th Ave., New York City.

Fellowship in Psychiatry. Open to holders of degree of Doctor of Medicine. Award available for traveling. For work in clinics conducted in different parts of the United States by the donor. Supported by Commonwealth Fund. Two; each $3000-$3500.

National Lime Association, Washington, D. C.

National Lime Association Fellowships. Industrial Chemistry. For research in the utilization of lime. At present these fellowships are represented in the following institutions: U. S. Bureau of Standards, Ohio State University, Indiana University, Massachusetts Institute of Technology. Four; each $600-$1800.

National Research Council, Washington, D. C.

NATIONAL RESEARCH FELLOWSHIPS.

Physics or Chemistry. In general open to citizens of the United States having the degree of Ph. D. or equivalent preparation. Total fund $500,000 for six years beginning 1919, contribution of The Rockefeller Foundation. Variable number at minimum stipend of $1800.

Medicine—any branch. Open to citizens of the United States or Canada who have the degree of M. D. or Ph. D. or equivalent qualifications. Total fund $500,000 for five years, beginning 1922, contribution of the General Education Board and The Rockefeller Foundation. Variable number at minimum stipend of $1800.

Biological Sciences—Botany, Zoology, Anthropology, Psychology. Open to citizens of the United States or Canada having the degree of Ph. D. or equivalent preparation. Total fund $325,000 for five years beginning 1923, contribution of The Rockefeller Foundation. Not more than $75,000 available in any one year. Variable number at minimum stipend of $1800.

These National Research Fellowships are awarded to individuals by the appropriate boards of the Council (Research Fellowship Board in Physics and Chemistry, Medical Fellowship Board, and Board of Fellowships in the Biological Sciences) for study and research in institutions chosen by the fellows and approved by these boards. They are awarded for one year with possibility of renewal. In applying address the particular board.

SULFUR FELLOWSHIPS. Agriculture—applications of sulfur. For research. Administered by the National Research Council through its Division of Biology and Agriculture. Open to residents of the United States or Canada. Extra funds provided for travel when necessary. Given by Texas Gulf Sulphur Company. Apply to A. G. McCall, University of Maryland, College Park, Md. Six; each $1000.

GERMINATION FELLOWSHIPS. Agriculture—field value of hard seeds of clovers and alfalfas. For research. Administered by the National Research Council through its Division of Biology and Agriculture. Given by American Seed Trade Association. Apply to Wm. Crocker, Thompson Institute for Plant Research, Yonkers, N. Y. Undetermined number, each at probable minimum stipend of $1000.

Nelson Morris Memorial Institute for Medical Research. Michael Reese Hospital, 29th St. and Ellis Ave., Chicago, Ill.

OTTO BAER FUND FELLOWSHIPS. Medicine, research. Open to holders of degree of M. D., Ph. D., or Sc. D. for work at the Michael

Reese Hospital. Capital fund $200,000. Annually available $12,000 to be distributed among half-time fellowships at stipend of $720 and full-time fellowships at stipend of $1800.

HERTZ FUND FELLOWSHIP. Chemistry, research. Open to holders of degree of M. D., Ph. D., or Sc. D. for work at the Nelson Morris Institute. Capital fund $30,000. Stipend, $1800.

KUPPENHEIMER FUND FELLOWSHIP. Bacteriology and Serology, research. Open to holders of degree of M. D., Ph. D., or Sc. D. for work at the Nelson Morris Institute. Capital fund $30,000. Stipend, $1800.

SNYDACKER FUND FELLOWSHIPS. Pathology, research. Open to holders of degree of M. D., Ph. D., or Sc. D. for work at the Nelson Morris Memorial Institute. Capital fund $30,000. Two; one with stipend of $1800 (the Hospital paying half), and one with stipend of $900 (the Hospital paying all).

New York University, New York City.

JOSEPH D. BRYANT MEDICAL SCHOLARSHIPS. Medicine. Open to graduates of New York University, preferably from Norwich, Chenango Co., N. Y. Founded by will of the late Professor Joseph D. Bryant. One or more; stipend, variable.

RICHARD H. BULL GRADUATE SCHOLARSHIP. Unrestricted. Founded by Richard H. Bull. Stipend, $45.

CLASS OF 1890 SCHOLARSHIP. Unrestricted. Available in College of Arts and Pure Science, College of Engineering, or Graduate School. Capital fund $1000, endowed by Class of 1890. Stipend, $100.

DURYEA FELLOWSHIP. Engineering. Open to members of the graduating class who have attained high standing in undergraduate course in engineering. Fellow must register for engineer's degree and serve as assistant in the College of Engineering for one year. Capital fund $4350, bequest of Samuel Bowne Duryea. Stipend, $195.

FACULTY GRADUATE SCHOLARSHIPS. Unrestricted. Used to supplement other scholarships in the Graduate School. Gift of the Faculty. Variable number; stipend, $400 maximum.

GRADUATE SCHOLARSHIPS FOR GRADUATES OF OHIO COLLEGES. Unrestricted. These include: William L. Strong Scholarship, Kenyon College; A. D. Juilliard Scholarships, Wooster University; S. Loeb Scholarship, University of Cincinnati; Henry M. MacCracken Scholarship, Wittenberg College. Four; each $100 maximum.

CHRISTIAN A. HERTER RESEARCH SCHOLARSHIP. Chemical Pathology, Physiological Chemistry, or Pharmacology. Open to graduates of New York University who have demonstrated aptitude for physiological and chemical research. Stipend, $300.

WILLIAM H. INMAN FELLOWSHIP. Science. Open to a student graduating in foremost third of his class. Fellow must register as candidate for Master's degree in the Graduate School and serve for one year in the laboratory of analytical chemistry. Capital fund $5000, gift of Miss Inman. Stipend, income.

EGBERT LEFEVRE SCHOLARSHIP. Medicine. Open to a member of the graduating class of Rutgers. Stipend, variable.

REMSEN GRADUATE SCHOLARSHIP. Unrestricted. Founded by R. G. Remsen. Stipend, $45.

UNIVERSITY FELLOWSHIPS. Unrestricted. Supported by University funds. Two at $1000 each; two at $800 each; and three at $500 each.

Newcomb (H. Sophie) Memorial College. See TULANE UNIVERSITY.

Nevada, University of, Reno, Nevada.

UNIVERSITY TEACHING FELLOWSHIPS. Chemistry, Physics, Biology. Candidates must be citizens of the United States. Maintained by general University fund. One appointment in each subject. Three; each $600.

North Carolina State College of Agriculture and Engineering, West Raleigh, N. C.

GYPSUM INDUSTRIES FELLOWSHIP. Research relative to use of Gypsum in Agriculture. Open to men, graduates of recognized colleges who have had scientific training dealing with problems of soil and plant nutrition. Maintained by the Gypsum Industries. Stipend, $1000.

Northwestern University, Evanston, Ill.

Medical fellowships and scholarships awarded by Medical School at Chicago, Ill.

ASSISTANTSHIPS. Astronomy, Botany, Chemistry, Geology, Physics and Zoology. Maintained by the University. Indeterminate number; each $100-$600.

MARCY SCHOLARSHIP IN BIOLOGY. For research. Use of a table at Marine Biological Laboratory, Woods Hole, Mass., is granted.

JAMES A. PATTEN FELLOWSHIPS IN MEDICAL RESEARCH. Medical Research. Open to students of the University. Capital fund $50,000, established by Mr. James A. Patten, of Evanston. Four; each $500.

PUBLIC HEALTH INSTITUTE FELLOWSHIPS IN CHEMISTRY. Chemistry. For research in organic mercury or organic arsenic. Supported by the Public Health Institute of Chicago. Ten; each $500-$600.

RUFUS H. SAGE SCHOLARSHIPS. Medicine. Founded by bequest of Mrs. Ellen Sage. Stipend, $150.

UNIVERSITY FELLOWSHIPS. Unrestricted. Appointment for one year. Fellows may be required to give limited assistance in work of their department. Maintained by the University. Exemption from tuition fee. Ten; each $400.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Maintained by the University. Exemption from tuition fee. Six; each $150.

CATHERINE M. WHITE SCHOLARSHIPS. Unrestricted. Founded by bequest of Catherine M. White, of Evanston, Ill. Exemption from tuition fee. Three; each $100.

Oberlin College, Oberlin, Ohio.

AELIOIAN FELLOWSHIP. Unrestricted. Open to women who are graduates of Oberlin, for study elsewhere in this country or abroad. Available 1923-24 and every third year. Founded by the Aelioian Association. Stipend, $500.

GILCHRIST-POTTER PRIZE FUND FELLOWSHIP. Unrestricted. Open to women who are graduates of Oberlin, for study elsewhere in this country or abroad. Founded on bequest of Mrs. Ella J. Gilchrist-Potter, of Alpena, Mich. Stipend, $300.

GRADUATE SCHOLARSHIPS. Unrestricted. Offered by the Trustees of the College. Sixteen; stipend, tuition.

GRASSELLI RESEARCH FELLOWSHIP. Chemistry. Maintained by the Grasselli Chemical Co. of Cleveland. Stipend, $500.

JOHNSTON FELLOWSHIP. Unrestricted. Open to women who are alumnae of the L. L. S. Association, for work elsewhere in this country or abroad. Available in 1923-24 and alternate years. Founded by the L. L. S. Association. Stipend, $500.

Ohio State University, Columbus, Ohio.

BUREAU OF MINES FELLOWSHIP. Mining, Metallurgical and Chemical Technology. Open to graduates of Ohio State University who have the Bachelor's or Master's degree or equivalent preparation. Maintained by the University, research directed by the Bureau of Mines. Two; exemption from tuition.

DUPONT FELLOWSHIP. Chemistry. Open to candidates for Ph. D. Given annually by E. I. duPont de Nemours & Co., of Wilmington, Del. Stipend, $750.

GRADUATE ASSISTANTSHIPS. Unrestricted. Open to graduates of approved universities, especially in Ohio. For three-quarters of the college year they are required to devote not more than one-half of their time to assisting in their department. During other quarter they are free to carry on their own work at the University or elsewhere. Established by the University. Several; each $500.

GRASSELLI CHEMICAL COMPANY FELLOWSHIP. Chemistry. Open to candidates for Doctor's degree, for research. Established by the Grasselli Chemical Company of Cleveland. Stipend, $750.

NATIONAL LIME ASSOCIATION FELLOWSHIP. Industrial Chemistry. Open to graduates in industrial chemistry who are candidates for Master's or Doctor's degree. Fellow must spend at least one-half his time in research on Lime. Granted by the National Lime Association, Washington, D. C. Stipend, $650.

STILLMAN W. ROBINSON FELLOWSHIP. Engineering, research. Open to graduates in mechanical, civil or electrical engineering who are candidates for Master's or Doctor's degree. Fellow devotes entire time to the work. Endowed by Stillman W. Robinson, late Professor of Mechanical Engineering. Stipend, $750.

THE SHEPHERD'S SCHOLARSHIP. Sheep Husbandry. Offered to a disabled soldier by the Philadelphia Wool and Textile Association. Stipend, $150.

UNIVERSITY FELLOWSHIPS. Unrestricted. Open to students who have a Master's degree. Fellows must spend whole time on the work, including research. Exemption from tuition fee. Maintained by the University. Variable number; each $500.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Scholars must spend all their time on the work, including research. Exemption from tuition fee. Maintained by the University. Variable number; each $300.

Oregon, University of, Eugene, Oregon.

GRADUATE ASSISTANTSHIPS. Unrestricted. Maintained by the University. About twenty; each $500.

Palmolive Company, Milwaukee, Wis.

PALMOLIVE FELLOWSHIP. Chemistry. Open to men who hold a Master's degree or the equivalent, for study of the fundamental principles connected with the detergent action of soap. Work to be done in any university fitted to carry out work of this character. Held at Cornell University during 1923-24. Established by the Palmolive Company. Professor Victor Lenher, University of Wisconsin, Madison, Wis., chairman of committee in charge of this fellowship. Stipend, $2000.

Pennsylvania, University of, Philadelphia, Pa.

All fellows and scholars are exempt from payment of tuition fees, but must pay matriculation, laboratory and graduation fees, and must make the usual deposits. Fellows for Research are exempt from the payment of tuition and laboratory fees and are not required to make the usual deposits.

AMERICAN FIELD SERVICE FELLOWSHIPS FOR FRENCH UNIVERSITIES. Fund set aside by Trustees of the University toward support of these fellowships. *See* Society for American Field Service Fellowships for French Universities.

JOSEPH M. BENNETT FELLOWSHIPS. Unrestricted. Open to women qualifying as candidates for Doctor's degree, for research. Capital fund $15,000. Founded by Col. Joseph M. Bennett. Two; each $225.

SAMUEL DICKSON FELLOWSHIP. Bacteriology. Open to women qualifying as candidates for Doctor's degree, for research in the William Pepper Laboratory of Clinical Medicine. Capital fund $100,000 (Fannie Hazard Dickson Memorial Fund), bequest of Samuel Dickson. Part of income appropriated as stipend. Stipend, $900.

JOHN F. FRAZER FELLOWSHIP. Physics. For research. Alternates with the Hector Tyndale Fellowship. Capital fund $12,000, gift of Francis Barton and others and increased by gift of Mrs. Thomas K. Conrad. Stipend, $1000.

ROBERT M. GIRVIN FELLOWSHIP. Medicine. For research in the John Herr Musser Department of Research Medicine. Capital fund $16,000, gift of Dr. John H. Musser, Jr. Stipend, $1200.

GEORGE LIEB HARRISON FELLOWSHIPS. Unrestricted. Open to men who have had one year of graduate work and have sight reading knowledge of French and German. To be held not more than two years. Ten; each $1000.

GEORGE LEIB HARRISON FELLOWSHIPS FOR RESEARCH. Unrestricted. Open to men for post-doctorate research. To be held not more than three years. Two; each $1500.

GEORGE LEIB HARRISON SCHOLARSHIPS. Unrestricted. For research. To be held not more than two years. Six; each $200.

The Harrison fellowships and scholarships are supported from the income of the George Leib Harrison Foundation for the Encouragement of Liberal Studies and the Advancement of Knowledge, established in 1895 by Dr. Charles C. Harrison, Provost of the University, 1894-1910. Capital fund now over $920,000; over $41,000 annually available.

HENRIETTA HECKSCHER MEMORIAL FELLOWSHIP IN MEDICAL RESEARCH. Medical Research. Capital fund over $12,000. Founded by Stevens Heckscher. Stipend, income.

GEORGE H. MCFADDEN FELLOWSHIP FOR STUDY IN DENMARK. Unrestricted. Open to a graduate student of the University of Pennsylvania or a member of the senior class. Awarded annually for a period of five years beginning with 1920. Established by George H. McFadden and his brother. One of the forty exchange fellowships administered by the American-Scandinavian Foundation. (See page 7.) Stipend, $1000.

BLOOMFIELD MOORE FELLOWSHIPS FOR WOMEN. Unrestricted. Open to women who purpose to become teachers. For research. Capital fund over $17,000. Founded by the late Mrs. Bloomfield Moore. Two; each $200.

FRANCES SERGEANT PEPPER FELLOWSHIP. Unrestricted. Open to a woman qualifying as candidate for Doctor's degree. For research. To be held not more than three years. Capital fund over $6000. Endowed by the late William Pepper, M. D. Stipend, $225.

ROBERT ROBINSON PORTER FELLOWSHIP. Medicine. For research in the John Herr Musser Department of Research Medicine. Capital fund over $14,000. Founded by the late Mrs. Harriet C. Prevost. Stipend, income.

THOMAS A. SCOTT FELLOWSHIP. Sanitary Science, research. Open to men who desire to become investigators or teachers in this field. Founded by Mrs. Thomas A. Scott. Stipend, $500.

HECTOR TYNDALE FELLOWSHIP. Physics. For research here or, with sanction of faculty, abroad. Awarded every other year alternating with the John F. Frazer Fellowship. To be held not more than three years. Stipend, $1000.

UNIVERSITY FELLOWSHIP FOR RESEARCH. Unrestricted. Created by the Corporation from time to time. Variable number. Exemption from tuition.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Open to graduates of American or foreign institutions. Thirty; exemption from tuition.

WISTAR INSTITUTE SCHOLARSHIPS. Neurology, Embryology and Genetics. Open to graduate students of the University of Pennsylvania who are preparing for a Doctor's degree. For research in the laboratories of the Wistar Institute of Anatomy and Biology. Established by General Isaac J. Wistar. One or more; each $1000-$3000.

George B. Wood Teaching Fellowship in Public Hygiene. Sanitary Science and Hygiene. Supported by the George B. Wood Auxiliary Faculty of Medicine Fund of over $390,000. Stipend, $700.

Woodward Fellowship in Physiological Chemistry. Physiological Chemistry. For research, work being prescribed by Director of William Pepper Laboratory of Clinical Medicine. Appointment for term of two years. Capital fund over $22,000. Established by Dr. George Woodward. Stipend, $1000.

Phi Mu Fraternity.

See under American Association of University Women.

Philippines, University of the, Manila, P. I.

Fellowships. Unrestricted. Open to men or women (Filipinos) who are graduates of and teachers in the University of the Philippines, and who have not yet received the Doctor's degree but have shown special fitness in the line of work in which they are engaged. For study abroad, generally in the United States. Provided under Act No. 2095 of the Philippine Legislature. Ten; stipend, $70 a month plus $100 clothing allowance.

Scholarships. Unrestricted. Open to men or women (Filipinos) who are graduates of the University of the Philippines, for graduate work in the United States. Provided under Act No. 2785 of the Philippine Legislature. Ten; stipend of $70 a month plus $100 clothing allowance.

Pittsburgh, University of, Pittsburgh, Pa. (See also Mellon Institute.)

Mellon Fellowships.

Medicine. Open to men who have degree of M. D., for study of electrocardiography and diseases of the heart at St. Francis Hospital, at which holder is resident. Stipend, $1200.

Pathology. For research in the Department of Pathology. Stipend, $1000.

Founded by Mr. R. B. Mellon.

Princeton University, Princeton, N. J.

All fellowships exempt from tuition fees and open to men only. Endowments of ordinary fellowships supplemented by University appropriation.

American-Scandinavian Travelling Fellowship. (Terminating with 1924-1925.) Unrestricted. An advanced fellowship, open to a Princeton student for study in Norwegian or Danish university, or for a Scandinavian student at Princeton. Founded by Henry Goddard Leach for four years beginning with 1921-1922. Stipend, $1500.

CLASS OF 1860 EXPERIMENTAL SCIENCE FELLOWSHIP. Physics. For research. Founded by the Class of 1860. Stipend, $700.

CLASS OF 1877 FELLOWSHIP IN BIOLOGY. Biology. For research. Founded by the Class of 1877. Stipend, $700.

CLASS OF 1883 FELLOWSHIP. Unrestricted. Two; each $700.

THEODORE CUYLER GRADUATE SCHOLARSHIP. Unrestricted. Founded by the late C. C. Cuyler. Stipend, $700.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Delaware. Stipend, $750.

E. M. BIOLOGICAL FELLOWSHIP. Biology. Use of a table at the Marine Biological Laboratory, Woods Hole, Mass., is granted. No stipend.

GORDON MACDONALD FELLOWSHIP. Unrestricted. Established by Mr. James Speyer. Stipend, $700.

HARVARD FELLOWSHIP IN CHEMISTRY. Chemistry. For research. Founded by one of the Fellows of Harvard University. Stipend, $700.

J. S. K. MATHEMATICAL FELLOWSHIPS. Mathematics. For research. Founded by a resident of New York City. One or two; each $700.

PORTER OGDEN JACOBUS FELLOWSHIP. Unrestricted. An advanced fellowship, for research. Open to a graduate student of Princeton of at least one year's standing. Endowed by Mrs. Clara Cooley Jacobus. Stipend, $1200.

FRANCIS HINTON MAULE BIOLOGICAL FELLOWSHIP. Biology. For research. Founded by Mr. and Mrs. Francis I. Maule. Stipend, $700.

ORSON DESAIX MUNN FELLOWSHIP IN ELECTRICAL ENGINEERING. Electrical Engineering. For research. Founded by Charles A. Munn and Mrs. Henry Norcross Munn. Stipend, $400.

CHARLOTTE ELIZABETH PROCTER FELLOWSHIPS. Unrestricted. Advanced fellowships, for research. Open to unmarried men who are graduates of not more than six years standing of Princeton or some other approved college. Fellows subject to reappointment for second year. Founded by William Cooper Procter. Ten; each $1200.

JANE ELIZA PROCTER VISITING FELLOWSHIPS. Unrestricted. Advanced fellowships. Open to unmarried men not over thirty years old who hold a degree in distinctively liberal studies from the University of Oxford, the University of Cambridge, or l'École Normale Supérieure of Paris, for resident advanced study and investigation in the Graduate School. One from each. Possibility of renewal for second or third year. Founded. Three; each $2000.

JAMES W. QUEEN GRADUATE SCHOLARSHIP. Physics. For research. Founded by bequest of Mrs. Abby S. Queen. Stipend, $350.

SAYRE FELLOWSHIP IN APPLIED CHEMISTRY. Chemistry. Open to a member of the graduating class of Princeton, for research. Founded by bequest of the late Dr. John Stanford Sayre. Stipend, $700.

SAYRE FELLOWSHIP IN APPLIED ELECTRICITY. Electrical Engineering. Founded by bequest of the late Dr. John Stanford Sayre. Stipend, $700.

THAW FELLOWSHIP IN ASTRONOMY. Astronomy. Founded by Mrs. William Thaw. Stipend, $700.

Puget Sound Biological Station.

See under WASHINGTON, UNIVERSITY OF (Graduate Scholarships for summer session).

Purdue University, Lafayette, Ind.

ASSISTANTSHIPS. Unrestricted. For part time assistance in the University, allowing sufficient free time to obtain Master's degree in two years. Stipend increased to $1000 the second year. Maintained by the University. Twenty to thirty; each $900.

Radcliffe College, Cambridge, Mass.

ANNA B. CLEMENTSON SCHOLARSHIP. Science. Open to women. Stipend, $400.

HARVARD ANNEX SCHOLARSHIP. Unrestricted. Open to women. Founded by alumnae and students of Harvard Annex and their friends. Stipend, $300.

EMILY STRAUSS SCHOLARSHIP. Unrestricted. Open to women. Stipend, $320.

TRAVELLING FELLOWSHIP. Science. Open to women. Founded by an anonymous donor. Stipend, $1500.

WHITNEY FELLOWSHIPS. Unrestricted. Open to women. Available for travelling or work elsewhere. Two; each $1500.

AUGUSTUS ANSON WHITNEY AND BENJAMIN WHITE WHITNEY SCHOLARSHIPS. Unrestricted. Open to women. Founded by the late Mrs. Abigail W. Howe, of Cambridge. Five at $250 each; three at $500 each.

Rensselaer Polytechnic Institute, Troy, N. Y.

SAGE FELLOWSHIPS. Unrestricted. Open to men who are candidates for Doctor's degree. Endowed by Mrs. Russell Sage. Capital fund $15,000 for each. Two; each income.

Rochester, University of, Rochester, N. Y.

COLUMBIA UNIVERSITY FELLOWSHIP. Engineering or Chemistry. Open to men who are graduates of the University of Rochester, for work at Columbia School of Mines. Maintained by Columbia University. Stipend, $250.

Rockefeller Foundation, New York City.

See under NATIONAL RESEARCH COUNCIL (National Research Fellowships).

Rutgers College, New Brunswick, N. J.

Fellows required to render some assistance in teaching or laboratory practice.

BLODGETT FELLOWSHIP IN SCIENCE. Science. Open to men who are graduates of Rutgers for advanced study in the Scientific School. Founded by James H. Blodgett, of Washington, D. C. Awarded when accumulated income amounts to $200. Exemption from fees. Stipend, $200.

SENOR DON BELTRAN MATHIEU CHILEAN NITRATE RESEARCH FELLOWSHIP. Agriculture. Open to men, for research in transformation and utilization of nitrate nitrogen in the soil. Part of work may be done elsewhere. Given by Chilean Nitrate Committee for three years. Stipend, $1000.

NIAGARA SPRAYER COMPANY FELLOWSHIP. Agriculture. Open to men, for study of the question of a sticker for insecticides in dust form. Part of work may be done elsewhere. Provided by Niagara Sprayer Co. Stipend, $2000.

ORGANIC NITROGEN RESEARCH FELLOWSHIP. Agriculture. Open to men, for research in organic nitrogen. Part of work may be done elsewhere. Established by George C. Furness, of New York. Stipend, $1000.

POTASH RESEARCH FELLOWSHIP. Agriculture. Open to men, for research in changes produced in soils receiving applications of calcium silicates. Part of work may be done elsewhere. Established by Eastern Potash Corporation of New York. Stipend, $1000.

SULFUR RESEARCH FELLOWSHIPS. Agriculture. Open to men, for research in sulfur problems. Part of work may be done elsewhere. Established by Jacob G. Lipman. Two; each $1200.

VANDER POEL FELLOWSHIP IN CHEMISTRY. Chemistry, or other Science. Open to men who are graduates of Rutgers College. Founded by the late John A. Vander Poel. Stipend, $600.

Society of Sigma Xi. Prof. Edward Ellery, Secretary, Union College, Schenectady, N. Y.

SIGMA XI FELLOWSHIPS. Science, other than physics, chemistry or medical sciences. Open to holders of Doctor's degree, for work anywhere best suited to needs. Maintained by contributions from Society members. Several; stipend, $1600 minimum.

Smith College, Northampton, Mass.

ALUMNAE FELLOWSHIP. Unrestricted. Open to a woman the year following graduation from Smith College. Available for traveling and work elsewhere. Supported by the Alumnae Association. Stipend, $500.

TRUSTEE FELLOWSHIPS. Unrestricted. Open to women, graduates of Smith, for work there or elsewhere, or to graduates of an approved institution for work at Smith. Remission of tuition fees when used at Smith. Under certain conditions available for traveling. Two of these fellowships may be divided into four scholarships of $250 each. Maintained by the Board of Trustees. Six; each $500.

Society for American Field Service Fellowships for French Universities, Dr. I. L. Kandel, Secretary, 525 W. 120th St., New York City.

AMERICAN FIELD SERVICE FELLOWSHIPS FOR FRENCH UNIVERSITIES. Unrestricted. Awards made for season 1923-24 in Engineering, Architecture, Chemistry and Medicine, among other subjects. Open to men for study in France. Sixteen; each $200 + 10,000 francs.

South Dakota State College, Brookings, S. D.

GRADUATE SCHOLARSHIPS. Agriculture. Open to men. Supported by college funds. Variable number; stipend, $200-$600.

Stanford University, Stanford University, Cal.

BERTHA HYDE BRALY SCHOLARSHIP. Unrestricted. Open to a woman student who is a resident of either Fresno County or Santa

Clara County. Established by Mrs. J. M. Braly, of Fresno. Stipend, $250.

Buckel Fellowship. Psychological Research. For the study of backward and feeble-minded children. Fund from the estate of Dr. C. Annette Buckel, supplemented by appropriation by the University. Stipend, $1000.

Cyril F. Elwell ('07) Scholarship. Electrical Engineering. Established by Mr. Cyril F. Elwell. Stipend, $500.

Fellowships for Study of Superior Children. Study of Superior Children. Supported jointly by Professor E. P. Cubberley and L. M. Terman and the Board of Trustees. Two; each $500.

Food Research Fellowships. Research in milling and baking technology, agricultural crises and depressions, or cooperative marketing of food products, preferably. Open to holders of Bachelor's or Master's degree who have had one or more years of graduate work in a university of high standing. For work at the Food Research Institute. Tuition free. Supported from the endowment funds of the Institute provided by Carnegie Corporation. Five; each $600-$1200.

Lionel Redmond Lenox Fellowship in Chemistry. Chemistry. For advanced study and research. Established by Lionel Redmond Lenox. Stipend, $600.

Petroleum Research Fellowship. Geology, Chemistry and Physics. Supported by Theodore J. Hoover. Stipend, $675.

Rixford Scholarship in Surgery. Surgery, original research, anatomical, physiological, pathological or clinical. Open to a graduate of Stanford Medical School. Established by Dr. Emmet Rixford. Stipend, $500.

Thomas Welton Stanford Fellowship in Psychic Research. Psychic Research. Supported by "Psychic Fund" created by Mr. Thomas Welton Stanford, of Australia. Stipend, $2500.

John Maxson Stillman Scholarship in Chemistry. Chemistry. Supported by Stillman Testimonial Fund, which was presented to Professor John Maxson Stillman by graduates and former students in chemistry, and others, on his retirement from active service in the University. Stipend, $150.

Teaching Fellowships in Chemistry. Chemistry. Established by Board of Trustees. Two; each $600.

University Fellowships. Unrestricted. For advanced study and research. Usually not awarded to students in their first year of graduate study. Established by Trustees of the University. Stipend covers three quarters. Eight; each $750.

University Scholarships. Unrestricted. Supported by funds of the University. Stipend covers three quarters. Several; each $200.

William R. Weaver Scholarship. Chemistry. Supported by Miss Eva Ray Weaver. Stipend, $150.

Swarthmore College, Swarthmore, Pa.

Hannah A. Leedom Fellowship. Unrestricted. Open to Swarthmore graduates of at least one year's standing. Available for traveling or work elsewhere. Founded by bequest of Hannah A. Leedom. Stipend, $400.

Joshua Lippincott Fellowship. Unrestricted. Open to Swarthmore graduates of at least one year's standing. Available for traveling or work elsewhere. Founded by Howard W. Lippincott. Stipend, $450.

John Lockwood Memorial Fellowship. Unrestricted. Open to Swarthmore graduates of at least one year's standing, preferably members of the Society of Friends. Available for traveling or work elsewhere. Founded by bequest of Lydia A. Lockwood, of New York. Stipend, $450.

Lucretia Mott Fellowship. Unrestricted. Open to women graduates of Swarthmore the year following graduation, for advanced work at some other approved institution. Founded by the Somerville Literary Society and sustained by contributions of its life members. Stipend, $525.

Martha E. Tyson Fellowship. Unrestricted. Open to women graduates of Swarthmore who have taught for two years after graduation and expect to continue teaching. For work in some other approved institution. Founded by the Somerville Literary Society and sustained by contributions of its life members. Stipend, $450.

Syracuse University, Syracuse, N. Y.

Graduate Assistantships Unrestricted. Supported by departmental budgets. Several; stipend, $500-$750.

Graduate Fellowships. Unrestricted (any liberal arts branch). Supported by the University. Exemption from tuition fees. Ten; each $500.

Texas, University of, Austin, Tex.

UNIVERSITY ADVANCED FELLOWSHIPS. Unrestricted. Open to graduates of at least one year's standing, preference being given to those qualified to do research. Maintained by the University. Exemption from fees. Four; each $500.

UNIVERSITY FELLOWSHIPS. Unrestricted. Maintained by the University. Exemption from fees. Two; each $250.

UNIVERSITY SCHOLARSHIPS. Unrestricted. Maintained by the University. Variable number; stipend, $100.

Trinity College, Hartford, Conn.

RUSSELL FELLOWSHIP. Unrestricted. Open to men who are graduates of Trinity College for two years' work. Available for traveling and work elsewhere. Capital fund $13,250. Stipend, $500.

TERRY FELLOWSHIP. Unrestricted. Open to men who are graduates of Trinity College. Available for traveling and work elsewhere. Capital fund $18,000. Stipend, $600.

Edward L. Trudeau Foundation, Trudeau Sanatorium, Trudeau, N. Y.

EDWARD L. TRUDEAU FOUNDATION FELLOWSHIPS. Medicine. For research in tuberculosis or related subjects at the Trudeau Sanatorium. Open to candidates who have a technical training in chemistry, physiology or pathology, preference being given to qualified physicians or students under treatment for tuberculosis. Supported by the Foundation. Five; each $800-$1000.

TRUDEAU SCHOOL OF TUBERCULOSIS SCHOLARSHIPS. Medicine. For research in tuberculosis or related subjects at the Trudeau Sanatorium. Open to candidates who have a technical training in chemistry, physiology or pathology, preference being given to qualified physicians or students under treatment for tuberculosis. Stipend covers tuition. Supported by the Foundation. Five; each $100.

Tufts College, Tufts College, Mass.

SCHOLARSHIPS. Biology, Chemistry, Civil and Structural Engineering, Mathematics, Physics. One in each department. Maintained by Trustees' gratuities giving free tuition. Five; each $100.

Tulane University of Louisiana, New Orleans, La.

ALUMNI TEACHING FELLOWSHIP. Unrestricted. Maintained by the University. Stipend, $250.

TEACHING FELLOWSHIPS. Unrestricted. Open to women who register as candidates for advanced degrees. Fellows devote a small part of their time to assistance in the department. Maintained by the University. Four; each $250.

United Daughters of the Confederacy.

See under VIRGINIA, UNIVERSITY OF.

U. S. Bureau of Mines, Department of the Interior, Washington, D. C.

FELLOWSHIPS OFFERED FOR GRADUATE WORK BY INSTITUTIONS OF LEARNING COOPERATING WITH THE UNITED STATES BUREAU OF MINES. Mining, Metallurgical and Chemical Technology. To undertake solution of problems being studied by the Bureau of Mines of especial importance to regions in which the institutions of learning are located. Research done under direction of the Bureau. Supported by the particular universities. Offered at present (1923-24) in Universities of Alabama, Arizona, Idaho, Utah and Washington, Carnegie Institute of Technology, Ohio State University. More than twenty; stipend, up to $810.

U. S. Bureau of Plant Industry, Department of Agriculture, Washington, D. C.

See under WISCONSIN, UNIVERSITY OF (Forest Products Laboratory assistantships and fellowships).

U. S. Bureau of Standards, Washington, D. C.

See under GYPSUM INDUSTRIES.

U. S. Forest Service, 928-930 F St., N W., Washington, D. C.

See under WISCONSIN, UNIVERSITY OF (Forest Products Laboratory assistantships and fellowships).

Utah, University of, Salt Lake City, Utah.

METALLURGICAL RESEARCH FELLOWSHIPS. (Bureau of Mines.) Mining Engineering. Open to men, preferably graduates of mining schools, who are candidates for Master's degree. Maintained by the University, research directed by the Bureau of Mines. Five; each $720.

SCHOOL OF MINES FELLOWSHIPS. Mining or allied subjects. Open to men. Maintained by the University. Two; each $350.

Vassar College, Poughkeepsie, N. Y.

FELLOWSHIP OF ASSOCIATE ALUMNAE OF VASSAR COLLEGE. Unrestricted. Open to women who are graduates of Vassar or instructors in Vassar. Available for work either in this country or abroad. Provided by the Associate Alumnae. Stipend, $500.

GRADUATE FELLOWSHIPS. Unrestricted. To be competed for by members of the senior class. For advanced study elsewhere. Maintained by the University. Four; each $500.

MARY PEMBERTON NOURSE FUND FELLOWSHIP. Public Health. Open to women, for work at some other approved institution. Supported by the Mary Pemberton Nourse Fund. Stipend, $1000.

MARY RICHARDSON AND LYDIA PRATT BABBOTT FELLOWSHIP. Unrestricted. Open to women who are graduates of Vassar or members of the senior class. Founded by Mrs. Lydia Pratt Babbott. Stipend, $500.

SUTRO FELLOWSHIPS. Unrestricted. Open to women who are graduates of Vassar of not more than five years' standing or members of the senior class. Capital fund $20,000, established by bequest of Adolph Sutro. Two; each $500-$700.

LOUISE HART VANLOON FELLOWSHIP. Unrestricted. Open to women who are graduates of Vassar. Capital fund $10,800. Established by Vassar Students' Aid Society. Stipend, $550.

WOMAN'S MEDICAL COLLEGE OF PENNSYLVANIA SCHOLARSHIP. Medicine. Open to women who are members of graduating class, for work at the Woman's Medical College of Pennsylvania, which offers the scholarship. Award made by faculty of Vassar. Exemption from tuition.

Vermont, University of, Burlington, Vt.

RESEARCH FELLOWSHIPS. Unrestricted. Maintained by the University. Renewable for a second year, stipend being then increased to $800. Stipend, $700.

Virginia, University of, Charlottesville, Va.

ALABAMA FEDERATION OF WOMEN'S CLUBS SCHOLARSHIP. Unrestricted. Open to graduates of Alabama colleges of recognized standing, who are residents of Alabama and in need of financial assistance. Appointment made on recommendation of Scholarship Committee of the Federation. Exemption from tuition.

ALUMNI SCHOLARSHIPS. Unrestricted. Financial need considered. Exemption from university fee and, for non-Virginians, tuition fees. Maintained by the General Alumni Association. Several; exemption from tuition.

THOMPSON BROWN SCHOLARSHIP. Unrestricted. Holder recommended by the donor. Founded. Stipend, $90.

ISAAC CARY SCHOLARSHIPS. Unrestricted. Financial need considered. Founded upon bequest of Isaac L. Cary, of Richmond. Two or more; stipend, $570 in all.

COLLEGE FELLOWSHIPS. Unrestricted. Open to graduates of selected colleges who enter the University of Virginia the session following graduation. For non-Virginians the stipend corresponds to tuition and one-half the university fee. Several, stipend, $180.

DUPONT FELLOWSHIP. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

ELIZABETH B. GARRETT FELLOWSHIP. Unrestricted. Founded upon bequest of Mrs. Elizabeth B. White, of Baltimore. Stipend, $280.

BENNET WOOD GREEN FELLOWSHIPS. Unrestricted. Open to graduates of the University of Virginia of not more than four years standing, preference being given to native Virginians and then to native residents of other Southern States. Candidates should hold degree of Master of Arts from the University of Virginia, or, in addition, Doctor of Medicine. Fellowships awarded for four years tenure. Founded upon bequest of Dr. Bennet Wood Green, of Warwick Co., Va. Two; each $600.

MCCORMICK SCHOLARSHIP. Unrestricted. Founded. Exemption from university fee. Exemption from tuition.

JOHN Y. MASON FELLOWSHIP. Unrestricted. Financial need considered. Founded upon gift of Colonel Archer Anderson, of Richmond, Va. Stipend, $350.

THOMAS FORTUNE RYAN SCHOLARSHIPS. Unrestricted. These fellowships, when possible, are distributed among the ten Congressional districts of Virginia. Founded by Mr. Thomas Ryan, of New York. Ten; each $300.

TEACHING FELLOWSHIPS IN CHEMISTRY. Chemistry. Fellows required to give not more than fifteen hours a week to instructional work. Supported by funds of the University. Five; each $500.

UNITED DAUGHTERS OF THE CONFEDERACY SCHOLARSHIPS. Unrestricted. Open to lineal descendants of Confederate veterans. Financial need considered. One award made in each of the following states on the recommendation of the U. D. C. therein: Alabama, Arkansas, California, Colorado, District of Columbia, Florida, Georgia, Illinois, Kentucky, Louisiana, Maryland, Mississippi, Missouri, North Carolina, Ohio, Oklahoma, South Carolina, Tennessee, Texas, Virginia, Washington, and West Virginia. Twenty-two; exemption from tuition.

VANDERBILT FELLOWSHIPS. Astronomy. Open to graduate students taking Astronomy as major subject. Must devote part of the time to work connected with the Observatory. Supported by funds of the University. Exemption from fees. Three; each $350.

Washington, State College of, Pullman, Washington.

R. C. MCCROSKEY RESEARCH FELLOWSHIP. Agriculture or General Science. Capital fund $5000, gift of Regent R. C. McCroskey, of the State College. Stipend, $300-$350.

STATE EXPERIMENT STATION GRADUATE RESEARCH FELLOWSHIP IN SOILS. Agriculture, soils. Maintained by the College. Stipend, $500.

STATE EXPERIMENT STATION RESEARCH FELLOWSHIP IN FARM MANAGEMENT. Agriculture. Maintained by the College. Stipend, $900.

STATE EXPERIMENT STATION RESEARCH FELLOWSHIP IN PLANT PATHOLOGY. Plant Pathology. Maintained by the College. Stipend, $360.

TEACHING FELLOWSHIP IN DAIRY HUSBANDRY. Dairy Husbandry. Maintained by the College. Stipend, $360.

TEACHING FELLOWSHIP IN GEOLOGY. Geology. Maintained by the College. Stipend, $450.

TEACHING FELLOWSHIP IN MECHANICAL ENGINEERING. Mechanical Engineering. Maintained by the College. Stipend, $750.

Washington, University of, Seattle, Washington.

BON MARCHE INDUSTRIAL FELLOWSHIP. Home Economics. Stipend, $600.

BUREAU OF MINES FELLOWSHIPS. Research in Mining, Metallurgy and Chemical Technology. Open to students who have a Master's degree or are candidates therefor. All honorably discharged service men or women of the late world war are exempt from fees up to $25 per quarter. Maintained by the University, research directed by the Bureau of Mines. Four; each $810.

ARTHUR A. DENNY FELLOWSHIP IN CIVIL ENGINEERING. Civil Engineering. Applicants should be residents of Washington State. Founded by the Denny family. Stipend, $500.

ARTHUR A. DENNY FELLOWSHIP IN MINING AND METALLURGY. Mining and Metallurgy. Applicants should be residents of Washington State. Founded by the Denny family. Stipend, $500.

ARTHUR A. DENNY FELLOWSHIP IN PHARMACY. Pharmacy. Applicants should be residents of Washington State. Founded by the Denny family. Stipend, $540.

LORETTA DENNY FELLOWSHIPS. Unrestricted. Capital fund $25,000, established by bequest of Sarah Loretta Denny. Three; each $500.

DUPONT FELLOWSHIP. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $700.

GRADUATE SCHOLARSHIPS.

Bacteriology. Exemption from tuition fees. Three; each $180.
Botany. Exemption from tuition fees. Stipend, $300.
Chemistry. Exemption from tuition fees. Three; each $450.
Forestry. Exemption from tuition fees. Stipend, $180.

These graduate scholarships are maintained by the University. Scholars give part time service.

GRADUATE SCHOLARSHIPS FOR SUMMER SESSION. Botany and Zoology. For six weeks work at Puget Sound Biological Station, Friday Harbor. Eleven at stipend of $75 each; one at stipend of $30.

TEACHING FELLOWSHIPS. Botany (2), Chemistry (2), Mathematics (4), Psychology (2), Zoology (2). Maintained by the University. Exemption from tuition fees. Twelve; each $540.

TEACHING FELLOWSHIP IN PHYSICS. Physics. Maintained by the University. Exemption from tuition fees. Four; each $720.

Washington University, St. Louis, Mo.

ASSISTANTSHIP IN CHEMISTRY. Chemistry. Maintained by funds of the University. Stipend, $750.

ASSISTANTSHIPS IN ZOOLOGY. Zoology. Maintained by funds of the University. Four; each $700-$800.

JESSIE R. BARR FELLOWSHIPS. Unrestricted. Open to women. Capital fund $50,000, bequest of the late Mrs. Jessie R. Barr (Mrs. William Barr). Six; each $416.

CENTURY ELECTRIC COMPANY RESEARCH FELLOWSHIP. Electrical Engineering. Open to men who are graduates in Electrical Engineering. One-half time is devoted to instruction and one-half to advanced study and research. Maintained annually by the Century Electric Company of St. Louis. Stipend, $725.

GRADUATE SCHOLARSHIPS IN THE SCHOOL OF ARCHITECTURE. Architecture. Maintained by funds of the University. Two; each $250.

RUFUS J. LACKLAND RESEARCH FELLOWSHIPS IN BOTANY. Botany. Offered annually by the Board of Trustees of the Missouri Botanical Garden. Five; each $500.

RESEARCH FELLOWSHIP IN PHYSICS. Physics. Maintained by funds of the University. Stipend, $750.

SURGICAL FELLOWSHIP. Neurosurgery. Maintained annually by anonymous individual. Stipend, $1200.

SURGICAL FELLOWSHIPS. Surgery, general. Maintained by funds of the University. Stipend, $600, $500, and $300.

TEACHING FELLOWSHIPS IN BOTANY. Botany. One-half time given to assistance in undergraduate courses and one-half to advanced study and investigation. Maintained by funds of the University. Two; each $500.

TEACHING FELLOWSHIP IN GEOLOGY. Geology. Maintained by funds of the University. Stipend, $500.

TEACHING FELLOWSHIPS IN PHYSICS. Physics. Maintained by funds of the University. Three; each $750.

UNION ELECTRIC LIGHT AND POWER COMPANY RESEARCH FELLOWSHIP. Electrical Engineering. Open to men who are graduates in Electrical Engineering. One-half time is devoted to instruction and one-half to advanced study and research. Maintained annually by the Union Electric Light and Power Company of St. Louis. Stipend, $725.

Wellesley College, Wellesley, Mass.

FELLOWSHIP FOR STUDY OF ORTHOPEDICS. Orthopedics in relation to Physical Education. Open to women. Annual gift. Stipend, $1000.

GRADUATE SCHOLARSHIPS. Unrestricted. Open to women. Established by the Trustees. Stipend covers tuition. Eighteen; each $300.

HORTON-HALLOWELL FELLOWSHIP. Unrestricted. Open to women who are graduates of Wellesley, for travel in connection with serious study. Fund established by Alumnae Association. Stipend, $1000.

ALICE FREEMAN PALMER FELLOWSHIP. Unrestricted. Open to women who have done some graduate work, for travel in connection with serious study. Founded. Stipend, $1000.

Wesleyan University, Middletown, Conn.

KATHRIN MILLER CADY SCHOLARSHIP. Physics. Not always awarded. Capital fund over $2000. Stipend, income.

Western Reserve University, Cleveland, Ohio.

G. W. Crile Research Fellowships.

Surgery. Open to Doctors of Medicine, residents of Cleveland, as a traveling fellowship. Capital fund of $100,000 divided to cover this and fellowships in Medicine. Stipend, $2500.

Medicine, any branch. According to discretion of a Faculty Committee this stipend is divided. Two or more; stipend, in all $2500.

Williams College, Williamstown, Mass.

Horace F. Clark Prize Scholarships. Unrestricted. Open to men who are members of the senior class at Williams College. For graduate study and research leading to a Master's or Doctor's degree in a University having a recognized graduate school. May be continued for more than one year. In exceptional cases may be combined for one scholarship of $1000. Founded by bequest of Mme. Marie Louise Souberbeille. Two; each $500.

Wisconsin, University of, Madison, Wis.

In awarding fellowships preference is given to candidates who have competent reading knowledge of French and German. Fellows may be required to give a small amount of time to instructional duties.

Charles R. Crane Fellowship. Chemistry, research. Established by Charles R. Crane. Stipend, $1500.

duPont Fellowship. Chemistry. Given annually by E. I. duPont de Nemours & Co., Wilmington, Del. Stipend, $750.

Forest Products Laboratory Assistantships. Forestry. Fellows appointed by Forest Service and Bureau of Plant Industry. Entire time is given to lecture or class work and research in the Forest Products Laboratory, in which laboratory fees are remitted. Several; each $200-$500.

Forest Products Laboratory Fellowships. Forestry. Fellows appointed by Forest Service and Bureau of Plant Industry under Civil Service regulations. Exemption from laboratory fees at the Forest Products Laboratory. Two; each $200-$500.

Fritzche Brothers Fellowship in Pharmacy. Pharmacy. For research in plant chemistry with special reference to the Monardas and related problems. Given annually by the Fritzche Brothers. Stipend, $500.

Graduate Scholarships. Agriculture (6), Engineering (3), among other subjects not scientific. The faculties of the following colleges have the privilege of nominating one candidate each from their senior classes: Beloit, Carroll, Lawrence, Milton, Milwaukee-Downer, Ripon and St. Clara. Offered by the Regents of the University. Nine; each $250.

HOLLISTER FELLOWSHIP IN PHARMACY. Pharmacy. Open to a student recommended by the Department of Pharmacy. Capital fund over $10,000, established by separate bequests of Mr. and Mrs. Albert H. Hollister, of Madison. Stipend, income.

HONORARY FELLOWSHIPS. Unrestricted. Open to persons who have done at least one year's graduate work and have held academic honors such as fellowships or research or teaching appointments. No teaching service required. Established by the Regents of the University. Twenty-six; stipend, tuition.

INTERNATIONAL MILK DEALERS ASSOCIATION FELLOWSHIP. Agriculture. Given annually by the International Milk Dealers Association. Stipend, $600.

A. J. MARSCHALL FELLOWSHIP. Pharmacy. Founded by A. J. Marschall. Stipend, $500.

MEAD JOHNSON & CO. INDUSTRIAL FELLOWSHIP. Agricultural Chemistry. Given annually by the Mead Johnson Co., with $800 additional for equipment. Stipend, $2500.

NATIONAL FERTILIZER ASSOCIATION INDUSTRIAL FELLOWSHIP. Chemistry. For research in relation to soils. Given for a term of three years by the National Fertilizer Association of Chicago and Baltimore. Stipend, $2000.

NEWPORT CHEMICAL CO. FELLOWSHIP. Pharmacy. For research in synthetic new remedies. Given annually by the Newport Chemical Co. One or more; each $1000.

OMICRON NU, WISCONSIN CHAPTER, SCHOLARSHIP. Unrestricted. Maintained by the Wisconsin Chapter of Omicron Nu. Stipend, $200-$250.

SCHOLARSHIPS FOR FRENCH YOUNG WOMEN. Unrestricted. Open to women of French nationality who have the equivalent of a Bachelor's degree. Exemption from tuition fees. Two; each $500.

UNIVERSITY FELLOWSHIPS. Geology and Biological Sciences (2), Engineering (2), Agriculture (4), 1 each in Botany, Chemistry, Physics, Mathematics and Astronomy, Philosophy and Psychology, and Home Economics, among other subjects not scientific. Maintained by the University. Fourteen; each $500.

WISCONSIN GAS ASSOCIATION FELLOWSHIP. Gas Engineering, research. Given annually by the Wisconsin Gas Association. Stipend, $500.

WISCONSIN UTILITIES ASSOCIATION FELLOWSHIPS. Electrical Engineering, and Gas Engineering. Given annually by the Wisconsin Utilities Association, one in each subject. Two; each $500.

Woods Hole, Mass., Marine Biological Laboratory.

LUCRETIA CROCKER SCHOLARSHIP FOR TEACHERS IN BOSTON. Biology. Open to teachers in Boston. Use of a table, for research. Supported by income of a fund established by friends of the Laboratory in Boston.

NEW LONDON BRANCH OF THE AMERICAN ASSOCIATION OF UNIVERSITY WOMEN SCHOLARSHIP. Biology. Open to students of Connecticut College for Women. Use of a table, for research. Supported by annual gift of New London Branch of the American Association of University Women.

SCHOLARSHIP OF $100. Biology. Use of a table, for research. Supported by yearly contribution privately made.

See also NORTHWESTERN UNIVERSITY (Marcy Scholarship in Biology); PRINCETON UNIVERSITY (E. M. Biological Fellowship).

A list of institutions which have tables for use of their nominees is printed each year in the Laboratory's Annual Announcement.

Yale University, New Haven, Conn.

In general, fellowships are reserved for students who have already done graduate work and who are candidates for a degree. In cases of equal merit preference is given to applicants in residence at Yale.

(*) refers to fellowships which may be combined to yield an income for several fellowships of $800 and $500 each, which are awarded only to students of the highest attainments.

JOHN J. ABERNETHY FELLOWSHIP.* Unrestricted. Open to graduates of Yale College. Capital fund $10,000, bequest of Dr. John Jay Abernethy. Stipend, income.

BENJAMIN F. BARGE SCHOLARSHIP. Unrestricted. Occasionally awarded as a graduate scholarship to graduates of Yale College. Capital fund $5000 from Benjamin F. Barge. Stipend, income.

MARSHALL S. BIDWELL FELLOWSHIP.* Unrestricted. Open to graduates of Yale College. Fellow is to pursue non-professional studies in New Haven. Fellowship may be held for not more than two consecutive years. Capital fund $10,000, bequest of Benjamin Douglas Silliman. Stipend, income.

Bishop Museum Fellowships. Anthropology, Botany, Zoology, Geology or Geography. Open to students who have completed at least one year of graduate work at an institution of high standing. Preference is given to those who have a Doctor's degree. For study and research in the islands of the Pacific, the results of which must be submitted to the Bishop Museum for publication. Established under an agreement between The Bernice Pauahi Bishop Museum, of Hawaii, and Yale University. Four; each $1000.

Cheney Fellowship. Chemistry, organic. Awarded to a student who has shown special proficiency in Chemistry and Biochemistry, and demonstrated his ability to pursue research for the doctorate. Research shall be on some problem the solution of which will advance the knowledge of the chemistry of silk, and substances and processes used in silk industry. May not be held for a longer period than two years. Gift of the Cheney Brothers, silk manufacturers, S. Manchester, Conn. Stipend, $1000.

Clark Scholarship. Unrestricted. Awarded to the applicant from the Senior Class of Yale College who has attained highest rank in the studies of the course. For resident and non-professional study. Established by a gift of $1000 from Mr. Sheldon Clark of Oxford, Conn. and allowed to accumulate until it reached the sum of $4000. Stipend, income.

Carroll Cutler Fellowship. Unrestricted. Open to men. Capital fund over $11,000, bequest of Mrs. Frances E. Cutler. Stipend, income.

Susan Rhoda Cutler Fellowship. Unrestricted. Open to women. Capital fund over $11,000, bequest of Mrs. Frances E. Cutler. Stipend, income.

Cuyler Fellowship.* Unrestricted. Open to graduates of Yale College. Capital fund $10,000 established by a gift from Thomas DeWitt Cuyler, Cornelius C. Cuyler and Miss Eleanor deGraff Cuyler. Stipend, income.

James Dwight Dana Fellowship. Geology. Preference given to a graduate of Yale College or Sheffield Scientific School. With approval of the Professors of Geology, Fellow may pursue his studies outside of New Haven. Fellowship may be held not more than two years. Founded upon the James Dwight Dana Memorial Fund, one-half of the income from which is devoted to this purpose, the other half being used to promote original research by the members of the Geological Faculty or by advanced students. Stipend, $500.

DOUGLAS FELLOWSHIP.* Unrestricted. Open to graduates of Yale College, for pursuit of non-professional studies in New Haven. May be held not more than three years. Capital fund $10,000 established by a gift from Mrs. Samuel Miller of New Haven. Stipend, income.

DUPONT FELLOWSHIP IN CHEMISTRY. Chemistry. Open to men who give promise of ability to conduct original research in Chemistry and who are candidates for Doctor's degree. Given annually by E. I. duPont de Nemours & Co., Wilmington, Delaware. Stipend, $750.

DANIEL C. EATON GRADUATE SCHOLARSHIP IN BOTANY. Botany. Open for competition only to members of the senior classes in Yale College and the Sheffield Scientific School. Capital fund $2000, established by the gift of Mrs. Caroline K. Eaton, now amounting to almost $5000. Stipend, income.

ELLEN BATTELL ELDRIDGE FELLOWSHIPS.* Unrestricted. Open to graduates of Yale College for resident study in New Haven. May be held not longer than three years. Capital fund $12,000 each, bequest of Mrs. Azariah Eldridge of Yarmouth, Mass. Two; stipend, income.

FOOTE FELLOWSHIPS.* Unrestricted. Open to graduates of Yale College, who must pursue graduate studies in New Haven. Capital fund $25,000, established by a bequest from Harry W. Foote of New Haven. Two or more; stipend, income.

ARTHUR TWINING HADLEY SCHOLARSHIP. Unrestricted. Awarded to assist students of the University in the publication of meritorious dissertations or results of investigations. Stipend sometimes divided. Stipend, $225.

HONORARY FELLOWSHIPS IN BIOCHEMISTRY. Biochemistry. Open to candidates for Ph. D. in organic chemistry or biochemistry who have shown special proficiency in these subjects and have demonstrated ability to pursue research for the doctorate.

AUSTIN F. HOWARD SCHOLARSHIPS. Unrestricted. Preference given to graduates of Yale College. Capital fund $10,000, established by a bequest from James T. Howard, of St. Johnsbury, Vermont. One or more; stipend, income.

HUNTINGTON FELLOWSHIP. Geography. Preference given to candidates for Ph. D., for which main part of the work is to be preparation of dissertation. Gift of Henry Strong Huntington. Stipend, $500.

SCOTT HURTT FELLOWSHIP.* Unrestricted. Open to graduates of not more than four years standing, of Yale College. May be held for three years, one of which may be spent in New Haven and the other two at any foreign university or the American School in Athens or Rome. Capital fund $12,000, established by a gift from Mrs. Sarah I. Hurtt of New York City. Stipend, income.

JOSEPH PAXON IDDINGS SCHOLARSHIP IN PETROLOGY. Petrology, research. Open to students competent to carry on research in Petrology. Capital fund over $25,000, gift of Mrs. Estelle Iddings Cleveland to the Sheffield Scientific School. Stipend, $500.

LARNED FELLOWSHIPS.* Unrestricted. Open to graduates of Yale College for pursuit of graduate studies in New Haven. Capital fund of each $7000, established by a bequest from Mrs. Irene Larned of New Haven, and augmented by a bequest from Mrs. Urania B. Humphrey of Norfolk, Conn. Three; stipend, income.

HENRY BRADFORD LOOMIS FELLOWSHIP IN CHEMISTRY. Chemistry. Open to graduates of Yale College and the Sheffield Scientific School, and to graduates of other institutions who have spent at least one year in the study of Chemistry in the Graduate School of Yale University. Granted to the candidate who passes the best competitive examination in Chemistry. Holder must be candidate for Ph. D. and make Chemistry his chief study. Capital fund $10,000, gift of Henry Bradford Loomis. Stipend, income.

LOOMIS FELLOWSHIP IN PHYSICS. Physics. Open to recent graduates of Yale College and Sheffield Scientific School and to graduates of other institutions who have spent one year in study of Physics in Yale Graduate School. Awarded to candidate passing best competitive examination in Physics. Holder must be candidate for Ph. D. and make Physics his chief study. Capital fund $10,000, gift of Francis E. Loomis. Stipend, income.

EDLA J. MCPHERSON FUND. Unrestricted. Income used in part in paying Graduate School fellowships and scholarships not provided for by any foundation. Capital fund $260,000, bequest of Mrs. Edla J. McPherson. Stipend, income.

MACY FELLOWSHIP.* Unrestricted. Open to graduates of Yale College. May be held for three years, for pursuit of non-professional studies in New Haven. Capital fund $10,000, established by a bequest from Rev. William A. Macy. Stipend, income.

PLAINFIELD SCHOLARSHIP. Unrestricted. Income used to help worthy students from the County of Windham, Conn., and preferably from the town of Plainfield. Occasionally awarded for graduate work. Gift of Edwin Miller of Plainfield, Conn. Stipend, $500.

Henry C. Robinson Fellowship. Unrestricted. Founded by gift of $5000, in the name of Mrs. Mary Robinson Cheney, of Hartford, Conn., and her sister, Miss Eliza Robinson. Stipend, income.

Seessel Fellowships. Biology. Preference given to men or women who have obtained their Doctorate and are able to carry on original research in one of the three departments of Biological studies: Physiology (including Physiological Chemistry), Zoology, and Botany. Established from the income of the Theresa Seessel Fund given by Dr. Albert Seessel. Usually two; stipend, $1500 each.

Silliman Fellowship.* Unrestricted. Open to graduates of Yale College. Recipient may hold the fellowship not more than three years. Capital fund $10,000. Stipend, income.

John Sloane Fellowship in Physics.* Physics. Open to graduates of Yale College for resident study in New Haven. Recipient acts as assistant in Sloane Physics Laboratory. May hold the fellowship not more than three years. Capital fund $10,000, gift from John Sloane, of New York. Stipend, income.

University Fellowships. Unrestricted. Preference given to those who have already spent at least one year in graduate study and have shown capacity for original work. Supported by income of various funds including Nathaniel Currier Scholarship Fund. Ten; each $350-$500.

University Scholarships. Unrestricted. Maintained by the University. About forty; exemption from tuition.

Thomas Glasby Waterman Scholarship. Unrestricted. Occasionally awarded to graduates of Yale College of not more than two years' standing. Income given to not more than three scholars. Capital fund $40,000 from estate of Thomas Glasby Waterman. Stipend, income.

LIST OF FELLOWSHIPS BY SPECIAL NAMES

SHOWING INSTITUTIONS UNDER WHICH THEY ARE DESCRIBED

Abernethy Fellowship (Yale University)
Acme White Lead and Color Works Fellowship (Michigan, University of)
Aelioian Fellowship (Oberlin College)
Alabama Federation of Women's Clubs Scholarship (Virginia, University of)
American Association of University Women European Fellowship (American Association of University Women)
American Association of University Women International Fellowship (American Association of University Women)
American Field Service Fellowships for French Universities (Pennsylvania, University of; Society for American Field Service Fellowships for French Universities)
American-Scandinavian Foundation Exchange Fellowships (American-Scandinavian Foundation)
American-Scandinavian Foundation Travelling Fellowship (Princeton University)
Ames Scholarship (Harvard University)
Appleton Travelling Fellowship (Harvard University)
Arnold Biological Fellowship (Brown University)
Associated Radiograph Laboratories Fellowship (California, University of)
Austin Fellowships (Harvard University)
Austin Fund (Massachusetts Institute of Technology)
Austin Research Fellowships (Massachusetts Institute of Technology)
Austin Scholarship in Landscape Architecture (Harvard University)
Austin Scholarships in Architecture (Harvard University)
Austin Teaching Fellowships (Harvard University)

Baer Fund Fellowships (Nelson Morris Memorial Institute for Medical Research)
Baldwin Fellowships (Cincinnati, University of)
Bardwell Memorial Fellowship (Mount Holyoke College)
Barge Scholarship (Yale University)
Barnard Fellowship (Columbia University)
Barnard Memorial Fellowship in Chemistry (Chicago, University of)
Barr Fellowships (Washington University, St. Louis)
Barringer Scholarships Nos. 1 and 2 (Harvard University)
Bartlett (Gordon) Scholarship (Harvard University)
Bartlett (Matthew and Mary E.) Scholarship (Harvard University)
Base Hospital No. 30 Scholarship (California, University of)
Bennet Fellowships (Pennsylvania, University of)
Berliner Research Fellowship (American Association of University Women)
Bidwell Fellowship (Yale University)
Billings Scholarship (Harvard University)
Bing Fellowship (Mount Sinai Hospital)
Bishop Museum Fellowships (Yale University)
Bliss Scholarships (Harvard University)

Blodgett Fellowship (Rutgers College)
Blumenthal Fellowships (Columbia University; Mount Sinai Hospital)
Blumenthal Scholarships (Columbia University)
Bon Marché Industrial Fellowship (Washington, University of)
Boring Fellowship in Architecture (Columbia University)
Boston Alumnae Fellowship (American Association of University Women)
Bradford Fellowship (Harvard University)
Braly Scholarship (Stanford University)
Braun Fellowship (Michigan, University of)
Bridgham Fellowship (Columbia University)
Bright Scholarship (Harvard University)
Brooks Fellowship (Cornell University)
Brown Fund (Massachusetts Institute of Technology)
Brown Scholarship (Virginia, University of)
Browne Scholarship (Harvard University)
Bruce Fellowship (Johns Hopkins University)
Bryant Medical Scholarships (New York University)
Buck Fellowships (Boston University)
Buckel Fellowship (Stanford University)
Buckley Scholarships (Harvard University)
Bull Graduate Scholarship (New York University)
Bullard Fellowships: (a) Shattuck Memorial (b) John Ware Memorial (c) Charles Eliot Ware Memorial (Harvard University)
Burchenal Fellowship (Cincinnati, University of)
Bureau of Mines Fellowships (Alabama, University of; Arizona, University of; Carnegie Institute of Technology; Idaho, University of; Missouri, University of; Ohio State University; U. S. Bureau of Mines; Utah, University of; Washington, University of)
Bureau of Personnel Research Fellowships (Carnegie Institute of Technology)
Burton Scholarship (Harvard University)
Butler Scholarship (Columbia University)

Cabot Fellowships (Harvard University)
Cady Scholarship (Wesleyan University)
Callender-Cornell Fellowship (Lehigh University)
Carnegie Fellowships (Carnegie Institute of Technology)
Carr Fellowship in Chemistry (Illinois, University of)
Cary Scholarships (Virginia, University of)
Case Fellowships (Case School of Applied Science)
Century Electric Co. Research Fellowship (Washington University)
Chamberlin Fellowship (Dartmouth College)
Chapin Fellowship in Highway Engineering (Michigan, University of)
Chapin Fellowship in Highway Transport (Michigan, University of)
Chapman Memorial Fellowship (Harvard University)
Cheney Fellowship (Yale University)
Choate Memorial Fellowship (Harvard University)
Clark Prize Scholarships (Williams College)
Clark Scholarship (Columbia University)
Clark Scholarship (Yale University)
Claypole Memorial Research Fellowship (California, University of)
Clementson Scholarship (Radcliffe College)

Coffin Fellowships (Charles A. Coffin Foundation)
Cole Fellowship (Michigan, University of)
Collamore Fund (Massachusetts Institute of Technology)
Columbia University Fellowship (Rochester, University of)
Coman Research Fund Fellowship (Chicago, University of)
Crane Fellowship (Wisconsin, University of)
Crile Research Fellowships (Western Reserve University)
Crocker Scholarship (Woods Hole, Massachusetts, Marine Biological Laboratory)
Cummings Scholarship (Harvard University)
Curtis University Scholarships (Columbia University)
Cutler (Carroll) Fellowship (Yale University)
Cutler (Susan Rhoda) Fellowship (Yale University)
Cutting Traveling Fellowships (Columbia University)
Cuyler Fellowship (Yale University)

Dalton Fund (Massachusetts Institute of Technology)
Dana Fellowships (Yale University)
Davis Teaching Fellowship (Cincinnati, University of)
Denny (Arthur) Fellowships (Washington, University of)
Denny (Loretta) Fellowships (Washington, University of)
Derby Scholarship (Harvard University)
Detroit Edison Fellowship in Highway Engineering (Michigan, University of)
Dickson Fellowship (Pennsylvania, University of)
Doe Scholarship (Harvard University)
Dorr Fellowships (Minnesota, University of)
Douglas Fellowship (Yale University)
Draper Scholarships (Boston University)
Drury Scholarships (Case School of Applied Science)
DuBois Fellowship (Columbia University)
duPont Fellowships (Brown University; California, University of; California Institute of Technology; Chicago, University of; Columbia University; Cornell University; E. I. duPont de Nemours & Company; Harvard University; Illinois, University of; Johns Hopkins University; Lafayette College; Massachusetts Institute of Technology; Michigan, University of; Minnesota, University of; Ohio State University; Princeton University; Virginia, University of; Washington, University of; Wisconsin, University of; Yale University)
duPont Scholarship (Delaware, University of; E. I. duPont de Nemours & Company)

E. M. Biological Fellowship (Princeton University)
Earle Memorial Fellowship (Cornell University)
Eaton Graduate Scholarship (Yale University)
Edwards Fellowship (Brown University)
Eldridge Fellowships (Yale University)
Eliot Travelling Fellowship in Landscape Architecture (Harvard University)
Elwell Scholarship (Stanford University)
Emerson Scholarships (Harvard University)
Emmons Geological Fellowship (Columbia University)
Escherich Fellowship (Mount Sinai Hospital)
Eveleth Scholarship (Harvard University)
Everett Scholarship (Bowdoin College)

Farnham Scholarship (Harvard University)
Ferguson Fellowship (Columbia University)
Firestone Tire and Rubber Co. Fellowship (Akron, Municipal University of)
Fleischmann Fellowship (Chicago, University of)
Folsom Teaching Fellowship (Harvard University)
Foote Fellowships (Yale University)
Forest Products Laboratory Assistantships (Wisconsin, University of)
Forest Products Laboratory Fellowships (Wisconsin, University of)
Foster Memorial Scholarship (California, University of)
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Frazer Fellowship (Pennsylvania, University of)
Friedberg Fellowship (Chicago, University of)
Fritzche Brothers Fellowship (Wisconsin, University of)
Furfural Fellowship (Crop Protection Institute)

Garrett (Elizabeth B.) Fellowship (Virginia, University of)
Garrett (Mary Elizabeth) Fellowship (Bryn Mawr College)
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Goldwin Smith Fellowship (Cornell University)
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Harvard Engineering Society of New York City Scholarship (Harvard University)
Harvard Fellowship in Chemistry (Princeton University)
Hayden Scholarship (Harvard University)
Heckscher Memorial Fellowship in Medical Research (Pennsylvania, University of)

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International Milk Dealers Association Fellowship (Cornell University; Wisconsin, University of)
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Jacobus Fellowship (Princeton University)
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LeFevre Scholarship (New York University)

Lenox Fellowship (Stanford University)
Lick Observatory Fellowships (California, University of)
Lincoln Scholarship (Harvard University)
Linder Scholarship (Harvard University)
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Lockwood Fellowship (Swarthmore College)
Loewenthal Fellowship (Chicago, University of)
Logan Research Fellowships (Chicago, University of)
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McCormick Scholarship (Virginia, University of)
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Meyer (Edgar J.) Memorial Fellowship (Cornell University)
Meyer (Eugene, Jr.) Fellowship (Mount Sinai Hospital)
Mitchell Fellowship (Columbia University)
Moore Fellowships for Women (Pennsylvania, University of)
Moore Scholarship Fund (Massachusetts Institute of Technology)
Morgan Edwards Fellowship (Brown University)
Moseley Travelling Fellowships (Harvard University)
Mowlson Scholarship (Harvard University)
Mott Fellowship (Swarthmore College)
Munn Fellowship (Princeton University)
Murphy Scholarship (Harvard University)

National Aniline and Chemical Company Fellowship (Michigan, University of)
National Association of Manufacturers of Carbonated Beverages, Fellowship of, (Iowa State College)
National Canners Association Fellowship (Cornell University)
National Fertilizer Association Industrial Fellowship (Wisconsin, University of)
National Lime Association Fellowships (Indiana University; Masachusetts Institute of Technology; National Lime Association; Ohio State University)
National Research Fellowships (National Research Council)
New London Branch of A. A. U. W. Scholarship.
Newport Chemical Co. Fellowship (Wisconsin, University of)

Niagara Sprayer Co. Fellowship (Rutgers College)
Nourse Fund Fellowship (Vassar College)

Oliver Scholarship (Harvard University)
Omicron Nu, Wisconsin Chapter, Scholarship (Wisconsin, University of)
Organic Nitrogen Research Fellowship (Rutgers College)

Pacific Coast Gas Association Fellowship (California, University of)
Paige Scholarship (Dartmouth College)
Palmer Fellowship (Wellesley College)
Palmer Memorial Fellowship (American Association of University Women)
Palmolive Fellowship (Palmolive Co.)
Parker Fellowship (Harvard University)
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Phi Mu Fellowship (American Association of University Women)
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Prime Scholarship (Brooklyn Institute of Arts and Sciences)
Princeton-Leach Fellowship (American-Scandinavian Foundation)
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Procter (Charlotte Elizabeth) Fellowships (Princeton University)
Procter (Jane Eliza) Visiting Fellowships (Princeton University)
Proudfit Fellowship (Columbia University)

Queen Graduate Scholarship (Princeton University)
Quincy Fund Scholarship (Johns Hopkins University)

Ransohoff Fellowship (Cincinnati, University of)
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Richards Memorial Research Fund (Massachusetts Institute of Technology)
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Roberts (Chamberlain) Fellowships (Columbia University)
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Rosenburg Memorial Scholarship (California, University of)
Rotch Travelling Scholarship (Boston Society of Architects)
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Ryan Scholarships (Virginia, University of)

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Sheldon Fund for Travelling Fellowships (Harvard University)
Shepherd's Scholarship (Ohio State University)
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Sibley Fellowship (Cornell University)
Sidgwick Memorial Fellowship (American Association of University Women)
Sigma Xi Fellowships (Society of Sigma Xi)
Silliman Fellowship (Yale University)
Simpson Fellowships and Lectureships (Amherst College)
Sleeper Fellowship (Boston University)
Sloane Fellowship (Yale University)
Smith (Goldwin) Fellowship (Cornell University)
Smith (Eugene Hanes) Scholarship (Harvard University)
Smyth Fund Scholarship (Dartmouth College)
Snydacker Fund Fellowships (Nelson Morris Memorial Institute for Medical Research)
Social Hygiene Board Fellowships (Illinois, University of)
Spanish Fellowship (American Association of University Women)
Speed Graduate Scholarship (Louisville, University of)
Stearns and Company Fellowship (Michigan, University of)
Stillman Scholarship (Stanford University)
Strauss (Emily) Scholarship (Radcliffe College)
Strauss (Peter E.) Scholarship (Harvard University)
Strong Scholarship (Harvard University)
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Sutro Fellowships (Vassar College)
Sweetser Scholarship (Harvard University)

Swett Fund (Massachusetts Institute of Technology)
Swift Fellowship (Chicago, University of)

Taft Fellowships (Cincinnati, University of)
Taussig Memorial Scholarship (California, University of)
Taylor Scholarship (Harvard University)
Temple Prime Scholarship (Brooklyn Institute of Arts and Sciences)
Terry Fellowship (Trinity College)
Thaw Fellowship (Princeton University)
Thayer Fellowship (Harvard University)
Thayer Scholarships (Harvard University)
Thomas Fellowship (Bryn Mawr)
Thomas Scholarship (Harvard University)
Thompson Memorial Scholarships (California, University of)
Townsend Scholarship (Harvard University)
Trowbridge Fellowship (Columbia University)
Trudeau Foundation Fellowships (Trudeau Foundation)
Trudeau Foundation Scholarships (Trudeau Foundation)
Trustee Fellowships (Smith College)
Tucker Fellowship (Dartmouth College)
Tyndale Fellowship (Pennsylvania, University of)
Tyndall Fellowship (Columbia University)
Tyndall Fellowship (Harvard University)

Union Electric Light and Power Co. Research Fellowship (Washington University, St. Louis)
United Daughters of the Confederacy Scholarships (Virginia, University of)
United States Radiator Company Fellowship (Michigan, University of)
United States Rubber Plantations Fellowship (Michigan, University of)
Upjohn Research Scholarship (Kalamazoo College)

Van Loon Fellowship (Vassar College)
Van Meter Alumnae Fellowship (Goucher College)
Vander Poel Fellowship (Rutgers College)
Vanderbilt Fellowships (Virginia, University of)

Walcott Fellowship (Harvard University)
Walker Fellowship (Chicago, University of)
War Memorial Graduate Scholarship (Alfred University)
Ware (Charles Eliot) Memorial Fellowship (Harvard University—*See* Bullard Fellowships
Ware (John) Memorial Fellowship (Harvard University)—*See* Bullard Fellowships
Waterman Scholarship (Yale University)
Watson Scholarship (California, University of)
Weaver Scholarship (Stanford University)
Webster Scholarship (Harvard University)
Weld Scholarship (Harvard University)
Western New York Fruit Growers Packing Association Fellowship (Cornell University)
White Fellowship (Cornell University)
White Scholarships (Northwestern University)

Whiting Fellowships (California, University of)
Whiting Fellowships (Harvard University)
Whitney Fellowships (Radcliffe College)
Whitney Fund (Massachusetts Institute of Technology)
Whitney Scholarships (Radcliffe College)
Whittier Fellowships (Michigan, University of)
Wigglesworth Scholarship (Harvard University)
Wilcox Fellowship (Lake Erie College)
Williams Memorial Fellowship (Johns Hopkins University)
Williamson Cooperative Vegetable Association Fellowship (Cornell University)
Winthrop Scholarship (Harvard University)
Wisconsin Gas Association Fellowship (Wisconsin, University of)
Wisconsin Utilities Association Fellowship (Wisconsin, University of)
Wistar Institute Scholarships (Pennsylvania, University of)
Wood Teaching Fellowship (Pennsylvania, University of)
Woods Hole Research Scholarships (Yale University)
Woodward Fellowship (Pennsylvania, University of)
Woolley Fellowship (Mount Holyoke College)
Woman's Medical College of Pennsylvania Scholarship (Vassar College)

LIST OF INDUSTRIAL CORPORATIONS AND TRADE ASSOCIATIONS SUPPORTING FELLOWSHIPS

Acme White Lead and Color Works, Detroit, Mich. (Michigan, University of)

Associated Radiograph Laboratories of San Francisco, San Francisco, Cal. (California, University of)

Century Electric Co. of St. Louis, St. Louis, Mo. (Washington University. St. Louis)

Cheney Brothers, South Manchester, Conn. (Yale University)

Chilean Nitrate Committee (Rutgers College)

Detroit Edison Co., Detroit, Mich. (Michigan, University of)

E. I. duPont de Nemours & Co., Wilmington, Del. (Brown University; California, University of; California Institute of Technology; Chicago, University of; Columbia University; Cornell University; Delaware, University of; Harvard University; Illinois, University of; Johns Hopkins University; Lafayette College; Massachusetts Institute of Technology; Michigan, University of; Minnesota, University of; Ohio State University; Princeton University; Virginia, University of; Washington, University of; Wisconsin, University of; Yale University)

Eastern Potash Corporation of New York, New York, N. Y. (Rutgers College)

Firestone Tire and Rubber Co., Akron, Ohio (Akron, Municipal University of)

Fleischman Company, Peekskill, N. Y. (Chicago, University of)

Freeport Sulphur Co., New York, N. Y. (Crop Protection Institute)

Fritzche Brothers, Inc., New York, N. Y. (Wisconsin, University of)

General Electric Co., Schenectady, N. Y. (Charles A. Coffin Foundation)

Goodyear Tire & Rubber Co., Akron, Ohio (Akron, Municipal University of)

Grasselli Chemical Co., Cleveland, Ohio (Case School of Applied Science, Cornell University; Indiana University; Massachusetts Institute of Technology; Oberlin College; Ohio State University)

Gypsum Industries, Chicago, Ill. (Chicago, University of; Cornell University; Iowa State College of Agriculture and Mechanic Arts; Kentucky, University of; North Carolina State College of Agriculture and Engineering)

Hillman Coal and Coke Co., Pittsburgh, Pa. (Carnegie Institute of Technology)

International Paper Co., New York, N. Y. (Lafayette College)

Kansas Flour Mills Co., Wichita, Kansas (Kansas State Agriculture College)

Mead Johnson & Co., Evansville, Ind. (Wisconsin, University of)

Mine Safety Appliances Co., Pittsburgh, Pa. (Carnegie Institute of Technology)

National Aniline and Chemical Co., Inc., New York, N. Y. (Michigan, University of)

Newport Chemical Co. (Wisconsin, University of)

Niagara Sprayer Co., Middleport, N. Y. (Rutgers College)

Palmolive Co., Milwaukee, Wis. (Cornell University)

B. G. Pratt Co., New York, N. Y. (Crop Protection Institute)

Quaker Oats Co., Chicago, Ill. (Crop Protection Institute)

Frederick Stearns & Co., Detroit, Mich. (Michigan, University of)

Texas Gulf Sulphur Co., New York, N. Y. (Crop Protection Institute; National Research Council)

Union Central Life Insurance Co. (Cincinnati, University of)
Union Electric Light and Power Co., St. Louis, Mo. (Washington University, St. Louis)
Union Sulphur Co., New York, N. Y. (Cornell University; Crop Protection Institute)
United States Radiator Co., Detroit, Mich. (Michigan, University of)
United States Rubber Plantations, Inc. (Michigan, University of)
Upjohn Company, Kalamazoo, Mich. (Kalamazoo College)

American Institute of Baking, 1135 Fullerton Ave., Chicago, Ill.
American Seed Trade Association, 101 High Ave., Cleveland, Ohio (National Research Council)
Clay Products Association, 913 Chamber of Commerce Bldg., Chicago, Ill. (Mellon Institute of Industrial Research)
Compressed Gas Manufacturers Association, 120 W. 42d St., New York, N. Y. (Mellon Institute of Industrial Research)
Container Club, 608 S. Dearborn St., Chicago, Ill. (Mellon Institute of Industrial Research)
Eastern Clay Products Association, 910-913 S. Penn. Square Bldg., Philadelphia, Pa. (Mellon Institute of Industrial Research)
Edible Gelatin Manufacturers of America, 81 Fulton St., New York, N. Y. (Mellon Institute of Industrial Research)
International Milk Dealers Association (Cornell University; Wisconsin, University of)
Laundryowners National Association, Pittsburgh, Pa. (Mellon Institute of Industrial Research)
Magnesia Association of America, 721 Bulletin Bldg., Philadelphia, Pa. (Mellon Institute of Industrial Research)
Mundatechnical Society of America (formerly International Technical Society of Cleaners and Dyers), Pittsburgh, Pa. (Mellon Institute of Industrial Research)
National Association of Manufacturers of Carbonated Beverages (Iowa State College of Agriculture and Mechanic Arts)
National Canners Association, 1739 H St., N. W., Washington, D. C. (Cornell University)
National Fertilizer Association, Philadelphia, Pa. (Wisconsin, University of)
National Lime Association, 918 G St. N. W., Washington, D. C. (Massachusetts Institute of Technology; Ohio State University)
National Wood Chemical Association, 76 Main St., Bradford, Pa. (Mellon Institute of Industrial Research)
Refractories Manufacturers Association, 840 Oliver Bldg., Pittsburgh, Pa. (Mellon Institute of Industrial Research)
Stove Founders' Research Association, Care of Moore Power Co., Joliet, Ill. (Mellon Institute of Industrial Research)
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THE QUANTUM THEORY
Second Edition. Revised and Enlarged

By

E. P. ADAMS

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PREFACE

The first edition of this Bulletin was put together from notes that had been used in giving an account of the development of the Quantum Theory before the Physical Colloquium of Princeton University. A new edition having been asked for, the former one has been completely revised with the object of attempting to give a more consistent and comprehensive account of typical applications of this theory to physical phenomena.

The development of the quantum theory, presented here, proceeds from the view that physical phenomena are governed by dynamical principles; our lack of precise knowledge of the ultimate structure and properties of matter, and its relation to radiation, requires certain hypotheses to be made to supplement the laws of dynamics in order to make any progress in coordinating the various phenomena. For this reason the first chapter contains a brief summary of the dynamical principles needed for the applications. The "momentum-energy tensor," introduced by Cartan, whose invariance, when integrated around a closed circuit, is a very general dynamical principle, is employed for obtaining the required transformations.

In the second chapter, after a brief account of the canonical ensembles of Gibbs, the hypotheses peculiar to the quantum theory are introduced; and in the following three chapters some typical applications of these hypotheses are made. Problems connected with atomic structure, in general, and the general theory of spectral series, are not discussed, being beyond the scope of the present work.

For those physical phenomena which can be described with the aid of models that form periodic, or conditionally periodic, dynamical systems, the hypotheses of the quantum theory, however artificial they appear, have proved of great value, particularly in the field of spectroscopy. There are, however, many phenomena, analogous to the photo-electric effect, which are undoubtedly governed by quantum principles of some kind, but which have not, as yet, been brought under any general hypotheses supplementary to dynamical principles. These phenomena are briefly discussed in the sixth chapter, where it is shown that their most natural interpretation leads to the view of "energy quanta." For simply periodic systems the view of energy quanta may be regarded as an alternative interpretation of the phenomena concerned; but for systems that are not simply periodic, as shown in the case of the asymmetrical linear oscillator, considered in the third chapter, the view of energy quanta is not consistent with the general hypotheses of the quantum theory applied to conditionally periodic systems. And the great mass of evidence from physical optics renders any quantum view of radiation very doubtful, although as a working hypothesis it has been, and will probably continue to be for some time, of considerable value.

The last chapter contains a brief account of the magnetic interpretation of the quantum theory, due, chiefly, to Allen. While this interpretation of the quantum theory does not lend itself to application to physical phenomena as well as the purely dynamical interpretation, it is of importance in indicating a possible direction for future development.

CHAPTER I

GENERAL DYNAMICAL THEORY

1. **Virtual Displacements.**—For a system of n free particles, the equations of motion are, by Newton's laws,

$$m_k \ddot{x}_k = X_k,\ m_k \ddot{y}_k = Y_k,\ m_k \ddot{z}_k = Z_k,\ k = 1, 2, 3, \cdots n. \qquad 1\cdot1$$

In these equations, the m_k are the masses of the particles, x_k, y_k, z_k their rectangular coordinates, the two dots indicating second time derivatives, and X_k, Y_k, Z_k are the components of the forces acting on the particles.

If the particles are not free, but are subject to constraints, the equations of motion may be written,

$$m_k \ddot{x}_k - X_k = X'_k,\ m_k \ddot{y}_k - Y_k = Y'_k,\ m_k \ddot{z}_k - Z_k = Z'_k, \qquad 1\cdot2$$

where X'_k, Y'_k, Z'_k are the components of the forces arising from the constraints.

Now multiply equations 1·1 by the arbitrary quantities, δx_k, δy_k, δz_k respectively, and take the sum for all the particles. In this way we get the equation,

$$\sum \left\{ (m_k \ddot{x}_k - X_k)\ \delta x_k + (m_k \ddot{y}_k - Y_k)\ \delta y_k + (m_k \ddot{z}_k - Z_k)\ \delta z_k \right\} = 0. \qquad 1\cdot3$$

Since δx_k, δy_k, δz_k are $3n$ wholly arbitrary quantities, equations 1·1 and 1·3 are equivalent to each other. From 1·3 equations 1·1 may be derived, just as 1·3 were derived from 1·1.

Now multiply equations 1·2 by δx_k, δy_k, δz_k and take the sum for all the particles. The result may be written,

$$\sum \left\{ (m_k \ddot{x}_k - X_k)\ \delta x_k + (m_k \ddot{y}_k - Y_k)\ \delta y_k + (m_k \ddot{z}_k - Z_k)\ \delta z_k \right\} =$$

$$\sum (X'_k\ \delta x_k + Y'_k\ \delta y_k + Z'_k \delta z_k). \qquad 1\cdot4$$

Let us now interpret δx_k, δy_k, δz_k as the components of displacements of the particles. Then the right-hand side of 1·4 is the expression for the work done on the system of particles in this displacement by the forces of constraint. It follows, therefore, that the left-hand side of 1·3 or 1·4 will vanish for a system of free particles, and also for a system of particles that is not free, provided that no work is done by the forces of constraint in the displacement represented by δx_k, δy_k, δz_k. Such displacements are called virtual displacements.

2. **The Principle of Stationary Action.**—Let the δx_k, δy_k, δz_k represent a virtual displacement of the particles of a system. We can then write for any system of particles,

$$\sum m_k(\ddot{x}_k\,\delta x_k+\ddot{y}_k\,\delta y_k+\ddot{z}_k\,\delta z_k)=\sum(X_k\,\delta x_k+Y_k\,\delta y_k+Z_k\,\delta z_k) \qquad 2\cdot 1$$

The right-hand side may be written $\delta' W$, and represents the work done by all the forces acting on the system in the displacement; the accent indicates that $\delta' W$ is not necessarily an exact differential. If the kinetic energy, T, of the system be defined by the equation,

$$2\,T=\sum m_k\,(\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2),$$

equation $2\cdot 1$ may be transformed into

$$\frac{d}{dt}\sum m_k\,(\dot{x}_k\,\delta x_k+\dot{y}_k\,\delta y_k+\dot{z}_k\,\delta z_k)=\delta T+\delta' W.$$

Now multiply this equation by dt and integrate between the limits t_0 and t_1. The result is,

$$\int_{t_0}^{t_1}(\delta T+\delta' W)\,dt=\Bigl[\sum m_k\,(\dot{x}_k\,\delta x_k+\dot{y}_k\,\delta y_k+\dot{z}_k\,\delta z_k)\Bigr]_{t_0}^{t_1}.$$

Let us now assume that the virtual displacements are zero at the two ends of the path of the system at t_0 and t_1. Then we have,

$$\int_{t_0}^{t_1}(\delta T+\delta' W)\,dt=0. \qquad 2.2$$

This equation expresses Hamilton's Principle, or the Principle of Stationary Action. If the components of the forces acting on the particles are given as the negative partial derivatives of a potential energy function, V,

$$X_k,\ Y_k,\ Z_k=-\left(\frac{\partial}{\partial x_k},\frac{\partial}{\partial y_k},\frac{\partial}{\partial z_k}\right)V,$$

then we can write,

$$\delta\int_{t_0}^{t_1}(T-V)\,dt=\delta\int_{t_0}^{t_1}L\,dt=0 \qquad 2\cdot 3$$

where $L=T-V$, the kinetic potential, or the Lagrangian function. This assumes that the variation represented by δ does not affect the time; that is, to each point, x_k, on the actual path the corresponding point, $x_k+\delta x_k$, on the varied path, is to be taken at the same time.

3. **Generalized Coordinates.**—In the expression for Hamilton's Principle, $2\cdot 2$ or $2\cdot 3$, all explicit reference to the rectangular coordi-

nates, used originally to specify the system, has disappeared. We may therefore introduce a set of generalized coordinates, $q_1, q_2, q_3, \cdots q_f$. The system is said to have f degrees of freedom if f coordinates are necessary and sufficient completely to specify its configuration, and if f velocities, $\dot{q}_1, \dot{q}_2, \dot{q}_3, \cdots \dot{q}_f$, in addition, are required to specify its state of motion. We may employ these coordinates and velocities in 2·2 and 2·3, provided that δq_k are virtual displacements. This means that it must be possible to make a variation, δq_k, in q_k, without varying any of the other coordinates; and this is only possible when the q's are all independent.

The generalized force components, Q_k, are defined by means of the work done in a displacement,

$$\delta' W = \sum Q_k \, \delta q_k.$$

When the generalized coordinates and velocities are introduced into the expression for the kinetic energy, we shall have,

$$\delta T = \sum \frac{\partial T}{\partial q_k} \delta q_k + \sum \frac{\partial T}{\partial \dot{q}_k} \delta \dot{q}_k.$$

Equation 2·2 now becomes,

$$\sum \int_{t_0}^{t_1} \left(\frac{\partial T}{\partial q_k} \delta q_k + Q_k \, \delta q_k \right) dt + \sum \int_{t_0}^{t_1} \frac{\partial T}{\partial \dot{q}_k} \delta \dot{q}_k \, dt = 0.$$

Since the time is not to be varied,

$$\delta \dot{q}_k = \frac{d}{dt} \delta q_k.$$

Therefore each term in the second sum may be integrated by parts; and if, as before, the variations δq_k vanish at the limits, t_0 and t_1, we get,

$$\sum \int_{t_0}^{t_1} \left(\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} - Q_k \right) \delta q_k \, dt = 0.$$

Since the interval $t_1 - t_0$ may be made as small as we please, and since the q_k are all independent of each other, we have,

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} = Q_k \qquad 3 \cdot 1$$

the Lagrangian equations of motion. If the forces arise from a potential energy function, V,

$$Q_k = -\frac{\partial V}{\partial q_k},$$

and introducing the kinetic potential, $L=T-V$, we can write

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k}-\frac{\partial L}{\partial q_k}=0. \qquad 3\cdot 2$$

4. **The Principle of Least Action.**—Let the sum of the kinetic energy and the potential energy, $H=T+V$, be constant throughout the motion. Then the principle of stationary action, 2·3, may be written,

$$\delta\int_{t_0}^{t_1}2T\cdot dt=(t_1-t_0)\,\delta H=0.$$

Since H is constant, and the time is not varied, we have,

$$\delta\int_{t_0}^{t_1}2T\cdot dt=0. \qquad 4\cdot 1$$

This is the Principle of Least Action. The integral

$$\int_{t_0}^{t_1}2T\cdot dt$$

is denoted the Action, while the integral,

$$S=\int_{t_0}^{t_1}Ldt$$

is Hamilton's Principal Function.

5. **The Canonical Equations of Motion.**—In the Lagrangian equations of motion, 3·2, put,

$$p_k=\frac{\partial L}{\partial \dot{q}_k}. \qquad 5\cdot 1$$

Since L is expressed as a function of the q_k, $\dot{q}_k$, and, in general, the time, t, we have, the time not being varied,

$$\delta L=\sum\frac{\partial L}{\partial \dot{q}_k}\delta\dot{q}_k+\sum\frac{\partial L}{\partial q_k}\delta q_k.$$

Therefore, using 5·1, we can write,

$$\delta\left\{\sum p_k\,\dot{q}_k-L\right\}=\sum(\dot{q}_k\,\delta p_k-\dot{p}_k\,\delta q_k).$$

Now by means of the f equations, 5·1, the $\dot{q}_k$ may be expressed in the terms of the p_k, q_k, and therefore $\sum p_k\dot{q}_k-L$ may be expressed in terms of the p_k, q_k, t. When so expressed, denote this function by H. Then,

$$\delta H=\sum\frac{\partial H}{\partial p_k}\delta p_k+\sum\frac{\partial H}{\partial q_k}\delta q_k=\sum\dot{q}_k\,\delta p_k-\sum\dot{p}_k\,\delta q_k.$$

And it follows from this that

$$\left.\begin{aligned} \dot{q}_k &= \frac{\partial H}{\partial p_k} \\ \dot{p}_k &= -\frac{\partial H}{\partial q_k} \end{aligned}\right\} \qquad 5\cdot 2$$

These are the Hamiltonian Canonical Equations of Motion.

In classical dynamics, the kinetic energy, T, is a homogeneous quadratic function of the velocities, $\dot{q}_k$, with coefficients depending upon the coordinates q_k. Then, by Euler's theorem,

$$2T = \sum \frac{\partial T}{\partial \dot{q}_k} \dot{q}_k = \sum p_k \dot{q}_k$$

and

$$H = 2T - L = T + V.$$

Therefore, the Hamiltonian function, H, represents the sum of the kinetic energy and potential energy of the system. This is not necessarily a constant, for the kinetic potential, L, may involve the time explicitly. In the canonical equations, 5·2, H must be expressed in terms of the p_k, q_k, and, in general, t.

If we take the principle of stationary action as a fundamental dynamical principle, it may be shown[1] that the canonical equations of motion result, even when L, the kinetic potential, is a much more general function of the coordinates, velocities, and time, than it is in classical dynamics. If L is any function of the independent variable t, any number of dependent variables, x_1, x_2, $\cdots$ and their derivatives with respect to t, up to any orders, then the canonical equations result when the variation of the integral $\int L dt$ is put equal to zero. The number of pairs of the canonical equations will be equal to the sum of the orders of the highest derivatives that appear in L.

6. The Principle of Varying Action.—The variation denoted by the symbol δ is one in which the time is not varied. Corresponding points on the actual and varied paths are compared at the same time, and, moreover, the end points of the paths have not been varied. We can, however, consider a more general variation, to be denoted by the symbol Δ. The point corresponding to q_k, at t, on the actual path we take to be the point $q_k + \Delta q_k$ at $t + \Delta t$, on the varied path. We shall also allow a variation of the initial and final points of the path, both in time and coordinates.

Besides depending upon the q_k, $\dot{q}_k$, t, the kinetic potential, L, will be supposed to contain certain parameters, to be denoted by a_k which are also subject to the variation Δ. In all the foregoing, such parameters were assumed to remain constant throughout the motion.

---

[1] Whittaker, Analytical Dynamics, Cambridge, 1917, p. 265.

We shall now form ΔS, where S is the Principal Function.

$$\Delta S = \Delta \int_{t_0}^{t_1} L\,dt = \int_{t_0}^{t_1} \Delta L \cdot dt + \int_{t_0}^{t_1} L \cdot \Delta dt$$

$$= \int \sum \frac{\partial L}{\partial \dot{q}} \Delta \dot{q} \cdot dt + \int \sum \frac{\partial L}{\partial q} \Delta q \cdot dt$$

$$+ \int \sum \frac{\partial L}{\partial a} \Delta a \cdot dt + \int \frac{\partial L}{\partial t} \Delta t \cdot dt + \int L \,\Delta dt.$$

Now,

$$\Delta dq = d\Delta q$$

$$\Delta dt = d\Delta t$$

$$\Delta \dot{q} = \Delta \frac{dq}{dt} = \frac{d}{dt} \Delta q - \dot{q} \frac{d}{dt} \Delta t.$$

We therefore have,

$$\int \frac{\partial L}{\partial \dot{q}} \Delta \dot{q} \cdot dt = \int \frac{\partial L}{\partial \dot{q}} \frac{d}{dt} \Delta q \cdot dt - \int \dot{q} \frac{\partial L}{\partial \dot{q}} \frac{d}{dt} \Delta t \cdot dt.$$

Integrating by parts,

$$\int \frac{\partial L}{\partial \dot{q}} \Delta \dot{q}\, dt = \left[\frac{\partial L}{\partial \dot{q}} \left(\Delta q - \dot{q}\, \Delta t \right]_{t_0}^{t_1}$$

$$- \int_{t_0}^{t_1} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) \left(\Delta q - \dot{q}\, \Delta t \right) dt + \int_{t_0}^{t_1} \ddot{q} \frac{\partial L}{\partial \dot{q}} \Delta t \cdot dt.$$

Similarly,

$$\int_{t_0}^{t_1} L\, \Delta dt = \int L d\, \Delta t = \Big[L\, \Delta t \Big]_{t_0}^{t_1} - \int \frac{dL}{dt} \Delta t \cdot dt$$

$$= \Big[L\, \Delta t \Big]_{t_0}^{t_1} - \int \sum \frac{\partial L}{\partial \dot{q}} \ddot{q}\, \Delta t \cdot dt - \int \sum \frac{\partial L}{\partial q} \dot{q}\, \Delta t \cdot dt$$

$$- \int \sum \frac{\partial L}{\partial a} \dot{a}\, \Delta t \cdot dt - \int \frac{\partial L}{\partial t} \Delta t \cdot dt.$$

We therefore find,

$$\Delta S = \left[\sum \frac{\partial L}{\partial \dot{q}} \left(\Delta q - \dot{q}\, \Delta t \right) + L\, \Delta t \right]_{t_0}^{t_1} + \int_{t_0}^{t_1} \sum \frac{\partial L}{\partial a} \left(\Delta a - \dot{a}\, \Delta t \right) dt$$

$$- \int_{t_0}^{t_1} \sum \left(\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} \right) \left(\Delta q - \dot{q}\, \Delta t \right) dt. \qquad 6 \cdot 1$$

The last integral vanishes on account of the Lagrangian equations of motion. And with

$$\frac{\partial L}{\partial \dot{q}_k} = p_k, \sum \dot{q}_k \frac{\partial L}{\partial \dot{q}_k} = 2T,\ H = T + V,$$

and since,

$$\Delta a = \delta a + \dot{a}\, \Delta t,$$

we have,

$$\Delta S = \left[\sum p_k\, \Delta q_k - H\, \Delta t \right]_{t_0}^{t_1} + \int_{t_0}^{t_1} \sum \frac{\partial L}{\partial a_k} \delta a_k \cdot dt. \qquad 6 \cdot 2$$

If the parameters, a_k, are not varied during the motion, we have,

$$\Delta S = \left[\sum p_k \, \Delta q_k - H \, \Delta t \right]_{t_0}^{t_1}. \qquad 6\cdot 3$$

This is the expression for the Principle of Varying Action.

7. INTEGRAL-INVARIANTS.—In multi-dimensional space of $2f+1$ dimensions, with q_k, p_k, t as coordinates, a single point represents completely the state and time of a dynamical system. In this space suppose a closed curve to be drawn, and the integral

$$\int \sum p_k \, \Delta q_k - H \, \Delta t \qquad 7\cdot 1$$

extended completely around the curve. Through each point of this closed curve suppose the dynamical paths of the system are traced. In this way a tube of paths may be supposed to be formed. Then, the principle of varying action, 6·3, shows that the integral 7·1 will have the same value when extended around any closed curve lying on this tube of paths. For denoting the integrand in 7·1 by I, and integrating around a closed curve so as to return to the starting point in the $2f+1$ – dimensional space, 6·3 gives,

$$\int \Delta S = 0 = \int [I]_t - \int [I]_{t_0}.$$

There will be no confusion now in denoting the symbol of variation by δ, for the presence of δt in 7·1 will indicate that the time is to be varied, in general. We thus arrive at an integral-invariant for closed curves, or a relative integral invariant,

$$\int \sum p_k \, \delta q_k - H \, \delta t. \qquad 7\cdot 2$$

If the time is not varied, so that $\delta t = 0$, we get the relative integral-invariant introduced by Poincaré. The more general integral-invariant 7·2 was introduced by Cartan.[1]

Denoting, as usual, differentiation along a dynamical path by d, and differentiation along the closed curve lying on the tube of paths by δ, we have, by what has just been proved,

$$d \int \sum p_k \, \delta q_k - H \, \delta t = 0$$

or

$$\int \sum dp_k \, \delta q_k + \int \sum p_k \, \delta dq_k - \int dH \, \delta t - \int H \, \delta dt = 0.$$

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[1] E. Cartan, Leçons sur les Invariants Integraux, Paris, 1922.

The second and last integral may be integrated by parts, with the result,

$$\int\sum dp_k\,\delta q_k-\int\sum\delta p_k\,dq_k-\int dH\,\delta t+\int\delta H\cdot dt=0,$$

and since,

$$\delta H=\sum\frac{\partial H}{\partial p_k}\delta p_k+\sum\frac{\partial H}{\partial q_k}\delta q_k+\frac{\partial H}{\partial t}\delta t,$$

we get,

$$\begin{aligned} dp_k &= -\frac{\partial H}{\partial q_k}dt\\ dq_k &= \frac{\partial H}{\partial p_k}dt\\ dH &= \frac{\partial H}{\partial t}dt \end{aligned}$$

or

$$\left.\begin{aligned} \dot{p}_k &= -\frac{\partial H}{\partial q_k}\\ \dot{q}_k &= \frac{\partial H}{\partial p_k}\\ \frac{dH}{dt} &= \frac{\partial H}{\partial t} \end{aligned}\right\} \tag{7·3}$$

The first two equations are the canonical equations, and the third equation shows that if H does not contain the time explicitly, the energy, H, is constant.

We can thus conclude that if Q_k, P_k, K, are the coordinates, momenta, and energy in a dynamical system, and

$$\int\sum P_k\,\delta Q_k-K\,\delta t \tag{7·4}$$

is a relative integral-invariant, the equations of motion will be canonical in Q_k, P_k, K, that is,

$$\left.\begin{aligned} \dot{P}_k &= -\frac{\partial K}{\partial Q_k}\\ \dot{Q}_k &= \frac{\partial K}{\partial P_k}\\ \frac{\partial K}{\partial t} &= \frac{dK}{dt} \end{aligned}\right\} \tag{7·5}$$

8. Transformation of Variables.—It is frequently desirable to introduce a new set of variables, Q_k, P_k, t, to specify a dynamical system, in place of the old variables, q_k, p_k, t. With the aid of the relative integral-invariant, 7·2, the condition that the equations of motion remain canonical in the new variables may readily be found.

Consider the same tube of paths, referred either to q_k, p_k, t or to Q_k, P_k, t. Then both 7·2 and 7·4 are relative integral-invariants, if both sets of variables are canonical, for closed curves lying on the tube of paths. Therefore the difference of the integrands in 7·2 and 7·4 must be an exact differential, as it must vanish on integrating around any such closed curve. It therefore follows that

$$\sum p_k\,\delta q_k - \sum P_k\,\delta Q_k - (H-K)\,\delta t = \delta S, \qquad 8\cdot 1$$

where δS is the exact differential of some arbitrary function, S.

Now take S to be an arbitrary function of the q_k, Q_k, t. Then

$$\delta S = \sum \frac{\partial S}{\partial q_k}\delta q_k + \sum \frac{\partial S}{\partial Q_k}\delta Q_k + \frac{\partial S}{\partial t}\delta t,$$

and 8·1 gives,

$$\left.\begin{aligned} p_k &= \frac{\partial S}{\partial q_k} \\ P_k &= -\frac{\partial S}{\partial Q_k} \\ H-K &= -\frac{\partial S}{\partial t} \end{aligned}\right\} \qquad 8\cdot 2$$

If, then, S is taken as any arbitrary function of the q_k, Q_k, not involving the time explicitly, the first two of 8·2 may be taken as the equations of transformation to change from the q_k, p_k to the Q_k, P_k, and the last equation shows that the energy expressed in the new variables has the same value as when expressed in the old variables. The arbitrary function S is called a substitution function.

Next, suppose we have a substitution function given as a function of the q_k, P_k, t. In 8·1 we can write

$$\sum P_k\,\delta Q_k = \delta \sum P_k\,Q_k - \sum Q_k\,\delta P_k.$$

We then have,

$$\sum p_k\,\delta q_k + \sum Q_k\,\delta P_k - (H-K)\,\delta t = \delta W,$$

where δW is the exact differential of an arbitrary function which we assume is expressed in terms of the q_k, P_k, t. We thus derive,

$$\left.\begin{aligned} p_k &= \frac{\partial W}{\partial q_k} \\ Q_k &= \frac{\partial W}{\partial P_k} \\ K-H &= \frac{\partial W}{\partial t} \end{aligned}\right\} \qquad 8\cdot 3$$

The first two of these equations are the equations of transformation from the old to the new variables; and if W does not involve the time explicitly, the energy in the new variables has the same value as when expressed in the old variables.

9. **The Hamilton-Jacobi Method of Integration.**—The integral

$$\int \sum P_k \,\delta Q_k - K\,\delta t$$

taken around any closed curve, will certainly be invariant if the P_k and K are constant along any dynamical path. And by 7·5,

$$\dot{P}_k = 0 = -\frac{\partial K}{\partial Q_k}, \quad \dot{Q}_k = \frac{\partial K}{\partial P_k}. \tag{9·1}$$

The integral will still be invariant if we choose $K=0$. Then 9·1 gives,

$$P_k = \beta_k, \quad Q_k = \alpha_k$$

where the β_k and α_k are two sets of f constants along any dynamical path. Now since

$$\sum p_k\,\delta q_k - \sum \beta_k\,\delta\alpha_k - H\,\delta t = \delta S,$$

where S is the exact differential of a function S, which we shall assume to be expressed in terms of the q_k, α_k, t, it follows that,

$$\left.\begin{aligned} p_k &= \frac{\partial S}{\partial q_k} \\ \beta_k &= -\frac{\partial S}{\partial \alpha_k} \\ H &= -\frac{\partial S}{\partial t} \end{aligned}\right\} \tag{9·2}$$

H is expressed as a function of the q_k, p_k, t. Hence the first and last of 9·2 give,

$$\frac{\partial S}{\partial t} + H\left(\frac{\partial S}{\partial q_1}, \frac{\partial S}{\partial q_2} \cdots\cdot, q_1\ q_2 \cdots\cdot q_f, t\right) = 0. \tag{9·3}$$

This is the Hamilton-Jacobi partial differential equation. Its complete integral will contain $f+1$ integration constants, of which one will be an additive constant, and may be ignored. Let

$$S = S(q_1, q_2, \cdots q_f, t, \alpha_1, \alpha_2, \cdots \alpha_f)$$

be the complete integral of 9·3. Then the complete solution of the dynamical problem is contained in the equations 9·2, and will thus involve the $2f$ integration constants, α_k, β_k, required for a system of f degrees of freedom.

10. **The Energy is Constant.**—Suppose now that the time does not enter explicitly in the expression for the energy, H, so that,

$$H(q_k, p_k,) = \alpha_1 \qquad 10\cdot 1$$

where α_1 is the constant energy along any dynamical path. If

$$\int \sum p_k\, \delta q_k - H\delta t \text{ and } \int \sum P_k\, \delta Q_k - K\,\delta t$$

are relative integral-invariants, so are

$$\int \sum p_k\, \delta q_k + t\,\delta H \text{ and} \int \sum P_k\, \delta Q_k + t\,\delta K$$

since each differs from the former by an exact differential. By 7·4 we shall have,

$$\dot{Q}_k = \frac{\partial K}{\partial P_k}, \quad \dot{P}_k = -\frac{\partial K}{\partial q_k}.$$

If now we take, $K=H$, $Q_k=\alpha_k=$constant, $P_1=-(t+\beta_1)$, $P_j=-\beta_j$, $(j=2, 3, \cdots f)$, where the β_k are constants, then

$$\int \sum P_k\, \delta Q_k + t\,\delta K$$

will certainly be an integral invariant. The canonical equations in P_k, Q_k will be satisfied, and

$$\sum p_k\, \delta q_k + (t+\beta_1)\ \delta\alpha_1 + \sum \beta_j\, \delta\alpha_j = \delta W,$$

where δW is an exact differential of a function W which we shall assume is expressed in terms of the q_k, α_k. It therefore follows that

$$\left.\begin{aligned} p_k &= \frac{\partial W}{\partial q_k} \\ t+\beta_1 &= \frac{\partial W}{\partial \alpha_1} \\ \beta_j &= \frac{\partial W}{\partial \alpha_j} \end{aligned}\right\} \qquad 10\cdot 2$$

where W is a solution of the Hamilton-Jacobi differential equation,

$$H\left(\frac{\partial W}{\partial q_1}, \frac{\partial W}{\partial q_2}, \cdots \frac{\partial W}{\partial q_f}, q_1, q_2, \cdots q_f\right) = \alpha_1. \qquad 10\cdot 3$$

The complete integral of this differential equation will contain, besides α_1, f arbitrary constants of integration, of which one will be an additive constant and may be ignored. Writing the solution of this equation,

$$W = W(q_1, q_2, \cdots q_f, \alpha_1, \alpha_2, \cdots \alpha_f) \qquad 10\cdot 4$$

the complete solution of the dynamical problem is given by 10·2.

11. **Angle and Action Variables.**—Let $W = W(q_k, P_k)$ be an arbitrary substitution function so that, by 8·3, the equations of transformation from p_k, q_k to P_k, Q_k are

$$\left.\begin{aligned} p_k &= \frac{\partial W}{\partial q_k} \\ Q_k &= \frac{\partial W}{\partial P_k} \end{aligned}\right\} \qquad 11\cdot 1$$

Neither W nor H is supposed to contain the time explicitly, so that $H =$ constant. We know that the equations of motion remain canonical in P_k, Q_k, and that the energy has the same value when expressed in either p_k, q_k or P_k, Q_k. The energy equation is now,

$$H\ (P_k, Q_k) = H = \text{constant},$$

or, using the second of 11·1,

$$H\left(\frac{\partial W}{\partial P_k}, P_k\right) = H. \qquad 11\cdot 2$$

Let us assume that this partial differential equation can be solved, and write the integral,

$$W = W\ (P_1, P_2, \cdots\cdots P_f, \alpha_1, \alpha_2, \cdots\cdots \alpha_f), \qquad 11\cdot 3$$

where the α_k are f integration constants. As there is one more integration constant than we need, there must be a relation between them, which we shall take as,

$$H = H\ (\alpha_1, \alpha_2, \cdots\cdots \alpha_f). \qquad 11\cdot 4$$

That is, the whole energy of the system is expressed as a function of the f integration constants.

Let us now regard W, given by 11·3, as a substitution function, with the aid of which we transform from Q_k, P_k to w_k, α_k. Since

$$\sum P_k\, \delta Q_k - \sum \alpha_k\, \delta w_k = \delta S,$$

where δS is an exact differential, we shall have,

$$\begin{aligned} -\sum Q_k\, \delta P_k + \sum w_k\, \delta \alpha_k &= \delta S - \delta \sum P_k\, Q_k + \delta \sum \alpha_k\, w_k \\ &= \delta W = \sum \frac{\partial W}{\partial P_k}\, \delta P_k + \sum \frac{\partial W}{\partial \alpha_k}\, \delta \alpha_k. \end{aligned}$$

Therefore the equations of transformation are

$$\left.\begin{aligned} Q_k &= -\frac{\partial W}{\partial P_k} \\ w_k &= \frac{\partial W}{\partial \alpha_k} \end{aligned}\right\} \qquad 11\cdot 5$$

The equations of motion remain canonical after this transformation is made, and are,

$$\dot{w}_k = \frac{\partial H}{\partial \alpha_k}$$

$$\dot{\alpha}_k = -\frac{\partial H}{\partial w_k} = 0.$$

Since H is constant, and is a function of the constants α_k only,

$$\dot{w}_k = \omega_k = \text{const.}$$

and

$$w_k = \omega_k t + \delta_k$$

where ω_k and δ_k are constants. Thus the new coordinates, w_k, so introduced, are variables which increase indefinitely with the time, and are therefore called angle variables. The constants,

$$\omega_k = \frac{\partial H}{\partial \alpha_k} \tag{11·6}$$

are the mean motions, and the constants α_k the action variables of the system.

12. STÄCKEL'S THEOREM.—We have seen that the solution of a dynamical problem can always be reduced to the solution of a single partial differential equation of the first order, the Hamilton-Jacobi equation. No methods are available for integrating this equation in general. Stäckel[1], however, showed that if the Hamilton-Jacobi equation, 10·3, contains the $\frac{\partial W}{\partial q_k}$ only in squares, and if this equation can be integrated by the method of separation of variables, then the motion of the system can be described completely.

Assume, therefore, that 10·3 is of the form

$$\frac{1}{2}\left\{A_1\left(\frac{\partial W}{\partial q_1}\right)^2 + A_2\left(\frac{\partial W}{\partial q_2}\right)^2 + \cdots\cdot A_f\left(\frac{\partial W}{\partial q_f}\right)^2\right\} + V - \alpha_1 = O$$

in which the A's, as well as V, the potential energy, are functions of the q's. Assume a solution of this equation,

$$W = Q_1 + Q_2 + \cdots\cdots\cdots + Q_f$$

where Q_k depends upon q_k only. Writing,

$$\frac{\partial Q_k}{\partial q_k} = Q'_k$$

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[1] Charlier, Die Mechanik des Himmels, I, Leipzig, 1902.

the differential equation may be written,

$$\frac{1}{2}\sum_{k=1}^{f} A_k Q_k'^2 + V - \alpha_1 = 0.$$

Differentiate this equation in turn by α_1, α_2, $\cdots\cdot$ α_f, the integration constants, and write,

$$\frac{\partial Q_k'^2}{\partial \alpha_i} = \varphi_{ki}(q_k).$$

We thus get,

$$\sum_{k=1}^{f} A_k \varphi_{k_1}(q_1) = 2$$

$$\sum_{k=1}^{f} A_k \varphi_{k_j}(q_k) = 0, \qquad j = 2, 3, \cdots f.$$

This system of f linear equations may be solved for the A's,

$$A_k = \frac{2}{\Delta}\frac{\partial \Delta}{\partial \varphi_{k1}}$$

where Δ is the determinant:

$$\Delta = \begin{vmatrix} \varphi_{11} \; \varphi_{21} \; \varphi_{31} \cdots\cdots\cdots\cdots & \varphi_{f1} \\ \varphi_{12} \; \varphi_{22} \cdots\cdots\cdots\cdots\cdots & \varphi_{f2} \\ \cdots\cdots & \\ \cdots\cdots & \\ \varphi_{1f} \; \varphi_{2f} \cdots\cdots\cdots\cdots\cdots & \varphi_{ff} \end{vmatrix}$$

This determinant may be expanded:

$$\Delta = \sum_{k=1}^{f} \varphi_{k1} \frac{\partial \Delta}{\partial \varphi_{k1}}. \qquad 12\cdot 2$$

We can therefore write 12·1,

$$V = \sum_{k=1}^{f} \frac{1}{\Delta}\frac{\partial \Delta}{\partial \varphi_{k1}} (\alpha_1 \varphi_{k1} - Q_k'^2)\cdot$$

Now put

$$\psi_k(q_k) = \tfrac{1}{2}Q_k^2 - \alpha_1 \varphi_{k1}$$

Then,

$$V = -\sum_{k=1}^{f} \frac{1}{\Delta}\frac{\partial \Delta}{\partial \varphi_{k1}}(2\psi_k + \alpha_1 \varphi_{k1}) \cdot \qquad 12\cdot 3$$

We can now show that

$$W = \sum_{k=1}^{f} \int \left\{ 2\psi_k + 2\sum_{i=1}^{f} \alpha_i \varphi_{ki} \right\}^{\frac{1}{2}} dq_k \qquad 12\cdot 4$$

is the complete integral of the Hamilton-Jacobi equation. For,

$$\left(\frac{\partial W}{\partial q_k}\right)^2 = Q_k^2 = 2\psi_k + 2\sum_{i=1}^{f} \alpha_i \varphi_{ki}$$

and putting this in 12·1, and using 12·2 and 12·3, we get,

$$\sum_{k=1}^{f} \frac{2}{\Delta}\frac{\partial \Delta}{\partial \varphi_{k1}} \sum_{i=2}^{f} \alpha_i \varphi_{ki} = 0.$$

But this equation is identically true, for, from the properties of determinants,

$$\frac{1}{\Delta}\sum_{k=1}^{f} \varphi_{ki}\frac{\partial \Delta}{\partial \varphi_{k1}} \quad \begin{array}{c} = 0 \text{ if } i \neq 1 \\ \text{or} \\ = 1 \text{ if } i = 1 \end{array}$$

Therefore 12·4 satisfies the Hamilton-Jacobi equation, and as it contains f constants of integration it is the complete integral.

The component momentum corresponding to q_k is,

$$p_k = \frac{\partial W}{\partial q_k} = \pm \left\{ 2\psi_k + 2\sum_{i=1}^{f} \alpha_i \varphi_{ki} \right\}^{\frac{1}{2}} \cdot \qquad 12\cdot 5$$

The solution of the Hamilton-Jacobi equation may therefore be written,

$$W = \sum_{k=1}^{f} \int p_k \, dq_k \qquad 12\cdot 6$$

and p_k depends on q_k and the f integration constants, $\alpha_1, \alpha_2, \cdots \alpha_f$.

The complete solution of the dynamical problem can now be given with the aid of 10·2,

$$\left.\begin{aligned} t+\beta_1 &= \frac{\partial W}{\partial \alpha_1} = \sum_{k=1}^{f} \int \frac{\varphi_{k1}\, dq_k}{p_k} \\ \beta_j &= \frac{\partial W}{\partial \alpha_j} = \sum_{k=1}^{f} \int \frac{\varphi_{kj}\, dq_k}{p_k} \\ & j = 2, 3, \cdots f. \end{aligned}\right\} \tag{12·7}$$

where the β's are f new integration constants.

13. If equations 12·7 are differentiated we get,

$$\left.\begin{aligned} dt &= \sum_{k=1}^{f} \frac{\varphi_{k1}\, dq_k}{p_k} \\ 0 &= \sum_{k=1}^{f} \frac{\varphi_{kj}\, dq_k}{p_k} \end{aligned}\right\} \tag{13·1}$$

Let initially $q_k = q_k^0$, and $a_k < q_k^0 < b_k$, where a_k and b_k are two neighboring roots of

$$p_k^2 = 2\,\varphi_k - 2\sum_{i=1}^{f} \alpha_i\, \varphi_{ki} = 0.$$

We shall assume that these roots are simple, so that

$$p_k = \left\{ (q_k - a_k)\,(b_k - q_k)\,\Psi\,(q_k) \right\}^{\frac{1}{2}}.$$

Now introduce a change of variables by means of the equations,

$$\frac{dq_k}{\sqrt{(q_k - a_k)(b_k - q_k)}} = m_k du_k \tag{13·2}$$

and put,

$$F_{ki}(q_k) = \frac{\varphi_{ki}}{\sqrt{\Psi_k}}.$$

Then 13·1 gives,

$$\begin{aligned} dt &= F_{11}(q_1)m_1du_1 + F_{21}(q_2)m_2du_2 + \cdots\cdots F_{f1}(q_f)m_fdu_f \\ O &= F_{12}(q_1)m_1du_1 + F_{22}(q_2)m_2du_2 + \cdots\cdots F_{f2}(q_f)m_fdu_f \\ & \cdots\cdots\cdots \\ & \cdots\cdots\cdots \\ O &= F_{1f}(q_1)m_1du_1 + \cdots\cdots\cdots\cdots\cdots\cdots F_{ff}(q_f)m_fdu_f. \end{aligned}$$

Solving these equations:

$$\frac{dt}{E}=\frac{m_1du_1}{\dfrac{\partial E}{\partial F_{11}}}=\frac{m_2du_2}{\dfrac{\partial E}{\partial F_{21}}}=\dots\dots\dots=\frac{m_fdu_f}{\dfrac{\partial E}{\partial F_{f1}}} \qquad 13\cdot3$$

where E is the determinant formed from the F's. In this transformation of variables we have introduced new coordinates, the u's, with arbitrary constants, the m's. The latter can therefore be chosen in a special way. Let us take the m's to be ±1, the sign being so chosen that, according to 13·3, as t increases all the u's shall increase. This assumes that, as may be proved, neither E nor its derivatives can vanish. It then follows from 13·2 that if initially $a_k < q_k^0 < b_k$, q_k will always remain between these limiting values, and that dq_k will vanish for both $q_k=a_k$ and $q_k=b_k$.

The motion which is expressed by this solution, in the case of simple roots of the equation $p_k=0$, is therefore such that each coordinate oscillates between two limits. Such motions are called librations. If any of the roots between which the initial coordinates lie are multiple, it may be shown that the corresponding coordinate approaches, and reaches after an infinite time, a limiting value. Such motions are called limitation motions.

A particular case is furnished by cyclic coordinates, that is, coordinates which do not themselves appear in the energy function, but only the corresponding velocities or momenta. Such coordinates increase indefinitely with the time, but they have the property, in all the applications to be made, of repeating their values with the period 2π.

Let us now write,

$$I_k=2\int^{b_k} p_k\,dq_k \qquad 13\cdot4$$

where a_k and b_k are the neighboring roots of $p_k=0$ between which q_k lies initially. The I's are functions of the integration constants. Conversely, we can express the integration constants as functions of the I's, or,

$$\alpha_k=\alpha_k(I_1,\,I_2,\,\cdots\cdot\,I_f).$$

The solution of the Hamilton-Jacobi equation, 12·6, may therefore be put in the form,

$$W=W(q_1,\,q_2,\cdots\cdot q_f,\,I_1,\,I_2,\,\cdots\cdots I_f). \qquad 13\cdot5$$

Let us regard this as a substitution function, such as was considered in 8·3. Take the new coordinates to be the w's, and the new momenta to be the I's, divided by 2π. We then have,

$$\left.\begin{aligned} p_k &= \frac{\partial W}{\partial q_k}\\ w_k &= 2\pi\frac{\partial W}{\partial I_k}\end{aligned}\right\} \qquad 13\cdot6$$

And since the new variables form a canonical system,

$$\dot{w}_k = 2\pi \frac{\partial H}{\partial I_k} = 2\pi \frac{\partial \alpha_1}{\partial I_k}$$

$$\dot{I}_k = -2\pi \frac{\partial H}{\partial w_k} = -2\pi \frac{\partial \alpha_1}{\partial w_k} = 0.$$

It follows that

$$w_k = \omega_k t + \delta_k \qquad 13\cdot 7$$

where,

$$\omega_k = 2\pi \frac{\partial \alpha_1}{\partial I_k}. \qquad 13\cdot 8$$

The w's are thus angle variables, introduced in §11, with the ω's as the mean motions. Here, α_1 is the energy function, H, expressed as a function of the I's.

14. CONDITIONALLY PERIODIC SYSTEMS.—Systems of the kind that have just been considered are, under certain conditions, periodic systems. Let any one of the original coordinates, say q_k, increase from q_k^0 to b_k, decrease to a_k, and finally come back to q_k^0; then, observing 12·6, we see that W will increase by I_k. Therefore, by the second of 13·6, w_k will increase by 2π, while there will be no change in any of the other angle variables. It follows, therefore, that the q's are periodic functions of the w's, with the period 2π, and can accordingly be expanded in Fourier series for many variables,

$$q_k = \sum\sum \cdots \sum A_{\tau_1 \tau_2 \cdots} \exp. \left\{ i \sum_{k=1}^{f} \tau_k w_k \right\} \qquad 14\cdot 1$$

where the complex constants, $A_{\tau_1}, \tau_2, \cdots$ are determined by,

$$A_{\tau_1 \tau_2 \cdots} = \frac{1}{(2\pi)^f} \int_0^{2\pi} \cdots \int_0^{2\pi} q_k \exp. \left\{ -i \sum_{k=1}^{f} \tau_k w_k \right\} dw_1\, dw_2 \cdots dw_f \qquad 14\cdot 2$$

when q_k is expressed as a function of the w's. The summations in 14·1 are over all positive and negative integers, $\tau_1, \tau_2 \cdots$.

The same result follows from 12·7. If any one of the coordinates, say q_k, goes through its cycle of values, the left-hand sides of those equations will each increase by

$$\omega_{kj} = 2 \int_{a_k}^{b_k} \frac{\varphi_{kj}\, dq_k}{p_k}, \quad k, j = 1, 2, \cdots f. \qquad 14\cdot 3$$

In particular, the time will increase by ω_{k1}. We can regard equations 12·7 as expressing the q's in terms of $t+\beta_1$, β_2, $\cdots$ β_f, for the q's enter into the right-hand sides of those equations as the upper limits in the integrals. If, then, we introduce the change of variables,

$$\left.\begin{aligned} 2\pi(t+\beta_1) &= \sum_{k=1}^{f} \omega_{k1} w_k \\ 2\pi\beta_j &= \sum_{k=1}^{f} \omega_{kj} w_k \qquad j=2, 3, \cdots f \end{aligned}\right\} \tag{14·4}$$

we shall get the same angle variables, w, as were introduced before. For, solving these linear equations, we get

$$w_k = \omega_k t + \delta_k$$

where

$$\omega_k = \frac{2\pi}{\Delta}\frac{\partial\Delta}{\partial\omega_{k1}}$$

$$\delta_k = \frac{2\pi}{\Delta}\sum_{j=1}^{f} \beta_j \frac{\partial\Delta}{\partial\omega_{kj}}$$

$$\Delta = \begin{vmatrix} \omega_{11} \cdots\cdots\cdots \omega_{f1} \\ \cdots\cdots\cdots\cdots\cdots \\ \omega_{1f} \qquad\qquad \omega_{ff} \end{vmatrix}.$$

Now, since

$$I_k = 2\int_{a_k}^{b_k} p_k\, dq_k$$

we get

$$\frac{\partial I_k}{\partial\alpha_j} = 2\int_{a_k}^{b_k} \frac{\varphi_{kj} dq_k}{p_k} = \omega_{kj}.$$

The energy, α_1, has been expressed as

$$\alpha_1 = H\,(I_1, I_2, \cdots\cdots I_f).$$

Differentiating this expression in turn by α_1, α_2, $\cdots\cdots$ α_f,

$$1 = \sum_{k=1}^{f} \frac{\partial H}{\partial I_k}\frac{\partial I_k}{\partial\alpha_1} = \sum_{k=1}^{f} \frac{\partial H}{\partial I_k}\omega_{k1}$$

$$0 = \sum_{k=1}^{f} \frac{\partial H}{\partial I_k}\frac{\partial I_k}{\partial\alpha_j} = \sum_{k=1}^{f} \frac{\partial H}{\partial I_k}\omega_{kj}.$$

Now differentiate equations 14·4 by the time,

$$2\pi = \sum_{k=1}^{f} \omega_{k1} \dot{w}_k$$

$$0 = \sum_{k=1}^{f} \omega_{kj} \dot{w}_k, \quad j = 2, 3, \cdots f.$$

Hence it follows, by comparing these expressions, that

$$\dot{w}_k = 2\pi \frac{\partial H}{\partial I_k} = \omega_k, \qquad 14\cdot 5$$

in agreement with 11·6 since our action variables are now the $I_k/2\pi$.

We have seen that the motion of the original coordinates, q_k, is periodic in the angle variables, w_k, with the period 2π. We shall now find the conditions that systems of the type here considered may be periodic in the time. Assuming that the motion of the whole system is periodic in the time, let T be the periodic time. When t increases by T, each w_k will increase by $\omega_k T$. If, then, $\omega_k T$ is an integral multiple of 2π, each q_k will return to its initial value after the time T; in other words, the system is periodic in time. We therefore have the f equations,

$$\frac{2\pi T}{\Delta} \frac{\partial \Delta}{\partial \omega_{k1}} = 2\pi m_k$$

where the m's are integers. Multiplying by ω_{kj} and summing for all the k's,

$$\frac{T}{\Delta} \sum_{k=1}^{f} \frac{\partial \Delta}{\partial \omega_{k1}} \omega_{kj} = \sum_{k=1}^{f} m_k \omega_{kj}.$$

But by the properties of determinants,

$$\frac{1}{\Delta} \sum_{k=1}^{f} \frac{\partial \Delta}{\partial \omega_{k1}} \omega_{kj} \quad \begin{aligned} &= 1 \text{ if } j = 1 \\ &= 0 \text{ if } j \neq 1. \end{aligned}$$

It therefore follows that, for the system to be periodic in the time, it must be possible to find a set of integers, m_k, such that,

$$\begin{aligned} T &= m_1\omega_{11} + m_2\omega_{21} + \dots\dots\dots + m_f\omega_{f1} \\ O &= m_1\omega_{12} + m_2\omega_{22} + \dots\dots\dots + m_f\omega_{f2} \\ &\dots\dots\dots \\ &\dots\dots\dots \\ O &= m_1\omega_{1f} + m_2\omega_{2f} + \dots\dots\dots + m_f\omega_{ff}. \end{aligned}$$

15. Absolute Integral-Invariants.—The relative integral-invariant which was introduced in §7 may be interpreted in the following way. In the $2f+1$ dimensional space, the p_k, q_k, t are to be regarded as functions of a single parameter, α. When so expressed the integral-invariant may be written,

$$\int I\,(\alpha)\,d\alpha.$$

By letting α take a continuous succession of values, a one-dimensional manifold, or a "curve" is traced in multi-dimensional space. And this line integral is invariant for all closed one-dimensional manifolds, or curves, which lie wholly on the tube which is formed by tracing the dynamical paths through the sucessive points of any one of the curves. Suppose, now, that the p_k, q_k, t are expressed as functions of two parameters, α and β. Take four points in the plane of α and β whose rectangular coordinates are

A	$\alpha,$	β
B	$\alpha+\delta\alpha,$	β
C	$\alpha,$	$\beta+\delta'\beta$
D	$\alpha+\delta\alpha,$	$\beta+\delta'\beta.$

The symbol δ refers to a variation when only α is changed, and δ' to a variation of β only. Now take the line integral of

$$\sum p_k\,\delta q_k - H\,\delta t$$

around the closed curve $ABDC$. Since $\delta\alpha$ and $\delta'\beta$ are infinitesimal, we can remove the sign of integration, and we get the following contributions to this line integral:

Along AB	$\sum p_k\delta q_k - H\,\delta t$
" BD	$\sum p_k\delta' q_k - H\,\delta' t + \delta\left\{\sum p_k\delta' q_k - H\,\delta' t\right\}$
" DC	$-\left\{\sum p_k\delta q_k - H\,\delta t\right\} - \delta'\left\{\sum p_k\delta q_k - H\,\delta t\right\}$
" CA	$-\left\{\sum p_k\delta' q_k - H\,\delta' t\right\}.$

The integral taken around the closed curve is the sum of these, or,

$$\sum \delta p_k\delta' q_k - \sum \delta' p_k\delta q_k - \delta H\,\delta' t + \delta' H\,\delta t.$$

Written out in full, this is,

$$\sum\left(\frac{\partial p_k}{\partial\alpha}\,\frac{\partial q_k}{\partial\beta} - \frac{\partial p_k}{\partial\beta}\,\frac{\partial q_k}{\partial\alpha}\right)\delta\alpha\,\delta\beta - \left(\frac{\partial H}{\partial\alpha}\,\frac{\partial t}{\partial\beta} - \frac{\partial H}{\partial\beta}\,\frac{\partial t}{\partial\alpha}\right)\delta\alpha\,\delta\beta.$$

If we consider the two-dimensional manifold of p_k, q_k, we shall have,

$$\delta p_k\,\delta q_k = \frac{\partial(p_k, q_k)}{\partial(\alpha, \beta)}\,\delta\alpha\,\delta\beta = \left(\frac{\partial p_k}{\partial\alpha}\frac{\partial q_k}{\partial\beta} - \frac{\partial p_k}{\partial\beta}\frac{\partial q_k}{\partial\alpha}\right)\delta\alpha\,\delta\beta,$$

and similarly for the last term. Therefore the line integral taken around the elementary curve is

$$\sum \delta p_k\,\delta q_k - \delta H\,\delta t.$$

Now a finite closed curve can be divided into an infinite number of infinitesimal closed curves like the one we have just considered, and the line integral around the finite curve will be the sum of the line integrals taken in the same sense around all the infinitesimal curves. It follows, therefore, that the two-dimensional integral,

$$\int\int\sum \delta p_k\,\delta q_k - \delta H\,\delta t \qquad 15\cdot 1$$

is an absolute integral-invariant; that is, this integral is invariant for a manifold that is not closed. When the time is not varied, we get the corresponding absolute integral-invariant introduced by Poincaré,

$$\int\int\sum \delta p_k\,\delta q_k. \qquad 15\cdot 2$$

By continuing this process, we can derive integral-invariants of higher orders, and finally we obtain the integral-invariant, when the time is not varied,

$$\int \cdots 2f \cdots \int \delta p_1\,\delta p_2 \cdots\cdots \delta q_f. \qquad 15\cdot 3$$

The invariance of this integral is an expression of Liouville's theorem. Owing to its importance, we shall give an independent proof. If we introduce new variables, Q_k, P_k in place of the q_k, p_k, we get,

$$\int \cdots \int \delta q_1 \cdots\, \delta p_f = \int \cdots \int \frac{\partial(q_1 q_2 \cdots p_f)}{\partial(Q_1, Q_2, \cdots P_f)}\,\delta Q_1\,\delta Q_2 \cdots \delta P_f.$$

We must show that the Jacobian, or functional determinant, is equal to unity. Suppose the transformation of variables to be made by means of a substitution function

$$W = W(q_k, P_k).$$

Then, by 8·3

$$p_k = \frac{\partial W}{\partial q_k}, \qquad Q_k = \frac{\partial W}{\partial P_k}.$$

Any element of the functional determinant of the type $\frac{\partial p_k}{\partial Q_s}$ may be written

$$\frac{\partial p_k}{\partial Q_s} = \frac{\partial}{\partial Q_s}\frac{\partial W}{\partial q_k} = \sum_{i=1}^{f}\frac{\partial}{\partial q_i}\frac{\partial W}{\partial q_k}\frac{\partial q_i}{\partial Q_s} = \sum_{i=1}^{f}\frac{\partial p_i}{\partial q_k}\frac{\partial q_i}{\partial Q_s} = 0,$$

since the p's and q's are independent variables. The determinant therefore may be written as the product of two determinants,

$$\frac{\partial(q_1,q_2,\cdots q_f)}{\partial(Q_1,Q_2,\cdots Q_f)} \times \frac{\partial(p_1,p_2,\cdots\cdots p_f)}{\partial(P_1,P_2,\cdots P_f)}.$$

Any element of the latter may be written,

$$\frac{\partial p_k}{\partial P_i} = \frac{\partial}{\partial P_i}\frac{\partial W}{\partial q_i} = \frac{\partial Q_i}{\partial q_i}.$$

The product therefore becomes

$$\frac{\partial(q_1,q_2,\cdots\cdots q_f)}{\partial(Q_1,Q_2,\cdots Q_f)} \times \frac{\partial(Q_1,Q_2,\cdots Q_f)}{\partial(q_1,q_2,\cdots\cdots q_f)} = 1,$$

by the properties of functional determinants. A similar result may be obtained by using a substitution function $W = W(q_k, Q_k)$.

16. ADIABATIC INVARIANTS.—Let us now return to equation 6·2 and regard the parameters a_r as subject to variation. If we consider

$$S = \int_{t_0}^{t_1} L\,dt$$

as a definite integral, and therefore a function of its limits, we shall have, by the ordinary rules of varying a definite integral,

$$\Delta S = \Big[L\,\Delta t\Big]_{t_0}^{t_1} + \int_{t_0}^{t_1}\delta L\cdot dt\cdot$$

It is assumed here that L does not involve the time explicitly, so that under the sign of integration Δ may be replaced by δ. Since $\delta L = \delta T - \delta V = 2\delta T - \delta H$, we have,

$$\Delta S = \Big[L\;\Delta t\Big]_{t_0}^{t_1} + \int_{t_0}^{t_1} 2\;\delta T\cdot dt - \int_{t_0}^{t_1}\delta H\cdot dt. \qquad 16\cdot 1$$

In 6·2 we can write.

$$\Delta q_k = \delta q_k + \dot{q}_k\;\Delta t,$$

so that it becomes,

$$\Delta S = \Big[\sum p_k\,\dot{q}_k\,\Delta t + \sum p_k\,\delta q_k - H\;\Delta t\Big]_{t_0}^{t_1} + \int_{t_0}^{t_1}\sum\frac{\partial L}{\partial a_r}\,\delta a_r\cdot dt$$

and with

$$2T = \sum p_k \dot{q}_k$$

we get,

$$\Delta S = \Big[L\,\Delta t\Big]_{t_0}^{t_1} + \Big[\sum p_k\,\delta q_k\Big]_{t_0}^{t_1} + \int_{t_0}^{t_1} \sum \frac{\partial L}{\partial a_r}\,\delta a_r\,dt.$$

Comparing this with 16·1, we have,

$$\int_{t_0}^{t_1} 2\,\delta T \cdot dt = \Big[\sum p_k\,\delta q_k\Big]_{t_0}^{t_1} + \int_{t_0}^{t_1} \left(\delta T + \delta V + \sum \frac{\partial L}{\partial a_r}\,\delta a_r\right) dt.$$

We shall now assume that the parameters a_r enter in the expression for the potential energy, V, but not in the kinetic energy, T. Then

$$\frac{\partial L}{\partial a_r} = -\frac{\partial V}{\partial a_r} = A_r,$$

and A_r will be the generalized component of force exerted by the system. The work done by the system against the constraints represented by the a_r will then be

$$\sum A_r\,\delta a_r.$$

Now by the principle of the conservation of energy, or the first law of thermodynamics, if a quantity of heat, $\delta' Q$ is added to a system,

$$\delta' Q = \delta T + \delta V + \sum A_r\,\delta a_r$$

and we shall have,

$$\int_{t_0}^{t_1} 2\,\delta T \cdot dt = \Big[\sum p_k\,\delta q_k\Big]_{t_0}^{t_1} + \int_{t_0}^{t_1} \delta' Q \cdot dt. \qquad 16{\cdot}2$$

Consider now a dynamical system formed of molecules, or atoms. The q_k, p_k determine the configuration and motion of the individual atoms and are beyond our control. The a_r, however, we shall regard as coordinates over which we have control; they are the coordinates that determine the state of the system in so far as it is subject to our observation. Let us assume that the system is periodic in the q_k, p_k, with a periodic time τ. Choose the time interval $t_1 - t_0 = n\tau$, a whole number of periods. Then the configuration of the system, as far as the p_k, q_k, are concerned will be the same at t_1 as at t_0, but a change will, in general, have been made in the a_r. Denoting mean values by a bar, we get, then, from 16·2

$$\delta(n\tau\, 2\overline{T}) = n\tau\,\overline{\delta' Q}$$

or,

$$\frac{1}{\tau}\,\delta(\tau\, 2\overline{T}) = \overline{\delta' Q}.$$

Writing $\tau = 1/\nu$, where ν is the frequency,

$$\nu\delta\left(\frac{2\overline{T}}{\nu}\right) = \overline{\delta' Q}. \qquad 16\cdot 3$$

If no heat is added to or taken from the system during this variation, $\overline{\delta' Q} = 0$, and

$$\frac{2\overline{T}}{\nu} \qquad 16\cdot 4$$

is an adiabatic invariant.

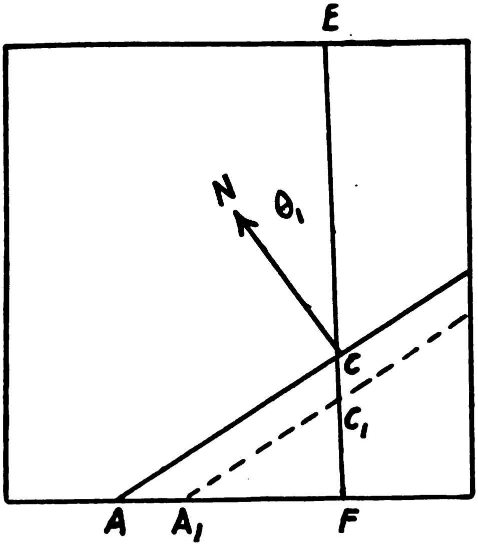
By the second law of thermodynamics, the heat, $\delta' Q$, added to a system in a small change is not the exact differential of a function, but the absolute temperature, θ, is an integrating factor, so that $\delta' Q/\theta$ is an exact differential. From 16·3 we therefore see that θ must be of the form,[1]

$$\theta = \nu f\left(\frac{\overline{T}}{\nu}\right)$$

or,

$$\overline{T} = \nu\varphi\left(\frac{\theta}{\nu}\right) \qquad 16\cdot 5$$

17. **Vibration Frequencies of a Continuous Medium.**—Given a continuous medium capable of transmitting waves of all frequencies, we can find the number of frequencies in the range from ν to $\nu + d\nu$ in a simple manner.[2]. Consider a cube of the medium whose edge is l.



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[1] Försterling, Annalen der Physik 47, p. 1127, 1915.

[2] Flamm, Phys. Zeit., 19, p. 116, 1918.

What we want to do is to find the distribution of the vibration frequencies regarding the medium as continuous. Let AB be any wave front, of either longitudinal or transverse waves, and CN the normal drawn in the direction of propagation. In order that AB may be the wave-front of a series of standing waves it is necessary that at E and F the phases should be equal or opposite. Let A_1B_1 be a parallel wave-front, one-half a wave-length, λ, behind AB. We then have:

$$CC_1 = \frac{\lambda}{2\cos\theta_1}.$$

The condition for standing waves is:

$$n_1 CC_1 = l$$

when n_1 is an integer. Taking into account also the two directions at right angles to FE, we find:

$$\frac{\lambda}{2} n_1 = l\cos\theta_1$$

$$\frac{\lambda}{2} n_2 = l\cos\theta_2$$

$$\frac{\lambda}{2} n_3 = l\cos\theta_3.$$

Squaring and adding, and putting $v = \lambda\nu$, where v is the velocity of the waves, we have

$$n_1^2 + n_2^2 + n_3^2 = \left(\frac{2\,l\,\nu}{v}\right)^2.$$

Now from any point as origin draw a system of rectangular axes. The number of points, $x = n_1$, $y = n_2$, $z = n_3$, which satisfy the above relation for which ν lies between 0 and ν will be the volume of an octant of the sphere of radius $2\nu l/v$. An octant is taken because n_1, n_2, n_3 are restricted to the positive integers. Therefore the number of frequencies per unit volume, between $\nu = 0$ and $\nu = \nu$, is

$$N = \frac{4}{3}\pi\left(\frac{\nu}{v}\right)^3.$$

The number for which ν lies between ν and $\nu + d\nu$ is,

$$dN = \frac{4\pi\nu^2}{v^2}\frac{d}{d\nu}\left(\frac{\nu}{v}\right).$$

If v is independent of ν this is simply,

$$dN = 4\pi\frac{\nu^2}{v^3}. \qquad 17{\cdot}1$$

But if v depends upon ν, we can introduce the group velocity, u, given by,

$$\frac{1}{u} = \frac{d}{d\nu}\left(\frac{\nu}{v}\right)$$

and get

$$dN = 4\pi\frac{\nu^2}{v^2 u}. \qquad 17\cdot2$$

If the medium can transmit both transverse and longitudinal waves, with velocities v_t and v_l respectively, we shall have for the number of frequencies in the range ν, $\nu + d\nu$,

$$dN = 4\pi\nu^2\left(\frac{2}{v_t^3} + \frac{1}{v_l^3}\right). \qquad 17\cdot3$$

The factor 2 comes from considering that the transverse waves may be polarized in two mutually perpendicular directions. In this, we have supposed that the velocity is independent of the frequency.

18. **Wien's Law.**—Now consider electromagnetic radiation. This may be polarized in two mutually perpendicular directions, and the velocity of the waves is c, the velocity of light. The number of frequencies in the range ν, $\nu + d\nu$ is therefore, by 16·3,

$$dN = \frac{8\pi\nu^2}{c^3}.$$

In 15·5 we obtained an expression for the mean kinetic energy of a periodic system. If we regard the ether as a dynamical system, capable of simple harmonic vibrations, the whole energy will be twice the mean kinetic energy, and therefore the energy in unit volume, corresponding to frequencies in the range ν, $\nu + d\nu$, will be of the form

$$u_\nu\, d\nu = \frac{\nu^3}{c^3}\varphi\left(\frac{\theta}{\nu}\right)d\nu. \qquad 18\cdot1$$

This is the expression of Wien's law. Classical dynamics has never been successful in obtaining an expression for the function $\varphi(\theta/\nu)$ which agrees with experiment. The quantum theory had its origin in Planck's success in obtaining a satisfactory expression for this function.

CHAPTER II

THE HYPOTHESES OF THE QUANTUM THEORY

19. Canonical Ensembles.—When the attempt is made to apply the principles of dynamics to systems formed of very large numbers of elements, each element itself being a dynamical system of a limited number of degrees of freedom, the difficulty of dealing with the enormous number of equations of motion is, in general, insurmountable. It is only in special cases, like crystals, where there is a regularity in the distribution of the elements, that progress can be made without introducing hypotheses to supplement the general principles of dynamics.

Let us consider a dynamical system, consisting of a very large number, N, of elements. These elements may be supposed to be the molecules of a gas, or atoms of a solid, each with f degrees of freedom. To specify the configuration and motion of the whole system at any time, fN generalized coordinates and an equal number of generalized velocities, or momenta, are, in general, required. These we shall take as the canonical variables and so we can make use of the canonical equations of motion introduced in §5. A point in multi-dimensional space of $2n = 2Nf$ dimensions will then represent the instantaneous state, or phase, of the system. In this multi-dimensional space, $dq_1dq_2 \cdots\cdots dq_ndp_1dp_2 \cdots\cdots dp_n$ will be a differential element of extension-in-phase.

Suppose now that we take a very large number, ν, of systems, an ensemble of systems, all similarly constituted, and distribute their representative points throughout the whole extension-in-phase. Let the number of systems whose representative points lie in the element of extension-in-phase $dq_1 \cdots\cdots dp_n$ be

$$d\nu = D\, dq_1 \cdots\cdots dp_n.$$

D, a function of the q's and p's, is termed the density-in-phase. Writing $P = D/\nu$, we get, integrating over the whole extension-in-phase,

$$1 = \int \cdots\cdots \int P\, dq_1 \cdots\cdots dp_n. \qquad 19{\cdot}1$$

Now Gibbs made the hypothesis that in the stationary state, P has the form,

$$P = e^{\frac{\psi - \epsilon}{\theta}}, \qquad 19{\cdot}2$$

where ϵ is the energy of any one of the systems, expressed as a function of $q_1, q_2, \cdots\cdots q_n p_1, p_2, \cdots\cdots p_n$, and ψ and θ are constants for the whole ensemble of systems. Such an ensemble Gibbs names a canonical ensemble, and P its coefficient of probability.

We get from 19·1, using this hypothesis,

$$e^{-\frac{\psi}{\theta}} = \int \cdots\cdots \int e^{-\frac{\epsilon}{\theta}} dq_1 \cdots dp_n. \qquad 19\cdot 3$$

Differentiating with respect to θ, we have,

$$\psi - \theta\frac{d\psi}{d\theta} = \int \cdots\cdots \int \epsilon\, e^{\frac{\psi-\epsilon}{\theta}}\, dq_1 \cdots dp_n = \bar{\epsilon},$$

where $\bar{\epsilon}$ is the average value of the energy of a system in the ensemble. And if we take the average of the energies of the systems in a canonical ensemble to be the same as the energy, E, of the particular system we are interested in, we shall have,

$$E = \psi - \theta\frac{d\psi}{d\theta}. \qquad 19\cdot 4$$

This formula is the same as the known thermodynamic formula, if we take ψ as the free energy, and θ as proportional to the absolute temperature of the system.

As in §6, we may suppose ϵ, the energy of a system in the ensemble, to depend, in addition to the q's and p's, upon certain parameters, a_k, limited in number; these may depend upon bodies external to our system. In any canonical ensemble, these parameters, like ψ and θ, are supposed to be the same for all the systems. Suppose now the external conditions to be varied so that ψ, θ and the a's are also varied. When a_k is varied,

$$\frac{\partial\epsilon}{\partial a_k}\,\delta a_k = -A_k\,\delta a_k$$

and the work done by the system in the variation of the a's is (*cf.* §16),

$$\sum A_k\,\delta a_k = \delta' W.$$

Taking the logarithms of both sides in 19·3,

$$-\frac{\psi}{\theta} = \log \int \cdots \int e^{-\frac{\epsilon}{\theta}} dq_1 \cdots dp_n \qquad 19\cdot 5$$

and taking the variations of ψ, θ, and the a's, we get,

$$\frac{\psi}{\theta^2}\,\delta\,\theta - \frac{1}{\theta}\,\delta\,\psi = \frac{1}{\theta}\,\overline{\delta' W} + \frac{\bar{\epsilon}}{\theta^2}\,\delta\,\theta.$$

The bars denote average values for the whole ensemble. If we write,

$$\eta = \log P,$$

so that,

$$\theta\,\bar{\eta} = \psi - \bar{\epsilon},$$

and take the variation of this equation, we get, combining with the former,

$$-\delta\,\bar{\eta} = \frac{\delta\,\bar{\epsilon} + \overline{\delta' W}}{\theta}.$$

By the law of the conservation of energy, the numerator on the right is the average heat supplied to the systems of the ensemble. If, then, we take $-\bar{\eta}$ as proportional to the entropy of the actual system we are interested in, we arrive at the equation expressing the second law of thermodynamics. If, in particular, we take

$$-\bar{\eta} = \frac{S}{k}$$

$$\theta = kT,$$

where S is the usual thermodynamic entropy, and T the absolute temperature on the usual scale, we get the usual thermodynamic relation, and in addition,

$$S = -k \log P.$$

This is analogous to Boltzmann's relation between entropy and probability, if it be remembered that our P is a mathematical probability, and therefore less than unity, so that its logarithm is negative.

20. The systems with which we shall be concerned are such that their energies may be expressed as the sums of the energies of their N elements. We therefore write,

$$\epsilon = \epsilon_1 + \epsilon_2 + \cdots\cdots + \epsilon_N,$$

where ϵ_1, say, depends only on $q_1, \cdots\cdot q_f, p_1, \cdots\cdot p_f$; ϵ_2 on $q_{f+1}, \cdots\cdot p_{2f}$, etc. We can therefore write 19·3,

$$e^{-\frac{\psi}{\theta}} = \prod_{s=1}^{N} \int \cdots\cdot \int e^{-\frac{\epsilon_s}{\theta}} dq_1 \cdots dp_f.$$

This, of course, assumes that the N elements are all similar dynamical systems of f degrees of freedom, so that the ϵ's are all the same functions of the $2f$ coordinates and momenta, which are now denoted, in general, by $q_1 \cdots\cdot p_f$, and that the limits of integration are the same for each of the separate products. We can therefore write, taking the logarithm,

$$-\psi = N\,\theta \log \int \cdots \int e^{-\frac{\epsilon}{\theta}} dq_1 \cdots dp_f \qquad 20\cdot1$$

where ψ is the free energy of the system consisting of N similar elements, ϵ is now the energy of a single element expressed as a function of the $2f$ variables $q_1, \cdots\cdot p_f$, and $\theta = kT$.

From 19·4 we now find,

$$E = N \frac{\int \cdots \int \epsilon e^{-\frac{\epsilon}{\theta}} dq_1 \cdots dp_f}{\int \cdots \int e^{-\frac{\epsilon}{\theta}} dq_1 \cdots dp_f}$$

where the integrations are to be taken over the whole range of values of the q's and p's. This is an expression for $E = N\bar{\epsilon}$, where $\bar{\epsilon}$ is now the average energy of the N elements. Let, now,

$$w\, dq_1 \cdots\cdots dp_f$$

be the number of elements (molecules) of our system of N elements whose coordinates and momenta lie within the element of extension-in-phase $dq_1\, dq_2 \cdots\cdot dp_f$. We then have,

$$E = \int \cdots\cdots \int w\, \epsilon\, dq_1\, dq_2 \cdots\cdots dp_f.$$

Comparing this with the former value of E, we find,

$$w = \frac{e^{-\frac{\epsilon}{\theta}}}{\int \cdots \int e^{-\frac{\epsilon}{\theta}} dq_1 \cdots dp_f}.$$

By 20·1 the denominator may be written $e^{-\frac{\psi}{N\theta}}$. Therefore,

$$w = e^{\frac{1}{\theta}\left(\frac{\psi}{N} - \epsilon\right)} \qquad 20\cdot 2$$

ψ is the free energy of the system of N molecules; so ψ/N may be interpreted as the free energy of a single molecule. We thus get the Maxwell-Boltzmann law of distribution expressed in the same form as the Gibbs Canonical distribution. In other words, a system of N molecules, where N is very large, may be regarded as an ensemble of similar systems, each system one of our elements, or molecules, of f degrees of freedom.

If the kinetic energy, ϵ_p, is a homogeneous quadratic function of the momenta, we can write, by Euler's theorem,

$$\epsilon_p = \frac{1}{2}\sum_{s=1}^{f} p_s \frac{\partial \epsilon}{\partial p_s}.$$

We can take

$$\frac{1}{2} p_s \frac{\partial \epsilon}{\partial p_s}$$

as the kinetic energy of one degree of freedom of our elements (molecules). Taking the average of this for all the N elements we get the result,

$$\overline{\frac{1}{2} p_s \frac{\partial \epsilon}{\partial p_s}} = \frac{1}{2}\theta = \frac{1}{2}kT. \qquad 20\cdot 3$$

This is the so-called theorem of the equipartition of energy. If the potential energy is a homogeneous quadratic function of the coordinates, with constant coefficients, the same theorem may be proved for the potential energy of each degree of freedom, but in general it does not hold for the potential energy. For simply periodic systems, where the average potential energy is equal to the average kinetic energy, the whole energy is, on the average, for each degree of freedom, kT.

21. THE THEORY OF STATIONARY STATES.—One of the fundamental hypotheses that Bohr introduced in his theory of spectral lines is that of stationary states. According to this hypothesis, an atom can exist only in a number of discrete stationary states; radiation takes place from an atom only when it passes from one to another stationary state. The remarkable success that has resulted from the application of this hypothesis to spectroscopy, in the region of Röntgen ray wave-lengths as well as ordinary light, makes it appear possible that this hypothesis may have a wider field of application.

Let us return to equation 20·1 which gives an expression for the free energy of a system consisting of N similar elements, each of f degrees of freedom. In that expression, ϵ is the energy of one of the elements of the system, and is a continuous function of the $2f$ variables, $q_1, \cdots p_f$. Suppose, now, that each element can exist only in discrete states. Let ϵ_n be the energy of an element in the nth state. The continuous $2f$-multiple integration in 20·1 may now be replaced by a summation taken over the different stationary states. We shall introduce a factor p_n, which may be regarded as a weight factor for the stationary states. Thus we get,

$$F = -NkT \log \sum p_n e^{-\frac{\epsilon_n}{kT}} \qquad 21\cdot 1$$

as an expression for the free energy of the system. The whole energy, E, of the system is, by 19·4

$$E = N \frac{\sum p_n \epsilon_n e^{-\frac{\epsilon_n}{kT}}}{\sum p_n e^{-\frac{\epsilon_n}{kT}}}.$$

If, of the N elements, there are N_n of them in the nth stationary state, with energy ϵ_n, then,

$$E = \sum N_n \epsilon_n,$$

or writing $w_n = N_n/N$, the proportional number of elements in the nth state, we find on comparing the two values of E,

$$w_n = \frac{p_n e^{-\frac{\epsilon_n}{kT}}}{\sum p_n e^{-\frac{\epsilon_n}{kT}}}. \qquad 21\cdot 2$$

22. Quantum Conditions for Conditionally Periodic Systems.—The next problem to be considered is the method of specifying the stationary states. As the very idea of stationary states is opposed to general dynamical principles, it is clear that new hypotheses must be made.

In the $2f+1$ – dimensional space of the p's, q's, t, consider any tube of paths described in §7. The integral,

$$\int \cdots \int \delta p_1 \, \delta p_2 \cdots\cdots \delta q_f$$

extended over the limits bounding the tube, may be regarded as the section of the tube, of $2f$ dimensions, at any time. This integral we know is invariant. Not only has it a constant value along the tube, but it keeps the same value when any other canonical variables are substituted for the p's and q's. In a conditionally periodic system we have seen that it is always possible to introduce action and angle variables, denoted by the α_k and w_k respectively. So the constant section of the tube of paths may be written,

$$\prod_{k=1}^{f} \int \int \delta\alpha_k \, \delta w_k$$

the limits being taken so as to include the tube. Let us now suppose that the space belonging to our dynamical system of f degrees of freedom has such properties that there is a definite limit to its division into tubes of paths. Let the elementary tube, or the unit tube, which is incapable of being subdivided, have the section h^f, where h is a universal constant of dimensions, Action = Energy × Time. For a unit tube, therefore,

$$\prod_{k=1}^{f} \int \int \delta\alpha_k \, \delta w_k = h^f \qquad 22 \cdot 1$$

the integrals being taken so as to include the unit tube. Now the operation represented by δ is a virtual displacement. Our action and angle variables are independent. A virtual displacement can therefore be imagined such that only α_k and w_k are varied; in other words, the system reduced to one degree of freedom. But for such a system,

$$\int \int \delta w_k \, \delta\alpha_k = h, \qquad 22 \cdot 2$$

where $k = 1, 2, \cdots f$.

Consider now the relative integral-invariant,

$$\int \sum p_k \, \delta q_k - H \, \delta t$$

the integration being around any tube of paths. Let this be extended around the section of the unit tube at time t. Then,

$$\int \sum p_k \, \delta q_k$$

is invariant, since $\delta t = 0$. In a conditionally periodic system we have seen that each q_k oscillates between its two libration limits, a_k and b_k. Suppose, now, at time t, we give each q_k a series of virtual displacements such that it goes through its cycle of value. A closed curve thus results, and our relative integral-invariant becomes,

$$\int \sum p_k \, \delta q_k = \sum I_k \qquad 22 \cdot 3$$

where

$$I_k = 2 \int_{a_k}^{b_k} p_k \, \delta q_k.$$

The tube formed by constructing the paths through each point of this closed curve we shall take as the unit tube.

It has been shown that the relative integral invariant

$$\int \sum p_k \, \delta q_k$$

is equal to the absolute integral invariant

$$\int \int \sum \delta p_k \, \delta q_k$$

where each double integral is taken over the projection of the section of the tube on the corresponding $p_k - q_k$ plane. The value of this integral is unchanged when any other canonical variables are introduced. So use action and angle variables, which can always be done in a conditionally periodic system. We thus have, for a unit tube,

$$\sum \int \int \delta \alpha_k \, \delta w_k = \sum I_k = \sum h = fh \qquad 22 \cdot 4$$

by 22·2. As before, we can reduce our system to one degree of freedom, and so get, for a unit tube,

$$I_k = h.$$

By the assumed properties of the space belonging to our dynamical system, there can be only tubes of paths which are multiples of unit tubes. So, for any tube of paths, we get,

$$I_k = n_k h,$$

where n_k is any positive integer. We thus arrive at quantum conditions, or equations specifying the stationary states of our system, in the form,

$$2\int_{a_k}^{b_k} p_k\,\delta q_k = n_k h. \qquad 22\cdot 5$$

If any of the generalized coordinates, q_k, are angles, q_j, we get, in going around a tube once, the corresponding condition,

$$\int_0^{2\pi} p_j\,\delta q_j = n_j h. \qquad 22\cdot 6$$

In terms of the action and angle variables, we get, from 22·2, for any tube of paths,

$$\int\int \delta w_k\,\delta\alpha_k = n_k h. \qquad 22\cdot 7$$

Now the limits for w_k, the angle variable, are 0 and 2π. Hence it follows that

$$\alpha_k = \frac{n_k h}{2\pi} + \text{const.}$$

But it has been seen that $I_k/2\pi = \alpha_k$; it therefore follows that the value of the integration constant is zero, and the quantum conditions become

$$2\pi\alpha_k = n_k h. \qquad 22\cdot 8$$

The quantum conditions for systems of many degrees of freedom were first given, in a form equivalent to 22·5, by W. Wilson[1]; somewhat later they were given by Schwarzschild[2] in the form 22·8.

23. DEGENERATE SYSTEMS.—In the last section it was assumed that a conditionally periodic system of f degrees of freedom requires f angle variables, and an equal number of action variables, for its specification, and that the energy can be expressed as a function of the f action variables. It frequently happens, however, that certain of the action variables appear in the energy function in linear combinations. Remembering that in the integration method of §11 these action variables were introduced as integration constants, this means that the number of independent action variables is less than the number of degrees of freedom of the system. Such systems are named degenerate, "entartete." Since the mean motions are given by 11·6 degenera systems may be characterized by the existence of a certain number of commensurabilities between the mean motions. Let us assume that there are r such relations, of the type,

$$m_{1k}\,\omega_1 + m_{2k}\,\omega_2 + \cdots + m_{fk}\,\omega_f = 0, \quad k = 1, 2, \cdots r,$$

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[1] Phil. Mag. XXIX, p. 795, 1915.

[2] Berlin Berichte, 1916, p. 548.

with the m's any positive or negative integers. We can now make a canonical transformation of variables,

$$u_k = m_{1k} w_1 + m_{2k} w_2 + \cdots\cdots m_{fk} w_f$$

so that

$$\dot{u}_k = 0,$$

and a corresponding transformation of action variables, so that, in place of

$$\alpha_1 \ \alpha_2, \ \cdots\cdot \ \alpha_f, \quad w_1 \ w_2, \ \cdots\cdots \ w_f$$

the system is specified in terms of

$$\left.\begin{array}{l} \alpha_1, \alpha_2, \ \cdots\cdot \ \alpha_s, \quad w_1, w_2, \ \cdots\cdots \ w_s \\ \text{and} \\ \beta_1, \beta_2, \ \cdots\cdot \ \beta_r, \quad u_1, u_2, \ \cdots\cdots \ u_r \end{array}\right\} \qquad 23 \cdot 1$$

with $r+s=f$.

The new α's and w's are now independent action and angle variables; there are no relations of commensurability between them. The β's and u's, on the other hand, are now constants, not only along any dynamical path, but constants of the dynamical system, not subject to variation as we go to a neighboring path. The dynamical system is now specified in terms of s pairs of action and angle variables.

Instead of being a manifold of $2f$ dimensions, the section of the tube of paths is now a manifold of only $2s$ dimensions. Therefore, for a unit tube, we shall take,

$$\prod_{k=1}^{s} \int\int \delta\alpha_k \, \delta w_k = h^s.$$

As our variables are now independent, by proper constraints we can reduce the system to one degree of freedom, and get,

$$\int\int \delta\alpha_k \, \delta w_k = h, \quad k = 1, 2, \cdots s.$$

The unit tube of paths is defined as in the last section, and proceeding as before, we get,

$$\sum_{k=1}^{f} \int p_k \, \delta q_k = \sum_{k=1}^{f} I_k = \sum \int\int \delta p_k \, \delta q_k = \sum_{k=1}^{s} \int\int \delta\alpha_k \, \delta w_k = sh.$$

Therefore, we cannot conclude, as before, that $I_k = n_k h$, for any tube of paths, but we can only conclude that

$$\sum m_k I_k = nh \qquad 23 \cdot 2$$

where the m's are any positive or negative integers.

In a degenerate system there is no difficulty in expressing the energy in the different states in terms of a set of integers; it will readily be seen that the same energy will, in general, result from different combinations of these integers. But there is difficulty in assigning the weight, p_n, to the different stationary states. In a non-degenerate system, it is clear that the weight-factor, p_n, introduced in §20, should be the same for all the states, and so we can put $p_n=1$. For degenerate systems, on the other hand, the natural assumption to make is that the weight factor for any state, specified by its energy, is equal to the number of different combinations of the integers, n_k, or quantum numbers, that give that energy. We shall see later how Bohr's correspondence principle may be used to exclude certain combinations.

In a degenerate system it is possible to find more than one set of coordinates and momenta, the *q's* and *p's*, in which the Hamilton-Jacobi equation can be integrated by the method of separation of the variables. The result of this is that different interpretations can be given to the actual motions of the system in the various stationary states.

24. ADIABATIC INVARIANTS.—When external forces act upon the dynamical systems that we have been considering it is to be expected that the stationary states will undergo changes. In this connection Ehrenfest[1] has called attention to the importance of the adiabatic invariant introduced in §16. The effect of the external forces is supposed to be represented by the parameters, a_r, involved in the kinetic potential, L; for a periodic system we have seen that the average, taken over many periods of the motion, of the kinetic energy, divided by the frequency is invariant, provided that the increase in the energy of the system is, on the average, equal to the work done on the system; or, in other words, no heat added to or taken from the system. This condition may be supposed to be satisfied if the parameters, a_r, vary so slowly that the system passes through a succession of stationary states differing infinitesimally from each other. It is easy to show that the quantum conditions, for periodic systems, satisfy this requirement. Periodic systems are degenerate, in the sense of the last section, and for these the quantum conditions are given by 23·2. Now we have put,

$$I_k=2\int_{a_k}^{b_k} p_k\,\delta q_k.$$

If we let the virtual displacement, δq_k, be the actual displacement in the time dt, $\delta q_k=dq_k=\dot{q}_k\,dt$. Suppose that in one period of the motion, q_k goes through its cycle of values m_k times; then,

$$\int_0^\tau 2\,T dt=\int_0^\tau \sum p_k\,\dot{q}_k\,dt=\sum m_k I_k=\frac{2\overline{T}}{\nu},$$

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[1] Phil. Mag. XXXIII, 1917, p. 500.

where $\nu = 1/\tau$. It therefore follows that $\sum m_k I_k$ is invariant; the quantum conditions require this to have the discrete values nh, where n is an integer.

It has been shown by Burgers,[1] that for conditionally periodic systems, when there are no relations of commensurability between the mean motions, the I_k separately are adiabatic invariants, provided that during the variation of the parameters, a_r, the system does not pass through a degenerate state. Therefore the quantum conditions, 22·5, for such systems are consistent with this principle.

For dynamical systems in general, which are not conditionally periodic, very little progress has been made in formulating the quantum conditions necessary to specify the stationary states. And the importance of adiabatic invariants lies mainly in this, that they serve as restrictions which, it would seem, must be observed, in the formulation of such conditions.

There is, however, an important class of dynamical systems to which the quantum conditions can be applied even when the Hamilton-Jacobi equation cannot be integrated by the method of separation of the variables. These are systems that may be regarded, to a first approximation, as conditionally periodic systems, and the departure from such systems considered as perturbations which may be allowed for by the method of successive approximations. Born and Pauli[2] have given full details for carrying out this process, following a method first used by Bohr. A simple application of this process, to one degree of freedom, will be given in the next chapter. Epstein,[3] also, has developed a different method for dealing with such cases.

25. Planck's Quantum Conditions.—Without making use of the theory of stationary states, Planck[4] has developed a different method for introducing the discontinuities inherent in a quantum theory. Formally, his conditions apply to any dynamical system, whether conditionally periodic or not.

Let us consider, as before, a dynamical system of f degrees of freedom, specified by the canonical coordinates and momenta, $q_1 \cdots\cdots p_f$. The first problem is to divide the whole extension-in-phase into cells. A single equation,

$$g_1(q_1, q_2, \cdots\cdots p_f) = g_1 = \text{const.}$$

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[1] Phil. Mag. XXXIII, 1917, p. 514.

[2] Zeit. für Physik, 10, 1922, p. 137.

[3] Zeit. für Physik, 8, pp. 211, 305; 9, p. 92, 1922; Physical Review, 19, 578, 1922.

[4] Berichte deut. Phys. Ges., 1915, 438; Annalen der Physik, 50, 1916, p. 385.

represents a manifold of $2f-1$ dimensions in the $2f$-dimensional space; we may call it a hypersurface. Let us construct the hypersurfaces corresponding to the values of the constant,

$$g_1 = 0,\ h,\ 2h,\ \cdots\cdot\ n_1h,\ \cdots$$

where n_1 is any integer, and h a universal constant. Similarly, we may construct other families of hypersurfaces,

$$\begin{aligned} g_2 &= 0,\ h,\ 2h,\ \cdots\cdots\ n_2h,\ \cdots \\ g_3 &= 0,\ h,\ 2h,\ \cdots\cdots\ n_3h,\ \cdots \\ &\cdots\cdots \\ &\cdots\cdot \end{aligned}$$

In this way the whole extension-in-phase may be divided into cells. These hypersurfaces are to be determined by the solution of the equations of motion of the system. This solution will, in general, show that there are certain singular hypersurfaces which the representative point of the system cannot cross during the motion. Such singular surfaces it is natural to take as the bounding hypersurfaces. It may happen that two or more degrees of freedom of the system lead to the same hypersurface. Such degrees of freedom Planck calls coherent; they correspond to the degrees of freedom which, in a degenerate system, become merged into one.

We have arrived at a division of the whole extension-in-phase into cells bounded by these hypersurfaces. The cell bounded by the hypersurfaces,

$$\begin{aligned} g_1 &= n_1h \text{ and } g_1 = (n_1+1)h \\ g_2 &= n_2h \text{ and } g_2 = (n_2+1)h \\ &\cdots\cdots \\ &\cdots\cdots \end{aligned}$$

will be denoted as the cell $(n_1,\ n_2,\ \cdots)$. Consider, now, a differential element, dG, of extension-in-phase, bounded by the surfaces g_1 and g_1+dg_1, g_2 and g_2+dg_2, $\cdots\cdot$. Its value is,

$$dG = \int\int_{g_1}^{g_1+dg_1} \int\int_{g_2}^{g_2+dg_2} \cdots\cdot \int\int dq_1 \cdots\cdot dp_f.$$

Assume that it is possible to express this in the form,

$$dG = (dg_1)^i\,(dg_2)^j\,(dg_3)^k \cdots\cdots$$

where $i,\ j,\ k,\ \cdots$ are the numbers of coherent degrees of freedom, so that

$$i+j+k+\ \cdots\cdot\ =f,$$

and g_1, g_2, $\cdots$ are all independent of each other. Then the extension-in-phase bounded by the surfaces $g_1=0$ and $g_1=n_1h$, $g_2=0$ and $g_2=n_2h$, $\cdots\cdot$ is,

$$G=\int_0^{n_1h}\int_0^{n_2h}\cdots\cdots\int(dg_1)^i(dg_2)^j\cdots\cdot=(n_1h)^i(n_2h)^j(n_3h)^k\cdots\cdot.$$

The volume of the cell $(n_1, n_2, \cdots)$ is therefore,

$$G_{n_1n_2\cdots}=\left\{\left[(n_1+1)^i-n_1^i\right]\left[(n_2+1)^j-n_2^j\right]\cdots\cdots\right\}h^f=p_{n_1n_2\cdots}G_{o,o,o}\cdots \tag{25·1}$$

where

$$p_{n_1n_2\cdots}=\left[(n_1+1)^i-n_1^i\right]\left[(n_2+1)^j-n_2^j\right]\cdots\cdot$$

and

$G_{0000}=h^f=$ volume of cell $(0, 0, 0\cdots)$. In particular, if all the degrees of freedom are incoherent, so that

$$i=j=k=\cdots=1$$

then it follows that

$$p_{n_1,n_2,n_3\cdots}=1.$$

In the expression 20·1 for the free energy of a dynamical system, we now, as in the theory of stationary states, replace the continuous integration over the whole extension-in-phase by a summation over the finite cells. Let us denote by $\bar{\epsilon}_{n_1,n_2,n_3}$, $\cdots\cdot$ the average energy of one of the elements (molecules) of the system when it is in the cell $(n_1, n_2, \cdots)$. We then get,

$$\psi=-N\Theta\log\sum_{n_1}\sum_{n_2}\cdots\cdot\sum p_{n_1n_2\cdots}\exp.\left\{-\frac{\bar{\epsilon}_{n_1n_2\cdots}}{\Theta}\right\}. \tag{25·2}$$

$p_{n_1n_2}\cdots$ is the weight assigned to the cell $(n_1, n_2\cdots)$. The energy of the system of N elements, given by 19·4 becomes,

$$E=N\frac{\sum\cdots\sum\bar{\epsilon}_{n_1n_2\cdots}\,p_{n_1n_2\cdots}\exp.\left\{-\frac{\bar{\epsilon}_{n_1n_2\cdots\cdots}}{\Theta}\right\}}{\sum\cdots\sum p_{n_1n_2\cdots}\exp.\left\{-\frac{\bar{\epsilon}_{n_1n_2\cdots\cdots}}{\Theta}\right\}}.$$

But we also have,

$$E=N\sum\cdots\sum\bar{\epsilon}_{n_1n_2\cdots}\,w_{n_1n_2\cdots}$$

where $w_{n_1n_2n_3}\cdots$ is the proportional number of elements in the cell $(n_1, n_2, \cdots)$. Comparing these two expressions for the energy, there results,

$$w_{n_1n_2\cdots} = \frac{p_{n_1n_2\cdots}\exp.\left\{-\frac{\epsilon_{n_1n_2\cdots}}{\Theta}\right\}}{\sum\cdots\sum p_{n_1n_2\cdots}\exp.\left\{-\frac{\epsilon_{n_1n_2\cdots}}{\Theta}\right\}}. \qquad 25\cdot3$$

Planck's division of the extension-in-phase represents a somewhat less radical departure from classical dynamics than does the theory of stationary states. The representative point of the system can vary continuously throughout any one cell as the initial conditions of the system are varied. It cannot, however, vary continuously from one cell to an adjacent one, and so the idea of discontinuity is involved in this theory as well as in the theory of stationary states. But we shall see later that the theory of stationary states seems to be demanded in order to account for the sharp spectral lines that are actually observed.

We can now show the relation between this theory and that of stationary states when both are applied to conditionally periodic systems. For such systems, we have already made use of a division of the extension-in-phase, or section of the tubes of paths, into cells, in terms of the action and angle variables of the system. It is natural to make use of the same division in Planck's method. If, then, all the degrees of freedom are incoherent, there will be as many independent action variables as there are degrees of freedom, and the equations,

$$I_k = 2\pi\alpha_k = n_kh,$$

which define the stationary states, will, at the same time, define the bounding hypersurfaces employed in Planck's method.

26. **Bohr's Frequency Condition.**—In addition to the hypothesis of stationary states, Bohr introduced another hypothesis, known as his frequency condition. According to this second hypothesis, a dynamical system (atom) radiates energy only in passing from one to another stationary state. If ϵ_m and ϵ_n are the energies in the mth and nth states, then this frequency condition is,

$$\epsilon_m - \epsilon_n = h\nu. \qquad 26\cdot1$$

h is the same constant used in defining the stationary states, and ν is the frequency of the monochromatic radiation emitted.

Einstein[1] has shown how, with the aid of Bohr's two hypotheses, Planck's radiation formula may be obtained. Consider an enclosure, at absolute temperature T, containing atoms, or dynamical systems of any kind. Let N_m and N_n be the numbers of these atoms in unit volume in the mth and nth states respectively. In unit time a certain number, proportional to N_m, of the atoms in the mth state will spontaneously pass to the nth state, assuming that ϵ_m is greater than ϵ_n. Let this number be $A_m^n N_m$ where A_m^n is a constant, depending on the mth and nth states of the atoms. Each of these atoms, in the transition, emits monochromatic radiation of frequency ν, given by 26·1. Suppose now that radiation of this frequency is present in the enclosure; let u_ν be the amount in unit volume. Under its influence atoms will pass from the mth to the nth state, as well as from the nth to the mth state. Whether an atom absorbs energy from the radiation and passes to the mth state, or emits radiation of this frequency, under its influence, will depend upon the relative phases of the radiation and the atom. The numbers of atoms which pass, in unit time, from the mth to the nth state, under the influence of u_ν, we shall take to be $B_m^n u_\nu N_m$ and the number that pass from the nth to the mth state, $B_n^m u_\nu N_n$. In equilibrium as many atoms must pass from the mth to the nth state as from the nth to the mth state. It therefore follows that

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[1] Deut. Phys. Ges., 1916, p. 318.

$$A_m^n N_m + B_m^n u_\nu N_m = B_n^m u_\nu N_n \cdot$$

Now, by 21·2

$$\frac{N_n}{N_m} = \frac{p_n}{p_m} \epsilon^{\frac{\epsilon_m - \epsilon_n}{kT}} = \frac{p_n}{p_m} \epsilon^{\frac{h\nu}{kT}}.$$

Combining these two equations, we get,

$$A_m p_m = u_\nu \left(B_n^m p_n \epsilon^{\frac{h\nu}{kT}} - B_m^n p_n \right) \cdot$$

As T approaches infinity, we must assume that u_ν also approaches infinity. Therefore,

$$B_n^m p_n = B_m^n p_m.$$

If we write,

$$\alpha_{mn} = \frac{A_m^n}{B_m^n}$$

a constant, depending upon the two states, we get,

$$u_\nu = \frac{\alpha_{mn}}{\epsilon^{\frac{h\nu}{kT}} - 1} \cdot$$

By Wien's law, 18·1, which was obtained from dynamics and thermodynamics, we see that u_ν must be of the form,

$$u_\nu = \frac{A\nu^3}{c^3} \frac{1}{\epsilon^{\frac{h\nu}{kT}} - 1},$$

where A is a constant. This is Planck's formula. The constant A may be obtained by comparing the limiting value which this expression approaches at very high temperatures,

$$u_\nu = \frac{A\nu^2 kT}{hc^3}$$

with the value obtained from 17·3, assigning the energy kT to each vibration, assuming only transverse vibrations, with velocity of propagation c. This gives,

$$u_\nu = \frac{8\pi\nu^2 kT}{c^3}.$$

Therefore we have for the value of the constant,

$$A = 8\pi h.$$

CHAPTER III

THE SPECIFIC HEAT OF SOLIDS

27. **The Linear Oscillator.**—The simplest dynamical system to which the quantum conditions can be applied is a linear, simple-harmonic oscillator. Let its equation of motion be

$$\ddot{q} + \mu^2 q = 0. \qquad 27\cdot 1$$

According to dynamical principles, there will be one definite frequency of vibration, given by,

$$\nu_0 = \frac{\mu}{2\pi}\cdot \qquad 27\cdot 2$$

Let us now apply the quantum conditions. The energy equation is,

$$H = T + V = \tfrac{1}{2}\dot{q}^2 + \tfrac{1}{2}\mu^2 q^2 = \tfrac{1}{2}(p^2 + \mu^2 q^2) = \alpha_1,$$

where $p = \dfrac{\partial T}{\partial \dot{q}}$, and α_1 is the constant energy. The system being periodic, we may take the quantum condition 22·5 which gives,

$$2\int_a^b p\,dq = \frac{2\alpha_1}{\mu}\pi = nh,$$

where $b,\ a = \pm\sqrt{(2\alpha_1/\mu^2)}$. The energy in the nth state is

$$\epsilon_n = \alpha_1 = nh\nu_0. \qquad 27\cdot 3$$

Suppose, now, that our oscillator is capable of emitting radiation of some kind. According to dynamical principles we should expect the frequency of this radiation to have one definite value, not ν_0, but differing from this because we should have to take into account the reaction of the radiation upon the oscillator itself, and its equation of motion would therefore need to be altered. Using the hypotheses of the quantum theory, on the other hand, there would be an infinite number of frequencies in the radiation, given by Bohr's frequency condition,

$$h\nu = \epsilon_m - \epsilon_n = \nu_0 h(m - n),$$

or

$$\nu = n'\nu_0,$$

where n' is any positive integer. Now one of the serious difficulties of dynamical theory is to account for the fact that a system, like an atom, of a limited number of degrees of freedom, is capable of emitting radiation of a very large number of frequencies. The hypotheses of the quantum theory, added to dynamical theory, do result, as we see, in allowing an infinite number of frequencies from a system of a single degree of freedom. A linear oscillator cannot serve as a model of a source of light radiation, for the frequencies given by $\nu = n'\nu_0$ are not the frequencies of known spectral series. But this linear oscillator serves as an illustration of the necessity for introducing the hypotheses of the quantum theory. In order to write the complete equation of motion of such an oscillator we should need thorough knowledge of the reaction of the radiation upon it, and of all the other forces acting upon it. But we are not in possession of such knowledge even in the simplest cases. It is true that a model of an electron can be conceived that accounts for most of the phenomena in which electrons play a part. But what an electron is, what the forces are immediately surrounding it and throughout its interior, even how an electron can hold together, we do not know. With the aid of the hypotheses of the quantum theory we can obtain important results, in many cases, without such detailed knowledge. We can form simple models, obeying dynamical laws, supplemented by these hypotheses, and ignore that of which we are ignorant. But the models so conceived may not, and probably do not, have any resemblance to reality.

28. Having obtained the energy in the stationary states of a linear oscillator we can obtain the average energy of a large number of such oscillators. Suppose we have, in unit volume, N such oscillators, each of frequency ν. Let there be N_n of them in the nth stationary state. Then the whole energy is,

$$E = \sum nh\nu N_n.$$

With $N_n = Nw_n$, we obtain for the average energy,

$$\epsilon = \frac{E}{N} = h\nu \sum nw_n.$$

w_n is given by 21·2. As our system is of one degree of freedom, necessarily non-degenerate, we take $p_n = 1$. With $x = exp.(-h\nu/kT)$, we can write,

$$\epsilon = h\nu \frac{\Sigma nx^n}{\Sigma x^n}.$$

If we take $n=0$ as a possible state, that is, one in which the energy is zero, we have

$$\sum x^n = \frac{1}{1-x}, \quad \sum nx^n = \frac{x}{(1-x)^2},$$

leading to

$$\epsilon = \frac{h\nu}{\epsilon^{\frac{h\nu}{kT}} - 1}. \tag{28·1}$$

For very high temperatures this approaches,

$$\epsilon = kT,$$

and this is the value demanded by the principle of the equipartition of energy.

29. Planck's Method.—Let us now apply Planck's division of the extension-in-phase into cells to find the average energy of a simply-periodic oscillator. There is only one degree of freedom, and so only one action variable, α, given by

$$2\pi\alpha = I = \frac{2\pi}{\mu}\alpha_1,$$

where α_1 is the constant energy. The bounding surfaces of the cells are now a single family, given by

$$\alpha = \frac{nh}{2\pi}\cdot$$

The average energy of an oscillator in the cell (n) is,

$$\epsilon_n = 2\pi\nu_0\frac{\int\alpha d\alpha}{\int d\alpha}\cdot$$

Both integrals are to be taken between the limits $\frac{nh}{2\pi}$ and $\frac{(n+1)h}{2\pi}\cdot$ The result is

$$\epsilon_n = \nu_0 h(n+\tfrac{1}{2})\cdot \tag{29·1}$$

In 25·3 we are to take $p_n = 1$, and we find for the proportional number of oscillators in the cell (n),

$$w_n = \frac{\exp.\ \dfrac{-\nu_0 h\left(n+\dfrac{1}{2}\right)}{kT}}{\sum\exp.\left(\dfrac{-\nu_0 h\left(n+\dfrac{1}{2}\right)}{kT}\right)}.$$

Writing *exp.* $(-\nu_0 h/kT) = x$, we get for the average energy of a large number of such oscillators,

$$\epsilon = \nu_0 h\frac{\sum\left(n+\dfrac{1}{2}\right)x^{n+\frac{1}{2}}}{\sum x^{n+\frac{1}{2}}} = \nu_0 h\left(\frac{x}{1-x}+\frac{1}{2}\right)$$

or,

$$\epsilon = \nu_0 h\left\{\frac{1}{e^{\frac{\nu_0 h}{kT}}-1}+\frac{1}{2}\right\}. \tag{29·2}$$

This differs from the expression 28·1 obtained for the average energy of an oscillator by the method of stationary states, only by the added term, $\nu_0 h/2$; the latter represents the average energy of an oscillator, of the kind here considered, at the absolute zero of temperature.

30. **Debye's Theory.**—According to our customary ideas, the heat energy of a solid is the energy of vibrations of the individual atoms. We are not in possession, however, of sufficient knowledge of the forces acting on an atom in a solid, whether arising from other atoms, or the reaction of radiation upon it, to write the complete equations of motion of the atom. We therefore form a model of an atom, conceiving it, as far as heat energy is concerned, as a material particle with three degrees of freedom, and acted upon by forces, toward its equilibrium position, proportional to its displacement. Each atom may therefore be replaced by three linear oscillators of the kind we have considered. If there are N atoms in unit volume of the solid, and we assign the energy kT to each of the oscillators, the specific heat of the solid at constant volume will be $3Nk$. This result, expressing the Dulong-Petit law, is, at ordinary temperatures, in good agreement with experiment for many solids. At very low termperatures, however, the specific heat decreases rapidly with the temperature.

Debye's method of applying the quantum theory to the determination of the specific heat of solids is based upon a different order of ideas. Instead of looking upon the solid as formed of atoms, it may be regarded as a continuous medium, capable of transmitting both longitudinal and transverse elastic waves. The thermal energy of a bounded solid may thus be regarded as the energy of the stationary elastic vibrations that can exist in it. Instead of assigning three linear oscillators to each atom of the solid, one linear oscillator is assigned to each of these vibrations, whose number, in the frequency range from ν to $\nu + d\nu$, is given by 17·3. In order to set a limit to this number of vibrations, it is assumed that their whole number is $3N$ in unit volume, where N is the number of atoms, and that the vibrations which are effective in the thermal properties of the solid are those of lowest frequencies. Denoting the highest effective frequency by ν_m, 17·3 gives,

$$3N = 4\pi\left(\frac{2}{v_t^3} + \frac{1}{v_l^3}\right)\int_0^{\nu_m} \nu^2 d\nu = \frac{4}{3}\pi a \nu_m^3,$$

where

$$a = \frac{2}{v_t^3} + \frac{1}{v_l^3}.$$

It is assumed here that the velocity of the elastic waves is independent of the frequency. The average energy of an oscillator, whose frequency is ν, is given by 28·1 and so the whole thermal energy of the solid is,

$$E = 4\pi h a \int_0^{\nu_m} \frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}} - 1}.$$

Put $h\nu/kT = x$, and take a "characteristic temperature"

$$\Theta = \frac{h\nu_m}{k}.$$

We then have,

$$E = 9NkT\left(\frac{T}{\Theta}\right)^3 \int_0^{\frac{\Theta}{T}} \frac{x^3dx}{e^x - 1},$$

and for the specific heat at constant volume,

$$C_v = \frac{dE}{dT} = 9Nk\left\{4\left(\frac{T}{\Theta}\right)^3 \int_0^{\frac{\Theta}{T}} \frac{x^3dx}{e^x - 1} - \frac{\Theta}{T}\frac{1}{e^{\frac{\Theta}{T}} - 1}\right\}. \qquad 30\cdot 1$$

For high temperatures the limiting value of the specific heat is,

$$C_v = 3Nk,$$

in agreement with the Dulong-Petit law. At low temperatures, where Θ/T approaches infinity,

$$C_v = 36Nk\left(\frac{T}{\Theta}\right)^3 \int_0^{\infty} \frac{x^3dx}{e^x - 1}.$$

The definite integral is known to have the value $\pi^4/15$, and therefore,

$$C_v = \frac{12}{5}\pi^4 Nk\left(\frac{T}{\Theta}\right)^3. \qquad 30\cdot 2$$

The only constant at our disposal in 30·1 is the characteristic temperature, Θ. This may be determined from the elastic constants of the solid with the aid of,

$$\Theta^3 = \frac{9\,Nh^3}{4\pi k^3}\,\frac{2v_t^3 + v_l^3}{v_t^3\,v_l^3}.$$

The following table[1] shows the nature of the agreement between these two wholly different methods of determining Θ:

	By Specific Heat Measurements	From Elastic Constants
Lead	88	75
Cadmium	168	174
Silver	215	220
Copper	315	341
Aluminium	398	413
Iron	453	484
Sylvite	230	227
Rock Salt	281	305
Fluorite	474	510
Pyrite	645	696

Debye's formula for the specific heat of solids, 30·1, when evaluated for intermediate temperatures, is found to be in excellent agreement

[1] Schrodinger, Phys. Zeit., 20, 1919, 476.

with experimental results. At very low temperatures, the proportionality of the specific heat to the third power of the absolute temperature, expressed by 30·2, is found to be true, well within the limits of experimental error.

The method of deriving this formula, however, in particular, the substitution of a continuous medium for the atomic structure, seems, at first sight, to be of doubtful validity. In recent years great progress has been made in the analysis of the structure of crystalline solids with the aid of Röntgen ray diffraction effects. The results that are obtained in this way can be interpreted on the assumption that crystals have a "space-lattice" structure, with atoms, or the nuclei of atoms, and electrons, at the lattice points. The problem of determining the free periods of vibration of such a space-lattice, assuming forces depending upon the mutual distances of the elements occupying the lattice points, is a definite dynamical problem. This problem was first attacked, with success, by Born and Karman[1] for the purpose of determining the vibrations that are effective for the thermal properties of the crystal; and Born[2] has developed the theory, showing how the elastic properties of crystals, their piezoelectric behavior, and even their optical properties, are determined by their space-lattice structure. It is beyond the scope of this work to give an account of the results of these investigations; it must suffice to state that it is shown that Debye's procedure gives a very good approximation to the periods of free vibration effective for the thermal properties of these solids. One thing of particular interest follows from this work; statistical methods are not required, for the regularity in the distribution of the particles makes a definite dynamical solution possible for given laws of force between the particles. The hypotheses of the quantum theory, however, cannot be discarded, on account of our ignorance as to the exact nature of these forces.

31. THE ASYMMETRICAL LINEAR OSCILLATOR.—At very high temperatures there is evidence that the specific heat of solids rises above the value given by the Dulong-Petit law, $3Nk$. As the temperature rises, we should expect, according to our usual ideas of the thermal energy of a solid, that the amplitude of the vibrations of the atoms would increase; and at sufficiently high temperatures the model that we have employed, of a simple-harmonic linear oscillator, may not be adequate to account for the observed phenomena. Let us, therefore, take as our model oscillator, one whose equation of motion is,

$$\ddot{q} = -\mu^2 q - \tfrac{3}{2}\lambda a q^2 - 2\lambda^2 b q^3,$$

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[1] Phys. Zeit., 13, p. 294, 1912; 14, pp. 15, 65, 1913.

[2] Born, Dynamik der Kristallgitter, Leipzig, 1915.

where μ^2, a, and b are constants. The energy, H, is, since $p = \dot{q}$,

$$H = \tfrac{1}{2}(p^2 + \mu^2 q^2 + \lambda a q^3 + \lambda^2 b q^4).$$

The problem we have to solve is a special case of a much more general problem that has been solved by Born and Brody.[1] Their problem is that of a system of f degrees of freedom, with kinetic energy expressed as half the sum of the squares of the f momenta, and with potential energy expressed as the sum of polynomials in the coordinates of the second, third, and higher degrees. The special case considered here will serve as an illustration of a method that may be employed to apply the quantum conditions with the aid of successive approximations.

The dynamical problem will be solved by the method of §11. As we are going to employ successive approximations, the factors λ, λ^2 have been introduced into the energy function to indicate the relative order of magnitude of the corresponding terms. For a simple-harmonic oscillator a and b are zero; we assume that the term containing a is small of the first order, and that containing b small of the second order.

Take as the substitution function, W, to change from the variables p, q to P, Q,

$$W = \tfrac{1}{2}\,\mu q^2 \tan P.$$

Then, by, 8·3,

$$p^2 = 2\,\mu Q \sin^2 P$$

$$q^2 = \frac{2Q}{\mu}\cos^2 P.$$

In this method of integration, our object is to express the constant energy as a function of the single constant of integration. Assume, then, that the energy is of the form,

$$H = H_0 + \lambda H_1 + \lambda^2 H_2 + \cdots$$

The Hamilton-Jacobi equation is now,

$$\mu\frac{\partial W}{\partial P} + \lambda a\left(\frac{2}{\mu}\right)^{\frac{3}{2}}\left(\frac{\partial W}{\partial P}\right)^{\frac{3}{2}}\cos^3 P + \lambda^2 b\left(\frac{2}{\mu}\right)^2\left(\frac{\partial W}{\partial P}\right)^2\cos^4 P = H_0 + \lambda H_1 + \lambda^2 H_2 + \cdots.$$

This equation is to be solved by the method of successive approximations. We assume a solution,

$$W = W_0 + \lambda W_1 + \lambda^2 W_2 + \cdots$$

It will be sufficient for our purposes to stop with terms in λ^2, although there is no difficulty in carrying the approximation further. We can therefore write,

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[1] Zeit. fur Physik, 6, 1921, p. 140; 8, 1922, p. 205.

$$\frac{\partial W}{\partial P}=\frac{\partial W_0}{\partial P}\left\{1+\lambda\frac{\dfrac{\partial W_1}{\partial P}}{\dfrac{\partial W_0}{\partial P}}+\lambda^2\frac{\dfrac{\partial W_2}{\partial P}}{\dfrac{\partial W_0}{\partial P}}+\cdots\right\}$$

$$\left(\frac{\partial W}{\partial P}\right)^{\frac{3}{2}}=\left(\frac{\partial W_0}{\partial P}\right)^{\frac{3}{2}}\left\{1+\frac{3}{2}\lambda\frac{\dfrac{\partial W_1}{\partial P}}{\dfrac{\partial W_0}{\partial P}}+\cdots\right\}$$

$$\left(\frac{\partial W}{\partial P}\right)^2=\left(\frac{\partial W_0}{\partial P}\right)^2\left\{1+\cdots\right\}$$

The differential equation now becomes

$$\mu\frac{\partial W_0}{\partial P}+\lambda\mu\frac{\partial W_1}{\partial P}+\lambda^2\mu\frac{\partial W_2}{\partial P}+\cdots\cdot$$
$$+\lambda a\left(\frac{2}{\mu}\right)^{\frac{3}{2}}\left(\frac{\partial W_0}{\partial P}\right)^{\frac{3}{2}}\cos^3 P+\frac{3}{2}\lambda^2 a\left(\frac{2}{\mu}\right)^{\frac{3}{2}}\left(\frac{\partial W_0}{\partial P}\right)^{\frac{1}{2}}\frac{\partial W_1}{\partial P}\cos^3 P$$
$$+\lambda^2 b\left(\frac{2}{\mu}\right)^2\left(\frac{\partial W_0}{\partial P}\right)^2\cos^4 P+\cdots\cdot$$
$$=H_0+\lambda H_1+\lambda^2 H_2+\cdots\cdot$$

Equating terms of the same order on the two sides of this equation, we get the series of equations,

$$\mu\frac{\partial W_0}{\partial P}=H_0$$

$$\mu\frac{\partial W_1}{\partial P}=H_1-a\left(\frac{2}{\mu}\right)^{\frac{3}{2}}\left(\frac{\partial W_0}{P\partial}\right)^{\frac{3}{2}}\cos^3 P$$

$$\mu\frac{\partial W_2}{\partial P}=H_2-\frac{3}{2}a\left(\frac{2}{\mu}\right)^{\frac{3}{2}}\left(\frac{\partial W_0}{\partial P}\right)^{\frac{1}{2}}\frac{\partial W_1}{\partial P}\cos^3 P-\frac{4b}{\mu^2}\left(\frac{\partial W_0}{\partial P}\right)^2\cos^4 P$$

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The first equation may be integrated at once, and gives,

$$W_0=\alpha P,$$

where α is an arbitrary constant. It follows that

$$H_0=\mu\alpha.$$

No more arbitrary constants are to be introduced in the integration of the successive differential equations, as there is only one degree of freedom. The second differential equation is now, using the value found for W_0,

$$\mu\frac{\partial W_1}{\partial P}=H_1-A\cos^3 P$$

where

$$A=a\left(\frac{2\alpha}{\mu}\right)^{\frac{3}{2}}.$$

Put

$$\cos^3 P=\tfrac{3}{4}\cos P+\tfrac{1}{4}\cos 3P.$$

Assume as a solution a sum of trigonometric functions of multiples of P. As no more arbitrary constants are to be introduced, we find,

$$W_1=-\frac{3}{4}\frac{A}{\mu}\sin P-\frac{1}{12}\frac{A}{\mu}\sin 3P$$

$$\frac{\partial W_1}{\partial P}=-\frac{A}{\mu}\cos^3 P$$

and

$$H_1=0.$$

Using the values of W_0 and W_1, now found, in the third differential equation, we get,

$$\mu\frac{\partial W_2}{\partial P}=H_2+\frac{3}{2}a\left(\frac{2}{\mu}\right)^{\frac{3}{2}}\alpha^{\frac{1}{2}}\frac{A}{\mu}\cos^6 P-4b\frac{\alpha^2}{\mu^2}\cos^4 P.$$

As we are not going to carry the approximation any farther, but merely wish to get H_2, it will be observed that we can arrive at the result quickly by equating to zero the sum of the constants on the right-hand side of this equation. The constant term in the expansion of $\cos^6 P$ is $\frac{5}{16}$, and in $\cos^4 P$, $\frac{3}{8}$. We therefore have,

$$H_2=-\frac{15}{32}\alpha\left(\frac{2}{\mu}\right)^{\frac{3}{2}}\sqrt{\alpha}\frac{A}{\mu}+\frac{3}{2}\frac{b\alpha^2}{\mu^2}.$$

and accordingly,

$$H=\mu\alpha+\frac{3}{2}\frac{\alpha^2}{\mu^2}\left(b-\frac{5}{2}\frac{a^2}{\mu^3}\right). \qquad 31\cdot 1$$

We now regard $W=W(P,\ \alpha)$ as a substitution function, with the aid of which we transform variables from P, Q to α, w. As α, w are canonical variables,

$$\dot{a}=-\frac{\partial H}{\partial w}=0$$

$$\omega=\dot{w}=\frac{\partial H}{\partial\alpha}=\mu+\frac{3\alpha}{2\mu^2}\left(b-\frac{5}{2}\frac{\alpha^2}{\mu^2}\right)$$

and

$$w=\omega t+\delta.$$

Therefore α is the constant action variable of our system, and w the angle variable.

32. Stationary States.—Up to this point we have used only the methods of classical dynamics. We can now find the stationary states of the linear, asymmetrical oscillator, by 22·8. These are given by,

$$2\pi\alpha = nh,$$

where n is any integer. Therefore the energy in the nth stationary state is,

$$\epsilon_n = \frac{\mu}{2\pi} nh + \frac{3}{8\pi^2 \mu^2} n^2 h^2 \left(b - \frac{5}{2}\frac{\alpha^2}{\mu^2}\right). \qquad 32{\cdot}1$$

It is interesting to observe that the energy of the asymmetrical oscillator, in its stationary states, cannot be expressed as a multiple of $h\nu$, where ν is a frequency. The view, therefore, that energy always exists as multiples of a "quantum of energy" is inconsistent with the hypotheses of the quantum theory that have been developed here. It is only in simply periodic cases that it is possible to speak of quanta of energy.

Let us now write,

$$\epsilon_n = \frac{\mu}{2\pi} nh - \frac{h^2}{4\pi^2} n^2 \sigma \qquad 32{\cdot}2$$

where,

$$\sigma = \frac{3}{2\mu^2}\left(\frac{5}{2}\frac{a^2}{\mu^2} - b\right). \qquad 32{\cdot}3$$

If we have N such oscillators, the number N_n, in the nth state is given by $N_n = w_n N$, where, by 21·2.

$$w_n = \frac{e^{-\frac{\epsilon_n}{kT}}}{\sum e^{-\frac{\epsilon_n}{kT}}}.$$

For one degree of freedom, $p_n = 1$. The average energy of the oscillators is now,

$$\bar{\epsilon} = \frac{\sum \epsilon_n e^{-\frac{\epsilon_n}{kT}}}{\sum e^{-\frac{\epsilon_n}{kT}}}.$$

Since σ is assumed to be small, we can write,

$$e^{-\frac{\epsilon_n}{kT}} = \left(1 + \frac{h^2 n^2 \sigma}{4\pi^2 kT}\right) \exp.\left\{-\frac{\mu nh}{2\pi kT}\right\}.$$

and with,

$$x = \exp.\left\{-\frac{\mu h}{2\pi kT}\right\}$$

we have,

$$\bar{\epsilon}=\frac{\sum\left(\frac{\mu hn}{2\pi}-\frac{h^2\sigma n^2}{4\pi^2}\right)\left(1+\frac{h^2\sigma n^2}{4\pi^2 kT}\right)x^n}{\sum\left(1+\frac{h^2\sigma n^2\sigma}{4\pi^2 kT}\right)x^n}\,.$$

Neglecting σ^2, and noting that,

$$\sum x^n=\frac{1}{1-x},\ \sum nx^n=\frac{x}{(1-x)^2},\ \sum n^2x^n=$$
$$\frac{x(1+x)}{(1-x)^3},\ \sum n^3x^n=\frac{x(1+4x+x^2)}{(1-x)^4},$$

where $n=0$ is included as a possible stationary state, we get,

$$\bar{\epsilon}=\frac{x}{1-x}\left\{\frac{h\mu}{2\pi}-\frac{h^2\sigma}{4\pi^2}\frac{1-x}{1-x}+\frac{h^3\mu\sigma}{8\pi^3kT}\frac{1+3x}{(1-x)^2}\right\}.$$

For high temperatures we can take $x=1-\frac{\mu h}{2\pi kT}$ and we find,

$$\bar{\epsilon}=kT\left(1+\frac{2\sigma kT}{\mu^2}\right).$$

If, now, we assign three of these oscillators to each atom, we get, for the thermal energy of a solid containing N atoms,

$$E=3NkT\left(1+\frac{2\sigma kT}{\mu^2}\right)$$

and the specific heat at constant volume is,

$$C_v=3Nk\left(1+\frac{4\sigma kT}{\mu^2}\right).$$

If, then, σ, given by 32·3, is positive, an increase in the specific heat at high temperatures, above the value $3Nk$ may be accounted for; the result indicates that, to the degree of approximation employed, the increase of the specific heat above $3Nk$ should be proportional to the absolute temperature. There is not very much experimental evidence that can be used to test this result; what there is seems, on the whole, to confirm it[1].

33. THE METHOD OF STATISTICAL MECHANICS.—It will be observed that the constant h, characteristic of the quantum theory, does not appear in the expression for the specific heat of solids at high temperatures. It should, therefore, be possible to obtain this expression by the

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[1] Magnus, Zeit. für Physik, 7, 1921, p. 141.

methods of statistical mechanics. We have obtained (20·1) an expression for the free energy, ψ, of a system of N similar elements, each of f degrees of freedom. In that expression, in place of the coordinates and momenta, q, p, we may introduce any other canonical variables. Let us, therefore, use our action and angle variables, α, w, and we get, since we assume each element to be the equivalent of three oscillators, the energy of each being given by 31·1,

$$-\psi = 3NkT \log \int\int \exp.\left\{-\frac{\mu\alpha}{kT} + \frac{\sigma\alpha^2}{kT}\right\} d\alpha\, dw.$$

The limits of integration for w, the angle variable, are 0 and 2π, and for α, the action variable, zero and infinity. Accordingly,

$$-\psi = 3NkT \log 2\pi \int_0^\infty \exp.\left\{-\frac{\mu\alpha}{kT}\right\}\left(1+\frac{\sigma\alpha^2}{kT}\right) d\alpha,$$

since σ is small. The definite integral may easily be evaluated by parts, resulting in,

$$-\psi = 3NkT \log 2\pi \left(\frac{kT}{\mu} + \frac{2\sigma k^2T^2}{\mu^3}\right).$$

This may be written,

$$-\psi = 3NkT \left\{\log 2\pi + \log\frac{kT}{\mu} + 1 + \frac{2\sigma kT}{\mu^2}\right\}.$$

The specific heat at constant volume is,

$$C_v = -T\frac{d^2\psi}{dT^2} = 3\,Nk\left(1 + \frac{4\sigma kT}{\mu^2}\right).$$

This agrees with the value obtained by the method of stationary states.

CHAPTER IV

QUANTUM THEORY AND THE KINETIC THEORY OF GASES

34. Monatomic Gases.—The methods of statistical mechanics, applied to the kinetic theory of gases, lead, on the whole, to fairly satisfactory agreement with the results of experiment. In particular, the specific heat of a monatomic gas is accounted for by supposing that each molecule has three translational degrees of freedom. Assigning the energy $\frac{1}{2}kT$ to each degree of freedom, and ignoring any potential energy, the specific heat of the gas at constant volume is $\frac{3}{2}kN$, N being the number of molecules. Whatever internal energy, or energy of rotation, the molecules may have, this must be supposed independent of the temperature in order to lead to this result, which is found to hold over a wide range of temperature. There is, however, evidence[1] that at low temperatures the specific heat of helium is less than the predicted value, diminishing about two per cent. between $T=291°$ and $T=93°$.

In order to account for the departures from the laws for perfect gases in the phenomena of viscosity, conduction of heat, and diffusion, it is assumed that, in polyatomic as well as in monatomic gases, there are mutual forces between the molecules; and it is found that, except at very low temperatures, the assumption of a law of force varying as r^{-s}, r being the distance between two molecules, leads to results in fair agreement with observation. The mutual action between molecules is undoubtedly of a complicated nature, determined by their electric and magnetic properties, and the assumption of a single law of force, varying inversely as some power of the distance between molecules, must be regarded as a first approximation, only, to the truth. The exact nature of the action between molecules is unknown to us, and, as in other analogous cases, it would seem as if certain quantum hypotheses might prove to be useful in giving an account of all the phenomena involved. That the laws of statistical mechanics may not be sufficient for explaining all the phenomena concerned was clearly recognized by Gibbs.[2] As far as monatomic gases are concerned, however, very little progress has been made in applying quantum hypotheses so as to account for observed departures from the laws for perfect gases. Keesom[3] and Sommerfeld[4] have used the method of regarding the gas as a continuous fluid, counting the number of vibrations below a maximum determined by the number of molecules, and applying quantum hypotheses to the energy of each frequency, much as in Debye's theory of the specific heat of solids. Sommerfeld showed that the simple gas laws may be obtained in this way, but no satisfactory results have been obtained with regard to the departures from these simple laws.

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[1] Scheel and Heuse, Ann. der Physik, 40, 473, 1913.

[2] Statistical Mechanics, pp. 166, 167.

[3] Phys. Zeit. XIV, 665, 1913.

[4] Vorträge über die Kinetische Theorie der Materie und der Elektrizität, Göttingen, 1914, p. 125.

35. ABSOLUTE ENTROPY OF MONATOMIC GASES.—The thermodynamic definition of entropy leaves an additive constant undetermined; with the aid of Nernst's third law of thermodynamics this additive constant may be determined. A number of attempts have been made to derive the value of the absolute entropy by means of the hypotheses of the quantum theory. Keesom and Sommerfeld derived expressions for it, using the method of vibration frequencies of a continuous fluid, referred to in the preceding paragraph, but their results do not agree with the value obtained from Nernst's theorem. For a monatomic gas, Planck[1] obtained a satisfactory value for the absolute entropy, using his method of dividing the extension-in-phase into definite cells.

The difficulty of applying the hypotheses of the quantum theory to the translational motion of the molecules of a gas arises from the fact that the system is not conditionally periodic. But by a rather artificial device, analogous to that employed by Planck, Brody[2] has been able to derive a satisfactory value for the absolute entropy using the quantum conditions in the form 22·5. The gas is supposed contained in a cubical box whose edge is l. The walls of the box are supposed to be perfectly elastic to the collisions of the molecules, and each molecule is replaced by three, moving with the component velocities v_x, v_y, and v_z parallel to the edges, just as in the elementary treatment of the pressure of a gas from the kinetic point of view. For a single molecule of mass m, the quantum conditions may then be written,

$$2\int_0^l mv_x\,dx = nh = 2mv_xl$$

$$2\int_0^l mv_y\,dy = n'h = 2mv_yl$$

$$2\int_0^l mv_z\,dz = n''h = 2mv_zl$$

The energy, which is all kinetic for a perfect gas, is then, for a single molecule,

$$\epsilon = \frac{h^2}{8ml^2}\left(n^2 + n'^2 + n''^2\right)$$

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[1] Berlin Akad. 1916, p. 653. Also, Vorlesungen über die Theorie der Wärmestrahlung, 4th Ed. 1921, p. 204.

[2] Zeit. für Physik, 6, 79, 1921.

and for the N molecules, denoting the different molecules by subscripts,

$$E=\frac{h^2}{8ml^2}\left(n_1^2+n_1'^2+n_1''^2+n_2^2+\cdots\cdots+n_N''^2\right).$$

If the integers, n, be taken as rectangular coordinates in space of $3N$ dimensions, then the number of combinations of these integers which give an energy lying between E and $E+dE$ will be the volume, dV, of the spherical shell whose radii are

$$\left(\frac{8\,ml^2E}{h^2}\right)^{\frac{1}{2}}\text{and}\left(\frac{8\,ml^2\,(E+dE)}{h^2}\right)^{\frac{1}{2}},$$

divided by 2^{3N}, since the integers are restricted to positive values. The number of ways of getting an energy between the limits E and $E+dE$ is therefore,

$$W=\frac{1}{2^{3N}N!}\frac{dV}{dE}.$$

This has been divided by $N!$ because, as far as any observation is concerned, it makes no difference which one of the N molecules is characterized by any particular set of the quantum numbers n, n', n''.

The volume of a sphere in s-dimensional space, with radius r, is,

$$V=\frac{\pi^{\frac{s}{2}}r^s}{\Gamma\left(\frac{s}{2}+1\right)}.$$

We therefore get,

$$W=\left(\frac{8\,\pi ml^2}{h^2}\right)^{\frac{3N}{2}}\frac{E^{\frac{3N}{2}-1}}{2^{3N}N!\,\Gamma\left(\frac{3N}{2}\right)}.$$

Using the asymptotic values, for large values of x,

$$\Gamma(x)=e^{-x}x^{x-\frac{1}{2}}\sqrt{2\pi}$$
$$x!=x^{x+\frac{1}{2}}e^{-x}\sqrt{2\pi},$$

and Boltzmann's relation,

$$S=k\log W,$$

we find, neglecting small quantities,

$$S=\frac{3}{2}Nk\log\left(Ev^{\frac{2}{3}}\right)+S_0$$
$$S_0=\frac{3}{2}kN\log\left\{\left(\frac{4}{3}\frac{\pi m}{h^2}\right)\left(\frac{e}{N}\right)^{\frac{5}{3}}\right\},$$

where the volume, v, is l^3.

Since

$$\frac{\partial S}{\partial E}=\frac{1}{T},$$

the whole energy, E, is

$$E=\tfrac{3}{2}NkT,$$

so that we can write,

$$S=kN\log\left\{\frac{(2\pi mkT)^{\frac{3}{2}}ve^{\frac{5}{2}}}{Nh^3}\right\}.$$

36. Diatomic Gases.—In addition to the three translatory degrees of freedom that a monatomic molecule possesses, the results of the kinetic theory of gases indicate that a diatomic molecule has two more degrees of freedom; and this is interpreted most naturally by assuming the molecules to have axial symmetry, with three translatory and two rotational degrees of freedom. Rotation about the axis of the molecule is supposed to play no part in thermal phenomena, just as in the case of monatomic gases rotational energy, if it exists, must be supposed independent of the temperature. Now it has been found that at low temperatures a diatomic gas tends to behave, with respect to its specific heat, as a monatomic gas, losing the kinetic energy corresponding to two degrees of freedom; these two degrees of freedom can only be those corresponding to rotations.

The mutual actions between molecules at an encounter are unknown to us. Owing to their probable electric and magnetic nature, they are undoubtedly very complicated. Just as in the case of solids, therefore, we imagine a simple model to replace the molecule. And to this model we apply dynamical principles, supplemented by the hypotheses of the quantum theory. Our model must have two rotational degrees of freedom. We may suppose it to have one point fixed, as, for the present, we are concerned only with rotational energy.

37. The Two-Dimensional Rotator.—The simplest model we can imagine with two rotational degrees of freedom, is a system consisting of two mass particles at the ends of a rigid rod of negligible mass. The rod is the axis of the system. Rotation about this axis is ignored, and the two principal moments of inertia about axes perpendicular to the axis of the rotator are each equal to A. Now we know that the steady state of motion of such a rotator, under no forces, is a uniform rotation about an invariable axis in space, with angular velocity, ω. Its energy, therefore, in the steady motion, may be expressed in the form,

$$H=\tfrac{1}{2}K^2\omega^2,$$

where K is a constant. There is, therefore, only a single arbitrary constant at our disposal, while two are required in order to apply the quantum hypotheses to two degrees of freedom. Our model is therefore too simple.

We have supposed the molecule acted upon by no forces. Actually, however, we must suppose that there are forces acting upon it, arising, in part, from other molecules. We can therefore assume a given direction in space at the center of our molecule, the axis of some couple acting upon it. It will make no difference what we take as the nature of this couple; so we can assume that our model rotator includes, at its center, a magnetic doublet, of moment μ, and that there is a magnetic field, of intensity F, in a definite direction. We now have a definite direction in space, that of the magnetic field, and we can use the two coordinates, θ, ψ, to specify the configuration of the rotator; θ is the angle its axis makes with the direction of the field, F, and ψ is the azimuth. We can therefore write the energy of the rotator, the sum of the kinetic and potential energy,

$$H=\tfrac{1}{2}A\,(\dot{\theta}^2+\sin^2\theta\,\dot{\psi}^2)+\mu F\,(1-\cos\theta).$$

In this, we must replace velocities by corresponding momenta, with the aid of

$$p_\theta=\frac{\partial T}{\partial\dot{\theta}},\quad p_\psi=\frac{\partial T}{\partial\dot{\psi}},$$

and we get,

$$H=\frac{1}{2A}\left(p_\theta{}^2+\frac{1}{\sin{}^2\theta}p_\psi{}^2\right)+\mu F\left(1-\cos\theta\right).$$

We shall employ the method of separation of the variables (§12) to solve this problem. The Hamilton-Jacobi equation is

$$\frac{1}{2A}\left\{\left(\frac{\partial W}{\partial\theta}\right)^2+\frac{1}{\sin^2\theta}\left(\frac{\partial W}{\partial\psi}\right)^2\right\}+\mu F\left(1-\cos\theta\right)-\alpha_1-0,$$

where α_1 is the constant energy. The angle ψ is a cyclic coordinate, p_ψ is therefore constant. So we take $p_\psi=\alpha_2$, the second integration constant. We now get separation of the variables by assuming,

$$W=\alpha_2\psi+W_\theta$$

where W_θ is a function of θ only. When substituted into the differential equation, we find,

$$p_\theta=\frac{\partial W}{\partial\theta}=\left\{2A\alpha_1-\frac{\alpha_2{}^2}{\sin^2\theta}-2A\mu F\,(1-\cos\theta)\right\}^{\frac{1}{2}}.$$

The quantum conditions, 22·5 and 22·6 now give,

$$2\int_{\theta_1}^{\theta_2}p_\theta d\theta=n_1h$$

$$\int_0^{2\pi}p_\psi d\psi=2\pi\alpha_2=n_2h.$$

θ_1 and θ_2 are the two roots of $p_\theta=0$ between which θ lies initially. In the expression for p_θ we substitute $\cos\theta=x$, and the first quantum condition is,

$$2\int_{x_1}^{x_2}\frac{dx}{1-x^2}\left\{2\alpha_1 A-\alpha_2^2-2\alpha_1 Ax^2-2\mu FA\,(1-x)\,(1-x^2)\right\}^{\frac{1}{2}}=n_1 h.$$

Of the three roots of the cubic in x, two lie between $x=\pm 1$, and the third is less than -1. We must, therefore, for any real motion, consider only the first two roots.

The external magnetic field, F, has been introduced only to give a definite direction, and we shall assume it to vanish in the limit. We can therefore expand the radical as a power series in F, keeping only the first terms. This gives,

$$2\int_{x_1}^{x_2}\frac{dx}{1-x^2}\left\{2\alpha_1 A-\alpha_2^2-2\alpha_1 Ax^2\right\}^{\frac{1}{2}}$$
$$-2\mu FA\int_{x_1}^{x_2}\frac{(1-x)\,dx}{\left\{2\alpha_1 A-\alpha_2^2-2\alpha_1 Ax^2\right\}^{\frac{1}{2}}}=n_1 h.$$

And to this order of approximation, we can take for x_1, x_2, $\mp a$, where

$$a^2=\frac{2\alpha_1 A-\alpha_2^2}{2\alpha_1 A}$$

so that we have,

$$2\sqrt{2\alpha_1 A}\int_{-a}^{+a}\frac{\sqrt{a^2-x^2}}{1-x^2}\,dx-\frac{2\mu FA}{\sqrt{2\alpha_1 A}}\int_{-a}^{+a}\frac{(1-x)\,dx}{\sqrt{a^2-x^2}}=n_1 h.$$

The definite integrals are readily evaluated, resulting in,

$$2\pi\left\{\sqrt{2\alpha_1 A}-\alpha_2\right\}-\frac{2\pi\,\mu FA}{\sqrt{2\alpha_1 A}}=n_1 h.$$

We now suppose that $F=0$, and get for α_1, the energy,

$$\alpha_1=\epsilon_{n_1 n_2}=\frac{(n_1+n_2)^2 h^2}{8\pi^2 A}.$$

The energy in the various stationary states is characterized by two integers, n_1, n_2, and as they enter in a linear combination the system is degenerate (§23).

38. Rotational Specific Heat.—When we apply the theory of stationary states to a large number of such vibrators, we are met with the difficulty of specifying the various states. If we take each state to be characterized by a definite energy, so that the nth state has the energy ϵ_n, there are various possibilities to be considered. These possibilities have been discussed by Reiche[1], who has shown the nature

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[1] Annalen der Physik, 58, 1919, p. 657.

of the expressions for the rotational specific heat to which they lead. In the nth state, the energy may be expressed as $\epsilon_n = n^2\epsilon$, where

$$\epsilon = \frac{h^2}{8\pi^2 A}.$$

The hypothesis that leads to the best results is the following: No state with zero energy is to be counted as a possible state, nor is any state in which there is no azimuthal rotation to be counted as a possible state. Therefore $n_2=0$, is excluded. The energy $\epsilon_1=\epsilon$ we can get only by $n_1=0$, $n_2=1$ if we adopt this hypothesis. The energy $\epsilon_2=4\epsilon$ we can get by taking

$$\begin{array}{ll} n_1 = 1, & 0 \\ n_2 = 1, & 2 \end{array}$$

that is, in two ways. The energy $\epsilon_3=9\epsilon$ we can get by taking

$$\begin{array}{lll} n_1 = 1, & 2, & 0 \\ n_2 = 2, & 1, & 3, \end{array}$$

that is, in three ways. In general, the energy $\epsilon_n = n^2\epsilon$ we can get in n different ways. We can also consider states as distinct in which the azimuthal rotation has opposite directions. This would be allowed for by including the negative integers in n_2. In this way, the number of different ways of getting the energy ϵ_n would be $2n$. But as far as the specific heat is concerned, we shall see that this will make no difference in the final result. According to this view, then, we are led to take the weights to be assigned to the various states as $p_n = n$, or $2n$, according as to whether or not we count opposite azimuthal rotations as leading to distinct states.

The free energy of N such rotators is now, by 21·1,

$$\psi = -NkT \log \sum_{n=1}^{\infty} n e^{-n^2\sigma}$$

where

$$\sigma = \frac{h^2}{8\pi^2 kAT}.$$

Let us write,

$$Q(\sigma) = \sum_{n=1}^{\infty} n e^{-n^2\sigma}.$$

For the specific heat at constant volume, due to the two rotational degrees of freedom, we get,

$$C_r = -T\frac{d^2\psi}{dT^2} = Nk\sigma^2 \frac{d^2}{d\sigma^2} \log Q(\sigma).$$

For very low temperatures, σ is large, and taking the first two terms of the series for $Q(\sigma)$, we find,

$$C_r = 18\, Nk\sigma^2 e^{-3\sigma}.$$

This approaches zero with the temperature, and is in agreement with experimental results. For very high temperatures, σ is small. The series for $Q(\sigma)$ may be transformed into the definite integral,

$$Q(\sigma) = \int_0^{\infty} ne^{-n^2\sigma}\, dn = \frac{1}{2\sigma},$$

and this gives,

$$C_r = Nk,$$

which is the value to be expected on the classical theory for the specific heat due to two degrees of freedom.

For intermediate temperatures, Reiche (l.c.) has evaluated the series for $Q(\sigma)$ and computed the specific heat, C_r, for hydrogen. The following table shows the nature of the agreement between the measured values of C_r, obtained by subtracting $\frac{3}{2}Nk$ from the value of the specific heat at constant value, and the values of C_r computed from the series for $Q(\sigma)$. σ was determined from the observed value at $T=100$, and found to be $\sigma = 188.7$, and this gives for A, the moment of inertia of the model, $A = 2.095\ 10^{-41}$.

T	$\frac{C_r}{Nk}$ Computed	$\frac{C_r}{Nk}$ Experimental
35	0.000	0.000
40	0.000	0.000
45	0.001	0.010
50	0.003	0.015
60	0.014	0.005
65	0.026	0.030
70	0.041	0.060
80	0.084	0.081
82	0.095	0.106
85	0.113	0.116
90	0.147	0.141
91	0.154	0.136
100	0.222	0.222
110	0.302	0.323
126	0.433	...
135	0.504	...
145	0.578	...
157	0.654	...
172	0.727	...
189	0.795	...
196.5	0.817	0.711
236	0.901	...
273.1	0.936	0.937

The agreement is seen to be fair, except at the temperature 196.5, where the difference is much too large to be accounted for by errors of experiment. However, the general character of the behavior of the specific heat of hydrogen is accounted for. It is probable that the model selected to represent the molecule is not adequate to give a complete interpretation of the observed phenomena. It is interesting to observe that the moment of inertia of this model, computed from these results, is of the order of magnitude to be expected if the molecule consists of two hydrogen atoms rigidly held at a distance apart comparable with molecular diameters.

39. PLANCK'S METHOD.—The problem of the specific heat of a diatomic gas, in its dependence upon the two rotational degrees of freedom, has also been treated by Planck's method of dividing the extension-in-phase into definite cells, and taking the mean value of the energy in a cell.[1] We have seen, in §25, that in a conditionally periodic system the equations that determine the stationary states give the bounding hypersurfaces for the cells. The extension-in-phase has four dimensions; the two families of surfaces are,

$$g_1 = 2\pi\sqrt{2\alpha_1 A - \alpha_2}$$
$$g_2 = 2\pi\,\alpha_2,$$

where we have put $F=0$.

$g_2=0$ is equivalent to $p_\psi=0$. This surface therefore divides the whole extension-in-phase into two halves, a positive and a negative half. To each positive cell corresponds an equal negative cell. We need, therefore, consider only the positive cells, and double the result. The energy is given by,

$$\alpha_1 = \frac{(g_1+g_2)^2}{8\pi^2 A},$$

and its mean value in the cell (n_1, n_2) is,

$$\bar{\epsilon}_{n_1 n_2} = \frac{\int_{n_1 h}^{(n_1+1)h}\int_{n_2 h}^{(n_2+1)h} \alpha_1\, dg_1\, dg_2}{\int\int dg_1\, dg_2}$$
$$= \frac{1}{8\pi^2 h^2 A}\int\int (g_1+g_2)^2\, dg_1\, dg_2$$

with the same limits of integration. Integrating, we get,

$$\bar{\epsilon}_{n_1 n_2} = \frac{h^2}{8\pi^2 A}\left\{(n_1+n_2+1)^2 + \frac{1}{6}\right\}.$$

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[1] Rotsajn, Annalen der Physik, 57, 1918, p. 81.

By 25·2 the free energy of N molecules is,

$$\psi = -NkT \log \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} 2 \exp. \left\{ -\sigma(n_1+n_2+1)^2 - \frac{\sigma}{6} \right\}$$

where

$$\sigma = \frac{h^2}{8\pi^2 AkT} \cdot$$

We therefore find,

$$C_r = Nk\sigma^2 \frac{d^2}{d\sigma^2} \log Q(\sigma)$$

where

$$Q(\sigma) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} 2e^{-\sigma(n_1+n_2+1)^2}.$$

This can be transformed into a single sum.

$$Q(\sigma) = \sum_{n=1}^{\infty} 2ne^{-\sigma n^2}$$

and the final result for the rotational specific heat is the same as was obtained by the method of stationary states, when a particular hypothesis was made for the weights assigned to the various states.

CHAPTER V

SPECTRAL LINES

40. Central Motion.—Neither of the two models that we have made use of, the linear oscillator and the two-dimensional rotator, are capable of accounting for spectral lines when Bohr's frequency condition (§26) is applied to them. The former gives a series of frequencies proceeding as the integers, and the latter gives frequencies proceeding as the differences of the squares of integers. The simplest spectral series observed are those where the frequencies vary as the differences between the reciprocals of the squares of integers. In his earliest work on the theory of spectral lines, Bohr[1] made use of a model of an atom which, in its main features, had been proposed by Rutherford, and which has been remarkably successful. In its simplest form, this model consists of a positively charged nucleus, supposed to be at rest, with a negatively charged electron moving about the nucleus under a force of attraction varying inversely as the square of the distance. It is, in fact, the simplest case of central motion.

We shall suppose that there is an external electric force, E, acting upon the electron, in addition to the force of attraction to the nucleus. This will make it possible to apply the model to the Stark effect, the splitting up of spectral lines under the influence of an electric field. The solution of this problem which is given here is due to Epstein[2]; Sommerfeld[3] and Schwarzschild[4] have also obtained equivalent solutions by somewhat different methods.

Denote the charge on the positive nucleus by se, and on the negative electron by $-e$. The electric force, E, we take parallel to the z-axis. Let r be the distance of the electron from the nucleus, ρ its distance from the z-axis, and φ the angle the plane through the z-axis and the electron makes with a fixed plane containing the axis. The kinetic energy of the electron is

$$T=\tfrac{1}{2}m(\dot{z}^2+\dot{\rho}^2+\rho^2\dot{\varphi}^2)$$

m being the mass of the electron, and its potential energy is,

$$V=-\frac{se^2}{r}-Eez,$$

with

$$r^2=\rho^2+z^2.$$

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[1] Phil. Mag. XXVI, 1913, p. 1.

[2] Ann. der Physik, 50, 489, 1916.

[3] Ann. der Physik, 51, 1, 1916.

[4] Berl. Akad. 1916, pp. 435, 548.

In order to get coordinates in which the method of separation of the variables succeeds, introduce parabolic coordinates, in any meridian plane, defined by,

$$z+i\rho=\tfrac{1}{2}(\xi+i\eta)^2.$$

From this we obtain,

$$z=\tfrac{1}{2}(\xi^2-\eta^2)$$
$$r=\tfrac{1}{2}(\xi^2+\eta^2).$$

The element of path of the electron is given by

$$\begin{aligned} ds^2 &= dz^2+d\rho^2+\rho^2 d\varphi^2 \\ &= (dz+id\rho)(dz-id\rho)+\rho^2 d\varphi^2 \\ &= (\xi^2+\eta^2)(d\xi^2+d\eta^2)+\xi^2\eta^2 d\varphi^2. \end{aligned}$$

We therefore have,

$$T=\frac{1}{2}\,m(\xi^2+\eta^2)\left\{\dot{\xi}^2+\dot{\eta}^2+\frac{\xi^2\eta^2}{\xi^2+\eta^2}\,\dot{\varphi}^2\right\}$$

$$V=-\frac{4se^2+eE\,(\xi^4-\eta^4)}{2(\xi^2+\eta^2)}\,.$$

The components of momenta are:

$$p_\xi=\frac{\partial T}{\partial \xi}=m\,(\xi^2+\eta^2)\xi$$

$$p_\eta=\frac{\partial T}{\partial \dot{\eta}}=m\,(\xi^2+\eta^2)\dot{\eta}$$

$$p_\phi=\frac{\partial T}{\partial \dot{\varphi}}=m\xi^2\eta^2\dot{\varphi}.$$

The Hamilton-Jacobi equation is

$$\frac{1}{2m(\xi^2+\eta^2)}\left\{\left(\frac{\partial W}{\partial \xi}\right)^2+\left(\frac{\partial W}{\partial \eta}\right)^2+\left(\frac{1}{\xi^2}+\frac{1}{\eta^2}\right)\left(\frac{\partial W}{\partial \varphi}\right)^2 \right.$$
$$\left. -4sme^2-meE(\xi^4-\eta^4)\right\}+\alpha_1=0$$

where α_1 is the negative energy.

φ is a cyclic coordinate; hence

$$p_\phi=\alpha_2=\text{const.}$$

Separation of the variables is obtained by taking:

$$W=\alpha_2\phi+W_\xi+W_\eta$$

where W_ξ and W_η are functions of ξ and η respectively. Substituting this value, we get,

$$\left(\frac{\partial W_\xi}{\partial \xi}\right)^2 + \frac{\alpha_2^2}{\xi^2} - 2mse^2 - meE\xi^4 + 2m\alpha_1\xi^2$$
$$= -\left(\frac{\partial W_\eta}{\partial \eta}\right)^2 - \frac{\alpha_2^2}{\eta^2} + 2mse^2 - meE\eta^4 - 2m\alpha_1\eta^2.$$

The left-hand side of this equation is a function of ξ only and the right-hand side a function of π only. They are therefore both equal to a constant, α_3. We therefore find

$$W_\xi = \int d\xi \sqrt{(2mse^2 + \alpha_3) - \frac{\alpha_2^2}{\xi^2} - 2m\alpha_1\xi^2 + meE\xi^4}$$
$$W_\eta = \int d\eta \sqrt{(2mse^2 - \alpha_3) - \frac{\alpha_2^2}{\eta^2} - 2m\alpha_1\eta^2 - meE\eta^4}$$
$$W_\phi = \alpha_2\phi.$$

Let us change the variables by the substitution

$$\xi^2 = \xi'$$
$$\eta^2 = \eta'$$

and then write the equations without the accents:

$$W_\xi = \frac{1}{2}\int \frac{d\xi}{\xi} \sqrt{-\alpha_2^2 + (2mse^2 + \alpha_3)\xi - 2m\alpha_1\xi^2 + meE\xi^3}$$
$$W_\eta = \frac{1}{2}\int \frac{d\eta}{\eta} \sqrt{-\alpha_2^2 + (2mse^2 - \alpha_3)\eta - 2m\alpha_1\eta^2 - meE\eta^3}$$

The quantum conditions are now, by 22·5, 6,

$$2\int_{\xi_1}^{\xi_2} p_\xi d\xi = \int_{\xi_1}^{\xi_2} \frac{d\xi}{\xi} \sqrt{-\alpha_2^2 + (2mse^2 + \alpha_3)\xi - 2m\alpha_1\xi^2 + meE\xi^3} = n_1h$$

$$2\int_{\eta_1}^{\eta_2} p_\eta d\eta = \int_{\eta_1}^{\eta_2} \frac{d\eta}{\eta} \sqrt{-\alpha_2^2 + (2mse^2 - \alpha_3)\eta - 2m\alpha_1\eta^2 - meE\eta^3} = n_2h$$

$$\int p_\phi d\varphi = 2\pi\alpha_2 = n_3h.$$

In these expressions, ξ_1, ξ_2, and η_1, η_2 are the two roots of the corresponding radicals between which the coordinates of the initial position of the electron lie.

We shall now assume that the external field, E, is so small that the force due to it acting on the electron is small compared with the force arising from the nucleus. Expanding the radicals, and keeping terms of the first order only in E, we get

$$\int_{\xi_1}^{\xi_2}\frac{d\xi}{\xi}\sqrt{-\alpha_2^2+(2mse^2+\alpha_3)\xi-2m\alpha_1\xi^2}$$

$$+\frac{meE}{2}\int_{\xi_1}^{\xi_2}\frac{\xi^2d\xi}{\sqrt{-\alpha_2^2+(2mse^2+\alpha_3)\xi-2m\alpha_1\xi^2}}=n_1h$$

$$\int_{\eta_1}^{\eta_2}\frac{d\eta}{\eta}\sqrt{-\alpha_2^2+(2mse^2-\alpha_3)\eta-2m\alpha_1\eta^2}$$

$$-\frac{meE}{2}\int_{\eta_1}^{\eta_2}\frac{\eta^2d\eta}{\sqrt{-\alpha_2^2+(2mse^2-\alpha_3)\eta-2m\alpha_1\eta^2}}=n_2h.$$

In these integrals, for ξ_1, ξ_2 and η_1, η_2 we may take the two roots of the quadratics under the radicals, and our results will be valid as far as terms in E.

The integrals are all known forms, and we find

$$-\pi\alpha_2+\frac{2mse^2+\alpha_3}{2\sqrt{2m\alpha_1}}\pi+\frac{\pi meE}{64\,m^2\alpha_1^2\sqrt{2m\alpha_1}}\left\{3(2mse^2+\alpha_3)^2-8m\alpha_1\alpha_2^2\right\}=n_1h$$

$$-\pi\alpha_2+\frac{2mse^2-\alpha_3}{2\sqrt{2m\alpha_1}}\pi-\frac{\pi meE}{64\,m^2\alpha_1^2\sqrt{2m\alpha_1}}\left\{3(2mse^2-\alpha_3)^2-8m\alpha_1\alpha_2^2\right\}=n_2h$$

$$2\pi\alpha_2=n_3h. \qquad 40\cdot1$$

Suppose first that $E=0$. Then adding the first two equations we get

$$\alpha_1=\frac{2\pi^2ms^2e^4}{(n_1+n_2+n_3)^2h^2}. \qquad 40\cdot2$$

Subtracting them

$$\alpha_3=\frac{2mse^2(n_1-n_2)}{(n_1+n_2+n_3)}. \qquad 40\cdot3$$

Then using these values of α_1 and α_3 in the terms containing E, we get as far as terms of the first degree in E:

$$\alpha_1=\frac{2ms^2e^4\pi^2}{(n_1+n_2+n_3)^2h^2}+\frac{3}{8}\frac{Eh^2(n_1-n_2)(n_1+n_2+n_3)}{\pi^2mse}. \qquad 40\cdot4$$

41. **The Hydrogen Series.**—In equation 40·4 put $s=1$, $E=0$, and $n_1+n_2+n_3=n$. Then,

$$\alpha_1=\epsilon_n=\frac{2me^4\pi^2}{n^2h^2}.$$

This is the expression for the energy in the nth state of an atom consisting of a singly charged positive nucleus, and one electron of mass

m and charge e; it corresponds, accordingly, to the hydrogen atom. If we now apply Bohr's frequency condition, 26·1, we can write,

$$\nu = N\left(\frac{1}{n_1^2} - \frac{1}{n^2}\right) \qquad 41 \cdot 1$$

where

$$N = \frac{2me^4\pi^2}{h^3c}. \qquad 41 \cdot 2$$

The velocity of light, c, has been introduced in order to make ν, as customary in spectroscopy, represent the reciprocal of the wave-length.

n_1 and n may be any integers. If we take $n_1 = 1$, and $n = 2, 3, 4$ in succession, this formula gives the first three lines in the ultraviolet spectrum of hydrogen observed by Lyman. With $n_1 = 2$, and $n = 3, 4, 5, \cdots\cdot$ we get the Balmer series of hydrogen. And with $n_1 = 3$, and $n = 4, 5$, the first two lines in the infra-red spectrum of hydrogen observed by Paschen.[1]

The Rydberg constant, N, may be evaluated in two different ways, either by means of spectroscopic measurements, or by using the values of the electron constants in the equation defining it. With the values,

$$e = 4 \cdot 774 \; 10^{-10}$$

$$\frac{e}{m} = 5 \cdot 301 \; 10^{17}$$

$$h = 6 \cdot 545 \; 10^{-27}$$

$$c = 3 \cdot 0 \; 10^{10}$$

we get,

$$N = 1 \cdot 098 \; 10^5,$$

while spectroscopic measurements give an average value of

$$N = 1 \cdot 096783 \; 10^5.$$

It has been assumed that the mass of the nucleus is so large compared with the mass of the electron that the former may be regarded at rest. If this assumption is not made, it will be necessary only to multiply the expression for the energy by $\frac{M}{M+m}$, where M is the mass of the nucleus.

If in 40·4 we take $s=2$ instead of 1, we get an atom consisting of a doubly charged positive nucleus and a single electron. This is taken as the model of the positively charged helium atom, the neutral atom having lost one electron. The Rydberg constant, N, of the series formula, is now not exactly $4N$, N being its value for hydrogen, but a slightly different value which results from taking the mass of the nucleus into account. In 41·1, with $N=4\times 1{\cdot}09723{\cdot}22\ 10^5$, and $n_1=2$, $n=3, 4$, we have the Lyman series; with the same N and $n_1=3$, $n=4, 5, \cdots$ the "4686" series; and $n_1=4$, $n=5, 6, \cdots\cdot$, the Pickering series.

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[1] Fowler, Report on Series in Line Spectra. Physical Society of London, 1922.

42. THE STARK EFFECT.—Returning now to the general expression for the energy of the atom in the nth state (40.4) we get, by Bohr's freuqency condition, for the frequencies (reciprocal wave-lengths) emitted in an electric field, E,

$$\nu = N\left\{\frac{1}{(n_1+n_2+n_3)^2}-\frac{1}{(m_1+m_2+m_3)^2}\right\}$$
$$+\frac{3}{8}\frac{Eh}{\pi^2 msec}\left\{(n_1-n_2)\,(n_1+n_2+n_3)-(m_1-m_2)\,(m_1+m_2+m_3)\right\}.$$

The frequency change, $\Delta\nu$, in any of the spectral lines of the hydrogen series, will then be given by

$$\Delta\nu = -\frac{3}{8}\frac{h}{\pi^2 mec}EZ \qquad 42{\cdot}1$$

where

$$Z=(m_1-m_2)\,(m_1+m_2+m_3)-(n_1-n_2)\,(n_1+n_2+n_3). \qquad 42{\cdot}2$$

For the Balmer series we take $n_1+n_2+n_3=2$, and $m_1+m_2+m_3=3, 4, 5, \cdots\cdot$ for the successive lines. In terms of the wave-length, λ,

$$\Delta\lambda = \frac{3}{8}\frac{h\lambda^2}{\pi^2 mec}EZ. \qquad 42{\cdot}3$$

Stark, in his observations, used an electric force,

$$E=1{\cdot}04\ 10^5 \text{ volts per cm.}$$

With this value, we get, for the first four lines of the Balmer series,

	$n_1+n_2+n_3$	$m_1+m_2+m_3$	λ	$\Delta\lambda$
H_α	2	3	6562.8 Å	2.88 Z Å
H_β	2	4	4861.3	1.58 Z
H_γ	2	5	4340.5	1.26 Z
H_δ	2	6	4101.7	1.13 Z

If we take all possible combinations of the integers m_1, m_2, m_3 and n_1, n_2, n_3 that satisfy the relations,

$$m_1+m_2+m_3=2$$
$$n_1+n_2+n_3=3, 4, 5, 6,$$

we shall find that we get more different values of $\Delta\lambda$, that is, more components of a certain line in the electric field, than have actually been observed. In the following table only those values are given which lead to observed components, and we shall consider later the reason for the non-appearance of the others. In this table, under the heading "transition" are given the values of the integers m_1, m_2, m_3 and n_1, n_2, n_3. The components of each line are divided into two groups, marked "parallel" and "perpendicular" respectively. These refer to the observed state of polarization of the components, the former corresponding to the case wheie the electric vector is parallel, and the latter, perpendicular to the electric field. The reason for this polarization will be considered later.

	Transition	Z	$\Delta\lambda$ Calc.	$\Delta\lambda$ Obs.
		H_α 6562.8 Å.		
Parallel	111 ⟶ 011	2	5.8 Å	6.2 Å
	102 ⟶ 002	3	8.6	8.8
	201 ⟶ 101	4	11.5	11.5
Perp.	102 ⟶ 101	1	2.9	2.6
		H_β 4861.3 Å.		
Parallel	211 ⟶ 101	2	3.2	3.3
	220 ⟶ 020	4	6.3	6.7
	211 ⟶ 011	6	9.5	10.0
	202 ⟶ 002	8	12.6	13.2
	301 ⟶ 101	10	15.8	16.4
	301 ⟶ 110	12	19.0	19.4
Perpendicular	112 ⟶ 011	2	3.2	3.4
	103 ⟶ 002	4	6.3	6.6
	202 ⟶ 101	6	9.5	9.7
	202 ⟶ 110	8	12.6	13.2
	202 ⟶ 011	10	15.8	16.4
	301 ⟶ 002	12	19.0	19.3
		H_γ 4340 5 Å.		
Parallel	221 ⟶ 011	2	2.5	2.7
	212 ⟶ 002	5	6.3	6.6
	311 ⟶ 101	8	9.7	10.6
	311 ⟶ 011	12	15.1	15.9
	320 ⟶ 002	15	18.9	19.9
	401 ⟶ 101	18	22.3	23.9
	401 ⟶ 011	22	27.7	?
Perpendicular	212 ⟶ 101	3	3.8	3.9
	212 ⟶ 011	7	8.6	9.7
	203 ⟶ 002	10	12.6	13.3
	302 ⟶ 101	13	16.4	17.3
	302 ⟶ 011	17	21.4	22.8
	401 ⟶ 002	20	25.2	26.3

	Transition	Z	$\Delta\lambda$ Calc.	$\Delta\lambda$ Obs.
		H_δ 4101.7 Å.		
Parallel	$321 \longrightarrow 101$	4	4.5	5.2
	$321 \longrightarrow 011$	8	9.0	9.6
	$312 \longrightarrow 002$	12	13.6	14.4
	$411 \longrightarrow 101$	16	18.2	19.6
	$411 \longrightarrow 011$	20	22.6	24.2
	$402 \longrightarrow 002$	24	27.1	28.6
	$501 \longrightarrow 101$	28	31.6	33.4
	$501 \longrightarrow 011$	32	36.2	37.5
Perpendicular	$222 \longrightarrow 011$	2	2.3	2.4
	$213 \longrightarrow 002$	6	6.8	7.4
	$312 \longrightarrow 101$	10	11.3	11.9
	$312 \longrightarrow 011$	14	16.7	17.2
	$303 \longrightarrow 002$	18	20.3	21.3
	$402 \longrightarrow 101$	22	24.9	25.8
	$402 \longrightarrow 011$	26	29.4	30.4
	$501 \longrightarrow 002$	30	33.9	34.8

It will be seen that the agreement between the displacements observed by Stark and their computed values is all that could be desired.

43. BOHR'S PRINCIPLE OF CORRESPONDENCE.—In a conditionally periodic dynamical system of f degrees of freedom, we have found (§14) that any one of the generalized coordinates, q_k, which is used to specify the configuration of the system, may be expressed as a multiple Fourier series, represented by equation 14·1. If in that equation we replace the angle variables by their values in terms of the time,

$$w_k = \omega_k t + \delta_k$$

then, if the system is such that it radiates energy of some form, we should expect the frequencies of the radiation to be given, approximately, by

$$\nu = \frac{1}{2\pi}\left(\tau_1\omega_1 + \tau_2\omega_2 + \cdots\cdot\right). \qquad 43\cdot 1$$

In obtaining this result we have ignored any radiation from the system; the reaction of radiation upon the system would result, if the latter is small, in small changes in the frequencies.

Consider, now, two different states of the system which differ but little from each other. The difference in the energy of the system in the two states is,

$$H(\alpha_1 + \delta\alpha_1, \alpha_2 + \delta\alpha_2, \cdots\cdot) - H(\alpha_1, \alpha_2, \cdots)$$
$$= \sum \frac{\partial H}{\partial \alpha_k}\delta\alpha_k = \sum \omega_k \delta\alpha_k. \qquad 43\cdot 2$$

According to the hypotheses of the quantum theory, the α_k take only certain allowed values, given by,

$$2\pi\alpha_k = n_k h.$$

Suppose, now, that the integers, n_k, defining the stationary states, are large numbers. Then the difference in the values of the α_k for two neighboring states will be small compared with their values for either state, and in this case we may write,

$$2\pi\, \delta\alpha_k = (n_k'' - n_k')\, h = \tau_k h.$$

Accordingly, 43·2 may be written,

$$\epsilon'' - \epsilon' = \frac{h}{2\pi} \sum \tau_k \omega_k.$$

Comparing this with 43·1, we have,

$$\epsilon'' - \epsilon' = h\nu.$$

This is Bohr's frequency condition. We see, therefore, that in the limit of large quantum numbers, Bohr's frequency condition is consistent with general dynamical theory. Now by observing the expression obtained for the frequencies emitted by the simple model of the hydrogen atom,

$$\nu = N\left(\frac{1}{n_1^2} - \frac{1}{n^2}\right),$$

we see that when the quantum numbers, n_1 and n are large, the frequencies are small, and accordingly the wave-lengths large. Thus, in the limit of long wave-lengths, the results obtained with the aid of the hypotheses of the quantum theory should be in agreement with the results obtained by the methods of classical dynamics. This result is of great importance; it is known that in the region of long wave-lengths Planck's radiation formula reduces to the Rayleigh-Jeans formula which was deduced from dynamical principles.

Bohr's principle of correspondence is an additional hypothesis introduced into the quantum theory; making use of this principle it is possible to obtain information with respect to the relative intensity and polarization of the components of any given spectral line. According to this principle, although the frequencies are not determined by dynamical methods, nevertheless the amplitude of the vibrations corresponding to a given frequency may be determined by the coefficient of the corresponding term in the Fourier's series 14·1. Any given frequency is, by Bohr's frequency condition, determined by two of the stationary states. It is to be expected, therefore, that the amplitudes of the vibrations in both these stationary states will determine the amplitude of the vibration of the radiation emitted. As we have no knowledge of the mechanism of radiation, it is impossible to say just how the intensity of the radiation will depend upon the amplitudes of the vibrations in the two states;

it will be seen later, however, that good results may be obtained by taking the mean of the squares of the two amplitudes as a measure of the intensity of the resulting radiation.

44. INTENSITY AND POLARIZATION OF THE COMPONENTS IN THE STARK EFFECT.—In a valuable memoir[1], Kramers has carried through the calculation of the relative intensities of the several components of the first four lines of the Balmer series of hydrogen that appear when an electric field is applied. Owing to its importance as an illustration of, and test for, the hypotheses of the quantum theory, his method will be given here.

We have already obtained the solution of the Hamilton-Jacobi equation in parabolic coordinates in which this equation can be integrated by the method of separation of the variables. In this solution, instead of the three integration constants α_1, α_2, α_3, we must introduce the values, I_1, I_2, I_3. As we are now concerned only with the problem of determining the relative intensities we may, as a sufficient approximation, use the values of the integration constants for a zero external field. Remembering that

$$I_k = n_k h$$

we find from 40·1, 2, 3,

$$\alpha_1 = \frac{2ms^2e^4\pi^2}{I^2}$$

$$\alpha_2 = \frac{I_3}{2\pi}$$

$$\alpha_3 = \frac{2mse^2(I_1 - I_2)}{I}$$

where we have put

$$I = I_1 + I_2 + I_3.$$

We then get

$$2\pi W = I_3\phi + \frac{1}{2}\int\frac{d\xi}{\xi}\sqrt{-I_3^2 + 2\frac{2I_1 + I_3}{\sigma I}\xi - \frac{\xi^2}{\sigma^2 I^2}}$$
$$+ \frac{1}{2}\int\frac{d\eta}{\eta}\sqrt{-I_3^2 + 2\frac{2I_2 + I_3}{\sigma I}\eta - \frac{\eta^2}{\sigma^2 I^2}}$$

where we have put

$$\sigma = \frac{1}{4\pi^2 se^2 m}.$$

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[1] Memoires de l'Academie Royale des Sciences et des Lettres de Danemark, t.III, No. 3, 1919.

The angle variables, being given by

$$w_1 = 2\pi\frac{\partial W}{\partial I_1},\ w_2 = 2\pi\frac{\partial W}{\partial I_2},\ w_3 = 2\pi\frac{\partial W}{\partial I_3}$$

are

$$\begin{aligned}
w_1 = {} & \frac{1}{2\sigma I^2}\int\frac{d\xi}{\xi}\,\frac{\sigma I(2I_2+I_3)\xi+\xi^2}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_1+I_3)\xi-\xi^2}} \\
& +\frac{1}{2\sigma I^2}\int\frac{d\eta}{\eta}\,\frac{-\sigma I(2I_2+I_3)\eta+\eta^2}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_2+I_3)\eta-\eta^2}} \\
w_2 = {} & \frac{1}{2\sigma I^2}\int\frac{d\xi}{\xi}\,\frac{-\sigma I(2I_1+I_3)\xi+\xi^2}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_1+I_3)\xi-\xi^2}} \\
& +\frac{1}{2\sigma I^2}\int\frac{d\eta}{\eta}\,\frac{\sigma I(2I_1+I_3)\eta+\eta^2}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_2+I_3)\eta-\eta^2}} \\
w_3 = {} & \frac{1}{2\sigma I^2}\int\frac{d\xi}{\xi}\,\frac{-\sigma^2I_3I^3+\sigma I(I_2-I_1)\xi+\xi^2}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_1+I_3)\xi-\xi^2}} \\
& +\frac{1}{2\sigma I^2}\int\frac{d\eta}{\eta}\,\frac{-\sigma^2I_3I^3+\sigma I(I_1-I_2)\eta+\eta^2}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_2+I_3)\eta-\eta^2}}+\varphi.
\end{aligned}$$

The integrations may be most easily carried out by using the abbreviations

$$\left.\begin{aligned}
M_1 &= \sigma I(2I_1+I_3) & L_1 &= 2\sigma I\sqrt{I_1^2+I_3I_1} \\
M_2 &= \sigma I(2I_2+I_3) & L_2 &= 2\sigma I\sqrt{I_2^2+I_3I_2} \\
\sigma_1 &= \frac{L_1}{2\sigma I^2} & \sigma_2 &= \frac{L_2}{2\sigma I^2}
\end{aligned}\right\}$$

and making the substitution

$$\left.\begin{aligned}
\xi &= M_1+L_1\cos\psi \\
\eta &= M_2+L_2\cos\chi
\end{aligned}\right\} \qquad 44\cdot1$$

so that

$$\left.\begin{aligned}
\frac{d\xi}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_1+I_3)\xi-\xi^2}} &= d\psi \\
\frac{d\eta}{\sqrt{-\sigma^2I^2I_3^2+2\sigma I(2I_2+I_3)\eta-\eta^2}} &= d\chi.
\end{aligned}\right\}$$

The integrals for w_1 and w_2 can now be written immediately, and we have

$$w_1 = \psi+\sigma_1\sin\psi+\sigma_2\sin\chi+\pi \qquad 44\cdot21$$

$$w_2 = \chi+\sigma_1\sin\psi+\sigma_2\sin\chi+\pi \qquad 44\cdot22$$

$$w_3 = \varphi + \tfrac{1}{2}(\psi + \chi) + \sigma_1 \sin \psi + \sigma_2 \sin \chi + \pi$$
$$+ i \log \frac{\left[(M_1 + L_1) \cos \frac{\psi}{2} + i\sigma I_1 I_3 \sin \frac{\psi}{2}\right]}{\sqrt{(M_1 + L_1)(M_2 + L_2)}} \frac{\left[(M_2 + L_2) \cos \frac{\chi}{2} + i\sigma I_1 I_3 \sin \frac{\chi}{2}\right]}{\sqrt{(M_1 + L_1 \cos \psi)(M_2 + L_2 \cos \chi)}}. \qquad 44\ 23$$

The arbitrary constant, π, has been added to each w for future convenience. The w's being angle variables, any constants added to them can have no effect on the final result.

We first express the displacement of the electron along the axis, in the line of the external electric field, as a multiple Fourier's series. Since,

$$z = \tfrac{1}{2}(\xi - \eta),$$

and ξ and η, and therefore z, are functions of w_1 and w_2 only, we can write,

$$z = \sum A_{\tau_1\tau_2} e^{i(\tau_1 w_1 + \tau_2 w_2)},$$

where the summations are over all the positive and negative integers, τ_1 and τ_2. The coefficients in this expansion are determined by,

$$A_{\tau_1\tau_2} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \frac{\xi - \eta}{2} e^{-i(\tau_1 w_1 + \tau_2 w_2)} \, dw_1 \, dw_2,$$

where ξ and η are supposed to be expressed in terms of w_1 and w_2. Now make a change of variables from w_1, w_2 to ψ, χ.

$$z = \frac{\xi - \eta}{2} = \sigma I(I_1 - I_2) + \sigma I^2(\sigma_1 \cos \psi - \sigma_2 \cos \chi).$$

The constant term in z will have an effect only on the coefficient $A_{00}\cdot$, for all the other coefficients we can ignore it. The functional determinant of the transformation is,

$$1 + \sigma_1 \cos \psi + \sigma_2 \cos \chi,$$

and we therefore get,

$$A_{\tau_1\tau_2} = \frac{\sigma I^2}{4\pi^2}(-1)^{\tau} \int_0^{2\pi} \int_0^{2\pi} (\sigma_1^2 \cos^2 \psi + \sigma_1 \cos \psi - \sigma_2^2 \cos^2 \chi - \sigma_2 \cos \chi)$$
$$\times e^{-i(\tau_1\psi + \tau\sigma_1 \sin \psi + \tau_2\chi + \tau_2\sigma_2 \sin \chi)} \, d\psi d\chi,$$

where

$$\tau = \tau_1 + \tau_2.$$

If $\cos \psi$ and $\cos \chi$ are expressed in the exponential form, the integrals will all be of the type,

$$\int_0^{2\pi} e^{-in\theta + ix \sin \theta} \, d\theta$$

where n is a positive or negative integer, and x may be positive or negative.

The definite integral,

$$J_n(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{-in\theta + ix\sin\theta}\,d\theta,$$

where $J_n(x)$ is Bessel's function of order n, is valid for all positive and negative integral values of n, and all positive and negative values of x. All the integrals we have to evaluate are of this form, and with the aid of the recurrence formulæ,

$$J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x)$$

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$$

and

$$J_n(-x) = (-1)^n J_n(x)$$

we find

$$A_{\tau_1\tau_2} = \frac{\sigma I^2}{\tau}\left\{\sigma_2 J_{\tau_1}(\tau\sigma_1)J'_{\tau_2}(\tau\sigma_2) - \sigma_1 J'_{\tau_1}(\tau\sigma_1)J_{\tau_2}(\tau\sigma_2)\right\}.$$

For A_{00}, we find, making use of the constant term in the expression for z,

$$A_{00} = \tfrac{3}{2}\sigma I(I_1 - I_2).$$

A combination of integers giving $\tau = \tau_1 + \tau_2 = 0$ leads to an indeterminate result, but for such combinations we find $A_{\tau_1\tau_2} = 0$. We therefore get the Fourier expansion for the displacement of the electron in the line of the electric field, in the form,

$$z = \frac{3}{2}\sigma I(I_1 - I_2) + \sigma I^2 \sum \frac{1}{\tau}\left\{\sigma_2 J_{\tau_1}(\tau\sigma_1)J'_{\tau_2}(\tau\sigma_2) - \sigma_1 J'_{\tau_1}(\tau\sigma_1)J_{\tau_2}(\tau\sigma_2)\right\} e^{i(\tau_1 w_1 + \tau_2 w_2}.$$

We must next obtain the Fourier expansion for displacements in the plane perpendicular to the electric intensity. The radius vector to the electron in any plane perpendicular to the axis, is

$$x + iy = \rho e^{i\phi} = \sqrt{\xi\eta}\, e^{i\phi}.$$

In developing this in a Fourier series it is obvious that the periodic term must be of the form

$$e^{i(\tau_1 w_1 + \tau_2 w_2 + w_3)}$$

for otherwise the angle φ would appear in the expression for the coefficients; and owing to the symmetry of the system about the axis, φ being a cyclic coordinate, it cannot appear in the result.

The expression for the coefficients in the expansion is most easily obtained by expanding

$$(x+iy)\,e^{i(w_2-w_3)}=\sum B_{\tau_1\tau_2}e^{i(\tau_1 w_1+\overline{\tau_2+1}w_2)}.$$

We have, from 44·1, 44·22, 23,

$$(x+yi)\,e^{i(w_2-w_3)}=\sqrt{\xi\eta}\,e^{i(\varphi+w_2-w_3)}$$

$$=\frac{e^{-i\frac{\psi}{2}+i\frac{\chi}{2}}}{\sqrt{M_1+L_1}\quad\sqrt{M_2+L_2}}\left\{(M_1+L_1)\cos\frac{\psi}{2}+i\sigma II_3\sin\frac{\psi}{2}\right\}$$
$$\left\{(M_2+L_2)\cos\frac{\chi}{2}+i\sigma II_3\sin\frac{\chi}{2}\right\}$$

so that we have

$$B_{\tau_1\tau_2}=\frac{1}{4\pi^2\sqrt{(M_1+L_1)(M_2+L_2)}}\int_0^{2\pi}\!\!\int_0^{2\pi}\left\{(M_1+L_1)\cos\frac{\psi}{2}+i\sigma II_3\sin\frac{\psi}{2}\right\}$$
$$\times\left\{(M_2+L_2)\cos\frac{\chi}{2}+i\sigma II_3\sin\frac{\chi}{2}\right\}\times e^{-i\frac{\psi}{2}+i\frac{\chi}{2}}e^{-i(\tau_1 w_1+\tau_2 w_2+w_3)}dw_1dw_2.$$

As before, we change the variables from w_1, w_2, to ψ, χ. The integrals again all become Bessel's functions, and the result may be written

$$B_{\tau_1\tau_2}=-\frac{\sigma I^2}{\tau}\left\{\iota_{13}\iota_{23}J_{\tau_1}(\tau\sigma_1)\,J_{\tau_2}(\tau\sigma_2)-\iota_2\iota_2 J_{\tau_1+1}(\tau\sigma_1)\,J_{\tau_2+1}(\tau\sigma_2)\right\}$$

where the following notation is used

$$\left.\begin{array}{l}
\iota_1=\sqrt{\dfrac{I_1}{I}},\ \iota_2=\sqrt{\dfrac{I_2}{I}},\ \iota_{13}=\sqrt{\dfrac{I_1+I_3}{I}},\ \iota_{23}=\sqrt{\dfrac{I_2+I_3}{I}},\\
\sigma_1=\iota_1\iota_{13}\quad\sigma_2=\iota_2\iota_{23}\\
M_1=\sigma I^2(\iota_1^2+\iota_{13}^2)\quad L_1=2\sigma I^2\iota_1\iota_{13}\\
M_2=\sigma I^2(\iota_2^2+\iota_{23}^2)\quad L_2=2\sigma I^2\iota_2\iota_{23}\\
\qquad\tau=\tau_1+\tau_2+1
\end{array}\right\}$$

For $\tau=0$ the value given for $B_{\tau_1\tau_2}$ becomes indeterminate. As special cases, we find

$$B_{-1,0}=\frac{3}{2}\sigma I^2\iota_1\iota_{23},\quad B_{0,-1}=\frac{3}{2}\sigma I^2\iota_2\iota_{13},$$

while the coefficients for all other combinations that make $\tau=0$ vanish. The displacement of the electron in a plane perpendicular to the axis is therefore given by,

$$x+iy=\frac{3}{2}\sigma I^2\left\{\iota_1\iota_{23}e^{i(-w_1+w_3)}+\iota_2\iota_{13}e^{i(-w_2+w_3)}\right\}$$
$$-\sigma I^2\sum\frac{1}{\tau}\left\{\iota_{13}\iota_{23}J_{\tau_1}(\tau\sigma_1)\,J_{\tau_2}(\tau\sigma_2)-\iota_1\iota_2 J_{\tau_1+1}(\tau\sigma_1)\,J_{\tau_2+1}(\tau\sigma_2)\right\}$$
$$\times e^{i(\tau_1 w_1+\tau_2 w_2+w_3)}.$$

In terms of the action variables, $I_k/2\pi$, the energy of the system is, by 40.4

$$H = -\alpha_1 = -\frac{2ms^2e^4\pi^2}{I^2} - \frac{3}{8}\frac{EI(I_1-I_2)}{\pi^2mse}.$$

The mean motions, ω_k, given by 13·8 are,

$$\frac{\omega_1}{2\pi} = \frac{\omega_3}{2\pi} - \frac{3}{8}\frac{EI}{\pi^2mse}$$

$$\frac{\omega_2}{2\pi} = \frac{\omega_3}{2\pi} + \frac{3}{8}\frac{EI}{\pi^2mse}$$

$$\frac{\omega_3}{2\pi} = \frac{4ms^2e^4\pi^2}{I^3} - \frac{3}{8}\frac{E(I_1-I_2)}{\pi^2mse}.$$

It follows that,

$$\omega_1 + \omega_2 - 2\omega_3 = 0.$$

There is thus a linear relation between the mean motions and the system is degenerate, as there are only two independent mean motions. If higher powers of the electric field, E, had been retained the system would not be degenerate. For the two independent mean motions we shall take ω_3, given above, and

$$\omega' = \frac{3}{8}\frac{EI}{\pi^2sme},$$

and then we shall have,

$$\omega_1 = \omega_3 - \omega'$$

$$\omega_2 = \omega_3 + \omega'.$$

If we now use the relations,

$$w_k = \omega_k t + \delta_k,$$

the exponential terms in the displacement parallel and perpendicular to the electric field will be,

$$e^{i(\tau_1\omega_1+\tau_2\omega_2)t},$$

and

$$e^{i(\tau_1\omega_1+\tau_2\omega_2+\omega_3)t},$$

respectively, as we may take the constants δ_k to be zero.

We now apply Bohr's principle of correspondence. According to ordinary dynamics we should expect to find radiation polarized along the axis, of frequencies given by,

$$\nu = \frac{1}{2\pi}(\tau_1\omega_1 + \tau_2\omega_2),$$

and radiation polarized circularly, in planes perpendicular to the axis, of frequencies,

$$\nu = \frac{1}{2\pi}(\tau_1\omega_1 + \tau_2\omega_2 + \omega_3).$$

According to Bohr's frequency condition, on the other hand, the frequencies are given by,

$$\nu = \frac{1}{h}\left\{\epsilon_{n_1'n_2'n_3'} - \epsilon_{n_1''n_2''n_3''}\right\}$$

and in the region of long wave-lengths we have seen that this becomes

$$\nu = \frac{1}{2\pi}(\tau_1\omega_1 + \tau_2\omega_2 + \tau_3\omega_3),$$

where

$$\tau_1 = n_1' - n_1'', \quad \tau_2 = n_2' - n_2'', \quad \tau_3 = n_3' - n_3''.$$

Making use of the correspondence principle, we see that, as $\tau_3 = \pm 1$ are the only values of this integer, transitions from one state to another that result from n_3 changing by one unit should produce radiation polarized in planes perpendicular to the direction of the electric field; and that transitions in which n_3 remains unchanged should produce radiation polarized in the direction of the electric field.

In accordance with the principle of correspondence, the intensity of the light emitted is measured by the square of the coefficients in the Fourier expansion corresponding to the frequency of the radiation. These terms are,

$$R = \frac{2}{\tau}\left\{\sigma_2 J_{\tau_1}(\tau\sigma_1)\, J'_{\tau_2}(\tau\sigma_2) - \sigma_1 J'_{\tau_1}(\tau\sigma_1)\, J_{\tau_2}(\tau\sigma_2)\right\}$$

for the light polarized along the line of the electric field, and

$$R = \frac{1}{\tau}\left\{\iota_{13}\iota_{23} J_{\tau_1}(\sigma_1\tau)\, J_{\tau_2+1}(\sigma_2\tau) - \iota_1\iota_2 J_{\tau_1+1}(\tau\sigma_1)\, J_{\tau_2+1}(\tau\sigma_2)\right\}$$

for the light polarized in the plane perpendicular to the axis. As we are dealing with relative values only, the factor σI^2 has been taken as unity. The factor 2 in the first expression arises from the fact that equal and opposite rotations about the axis produce equivalent effects. The constants in these expression are determined by,

$$\sigma_1 = \frac{1}{n}\sqrt{n_1(n_1 + n_3)}$$

$$\sigma_2 = \frac{1}{n}\sqrt{n_2(n_2 + n_3)}$$

$$\iota_1 = \sqrt{\frac{n_1}{n}}, \quad \iota_2 = \sqrt{\frac{n_2}{n}}$$

$$\iota_{13} = \sqrt{\frac{n_1 + n_3}{n}}, \quad \iota_{23} = \sqrt{\frac{n_2 + n_3}{n}}$$

$$n = n_1 + n_2 + n_3, \qquad \iota = \iota_1 + \iota_2 + \iota_3$$

$$\iota_1 = n_1' - n_1'', \qquad \iota_2 = n_2' - n_2'', \qquad \iota_3 = n_3' - n_3''.$$

The accents and double accents refer to the initial and final states, respectively.

In the following table, taken from Kramer's memoir, the results of estimating the relative intensities of the components of the Stark effect for the first four lines of the Balmer series of hydrogen are given. The first column gives the transition, $n_1'n_2'n_3' \longrightarrow n_1''n_2''n_3''$. The second column gives the value of Z (42·2), which in our present notation is

$$Z = (n_1' - n_2')\,(n_1' + n_2' + n_3') - (n_1'' - n_2'')\,(n_1'' + n_2'' + n_3'').$$

The next three columns give the values of τ_1, τ_2, and τ_3. The value of the coefficient, R, for the initial state, denoted R', is given in the sixth column, and for the final state, R'', in the seventh column. Their squares are given in the next two columns. In the last column is given the relative intensity estimated by Stark from his observations.

H$_\alpha$ 6562·8 Å. 3 ⟶ 2

	Transition	Z	τ_1	τ_2	τ_3	R′	R″	R'^2	R''^2	Obs.
Parallel	111 ⟶ 011	2	1	0	0	0.46	0	0.21	0	1.0
	102 ⟶ 002	3	1	0	0	0.51	0	0.26	0	1.1
	201 ⟶ 101	4	1	0	0	0.62	0.57	0.38	0.33	1.2
	201 ⟶ 011	8	2	−1	0	0	0	0	0	...
Perpendicular	003 ⟶ 002 }	0	0	0	1	1.0	1.0	1.0	1.0	} 2.6
	111 ⟶ 002 }	0	1	1	−1	0.26	0	0.07	0	
	102 ⟶ 101	1	0	0	1	0.75	0.62	0.56	0.39	1.0
	102 ⟶ 011	5	1	−1	1	0	0	0	0	...
	201 ⟶ 002	6	2	2	−1	0.05	0	0.003	0	...

H$_\beta$ 4861·3 Å. 4 ⟶ 2

	Transition	Z	τ_1	τ_2	τ_3	R′	R″	R'^2	R''^2	Obs.
Parallel	112 ⟶ 002	0	1	1	0	0	0	0	0	1.4
	211 ⟶ 101	2	1	1	0	0.02	0	0.0004	0	1.2
		(4)								1.0
	211 ⟶ 011	6	2	0	0	0.15	0	0.021	0	4.8
	202 ⟶ 002	8	2	0	0	0.17	0	0.030	0	9.1
	301 ⟶ 101	10	2	0	0	0.21	0.17	0.045	0.030	11.5
		(12)								1.0
	301 ⟶ 011	14	3	−1	0	0	0	0	0	...
Perpendicular		(0)								1.4
	112 ⟶ 011	2	1	1	1	0.12	0	0.014	0	3.3
	103 ⟶ 002 }	4	1	0	1	0.19	0	0.036	0	} 12.6
	211 ⟶ 002 }	4	2	1	−1	0.06	0	0.004	0	
	202 ⟶ 101	6	1	0	1	0.19	0.19	0.037	0.037	9.7
		(8)								1.3
	202 ⟶ 011	10	2	−1	1	0	0	0	0	1.1
	301 ⟶ 002	12	3	0	−1	0.02	0	0.0004	0	1.0

$H\gamma$ 4340·5 Å. 5 ⟶ 2

	Transition	Z	τ_1	τ_2	τ_3	R′	R″	R'^2	R''^2	Obs.
Parallel	221 ⟶ 011	2	2	1	0	0.033	0	0.0011	0	1.6
	212 ⟶ 002	5	2	1	0	0.022	0	0.0005	0	1.5
	311 ⟶ 101	8	2	1	0	0.013	0	0.0016	0	1.0
	311 ⟶ 011	12	3	0	0	0.074	0	0.0055	0	2.0
	302 ⟶ 002	15	3	0	0	0.093	0	0.0086	0	7.2
	401 ⟶ 101	18	3	0	0	0.112	0.080	0.0125	0.0063	10.8
	401 ⟶ 011	22	4	−1	0	0	0	0	0	1 (?)
Perpendicular	113 ⟶ 002	0	1	1	1	0.065	0	0.0041	0	7.2
	221 ⟶ 002	0	2	−2	−1	0.031	0	0.0009	0	
	212 ⟶ 101	3	1	1	1	0.057	0	0.0032	0	3.2
	212 ⟶ 011	7	2	0	1	0.045	0	0.0020	0	1.2
	203 ⟶ 002	10	2	0	1	0.085	0	0.0072	0	4.3
	311 ⟶ 002	10	3	1	−1	0.025	0	0.0006	0	
	302 ⟶ 101	13	2	0	1	0.088	0.089	0.0077	0.080	6.1
	302 ⟶ 011	17	3	−1	1	0	0	0	0	1.1
	401 ⟶ 002	10	4	0	−1	0.014	0	0.0002	0	1.0

$H\delta$ 4101·7 Å. 6 ⟶ 2

	Transition	Z	τ_1	τ_2	τ_3	R′	R″	R'^2	R''^2	Obs.
Parallel	222 ⟶ 002	0	2	2	0	0	0	0	0	...
	321 ⟶ 101	4	2	2	0	0.008	0	0.0001	0	1.9
	321 ⟶ 011	8	3	1	0	0.027	0	0.0007	0	1.2
	312 ⟶ 002	12	3	1	0	0.020	0	0.0004	0	1.5
	411 ⟶ 101	16	3	1	0	0.016	0	0.0003	0	1.2
	411 ⟶ 011	20	4	0	0	0.045	0	0.0020	0	1.1
	402 ⟶ 002	24	4	0	0	0.060	0	0.0036	0	2.8
	501 ⟶ 101	28	4	0	0	0.066	0.043	0.0044	0.0019	7.2
	501 ⟶ 011	32	5	−1	0	0	0	0	0	1 (?)
Perpendicular	222 ⟶ 011	2	2	1	1	0.028	0	0.0008	0	1.3
	213 ⟶ 002	6	2	1	1	0.036	0	0.0013	0	3.2
	321 ⟶ 002	6	3	2	−1	0.017	0	0.0003	0	
	312 ⟶ 101	10	2	1	1	0.033	0	0.0011	0	2.1
	312 ⟶ 011	14	3	0	1	0.023	0	0.0005	0	1.0
	303 ⟶ 002	18	3	0	1	0.049	0	0.0024	0	2.0
	411 ⟶ 002	18	4	1	−1	0.013	0	0.0002	0	
	402 ⟶ 101	22	3	0	1	0.051	0.049	0.0026	0.0024	2.4
	402 ⟶ 011	26	4	−1	1	0	0	0	0	1.3
	501 ⟶ 002	30	5	0	−1	0.006	0	0.00004	0	1 (?)

In this table, transitions which lead to the same value of Z are bracketed together as they give the same component. No state for which $n_3=0$ is counted as a possible stationary state. Remembering that $n_3=0$ means that the angular momentum of the electron about the axis is zero, the necessity for this restriction becomes evident. In general, the agreement between the computed values of the intensities, taken as proportional to the means of R'^2 and R''^2, and the observed intensities, estimated by Stark, is remarkably good. There are, however, some exceptions, notably in the H_β line. For example, the transition

112 ⟶ 002, which, according to this theory, should not result in a component of appreciable intensity, is actually observed. And for this same line there are four values of Z, put in parentheses, which correspond to observed components, although there is no transition leading to them which does not violate the conditions which, in general, hold. It may be that these exceptions are the result of the neglect of taking into account the changes due to effects of relativity, to be mentioned later. But it must be remembered that there is an uncertainty in the experimental results, due to the long exposures necessary to obtain the photographs, and some of the observed components may be spurious. On the whole, these results afford most convincing evidence for the value of the principles which Bohr has introduced into the quantum theory.

45. THE ZEEMAN EFFECT.—Particular interest is attached to the Zeeman effect because its discovery was the result of an attempt to confirm the views held by Lorentz of the part played by charged ions in optical phenomena, even before the electron, as a physical entity, had been discovered through the researches of Sir J. J. Thomson. In Lorentz's theory, a model of a source of light radiation was a charged ion, or, as we say now, an electron, vibrating about a position of equilibrium. The value of the ratio of the charge to the mass of the electron, deduced from measurements of the Zeeman effect on the basis of this theory, was found to be in good agreement with the value deduced from direct measurements on electrons in rarefied gases. But we know now that in order to account for spectral series such a model of a source of light radiation is not adequate, and it is therefore of importance to see whether the hypotheses of the quantum theory will account for the Zeeman effect as well as the classical theory.

Keeping to the same model that has served so well to illustrate the main features of the Stark effect, we see at once that a somewhat different procedure is necessary, because the force acting upon an electron that arises from a magnetic field is always at right angles to the direction of motion of the electron, and so no work is done upon the electron by the magnetic force. The effect of the magnetic force cannot, therefore, be included in the potential energy of the electron. If the magnetic field intensity be denoted by F, and its direction be taken along the z-axis, the equations of motion of the electron will be,[1]

$$\begin{aligned} m\ddot{x} - \frac{eF}{c}\dot{y} &= -\frac{\partial V}{\partial x} \\ m\ddot{y} + \frac{eF}{c}\dot{x} &= -\frac{\partial V}{\partial y} \\ m\ddot{z} \qquad\quad &= -\frac{\partial V}{\partial z} \end{aligned}$$

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[1] Debye, Phys. Zeit. 17, 507, 1916.

where V is the potential energy of the electron arising from the central attraction of the nucleus,

$$V = \frac{e^2 s}{r} \cdot$$

If we now take a Hamiltonian function,

$$H = \frac{1}{2m}\left\{\left(p_1 + \frac{1}{2}\frac{eF}{c}y\right)^2 + \left(p_2 - \frac{1}{2}\frac{eF}{c}x\right)^2 + p_3^2\right\} + V,$$

where

$$p_1 = m\dot{x} - \frac{1}{2}\frac{eF}{c}y$$

$$p_2 = m\dot{y} + \frac{1}{2}\frac{eF}{c}x$$

$$p_3 = m\dot{z}$$

it will be seen that p_1, p_2, p_3, x, y, z form a canonical system of variables, and the canonical equations will lead to the equations of motion of the electron in the rectangular coordinates. We can now transform to spherical polar coordinates, so that the new canonical variables are

$$p_r.\ p_\theta.\ p_\varphi.\ r,\ \theta,\ \varphi,$$

and the Hamiltonian function becomes,

$$H = \frac{1}{2m}\left\{p_r^2 + \frac{1}{r^2}p_\theta^2 + \frac{1}{r^2 sin^2\theta}\ p_\varphi^2 - \frac{eF}{c}p_\varphi + \frac{1}{4}\frac{e^2F^2}{c^2}r^2\ sin^2\ \theta\right\} + \frac{e^2 s}{r} \cdot$$

It will be noted that, owing to the term in p_φ, this function is not of the form to which Stäckel's theorem applies. We may, however, consider the system as conditionally periodic, subject to a small perturbation resulting from the magnetic field. With this understanding, we may, for a first approximation, neglect the term involving F^2 in comparison with the other terms; then, observing that φ is a cyclic coordinate,

$$p_\varphi = \alpha_3,$$

where α_3 is one of the constants of integration. The Hamilton-Jacobi equation is now

$$\left(\frac{\partial W}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial W}{\partial \theta}\right)^2 + \frac{\alpha_3^2}{r^2 \sin^2\theta} - 2\omega m\alpha_3 + \frac{2mse^2}{r} + 2m\alpha_1 = 0,$$

where

$$\omega = \frac{1}{2}\frac{eF}{cm}$$

and α_1 is the negative constant energy. The variables are now separable, and we get at once,

$$p_r=\frac{\partial W}{\partial r}=\left\{2\omega m\alpha_3-\frac{2mse^2}{r}-2m\alpha_1-\frac{\alpha_2^2}{r^2}\right\}^{\frac{1}{2}}$$

$$p_\theta=\frac{\partial W}{\partial \theta}=\left\{\alpha_2^2-\frac{\alpha_3^2}{\sin^2\theta}\right\}^{\frac{1}{2}}$$

with α_2^2 as the third constant of integration. The quantum conditions are,

$$2\int_{r_1}^{r_2}\frac{1}{r}\left\{\left(2m\omega\alpha_3-2m\alpha_1\right)r^2-2mse^2r-\alpha_2^2\right\}^{\frac{1}{2}}dr=n_1h$$

$$2\int_{\theta_1}^{\theta_2}\frac{1}{\sin\theta}\left\{\alpha_2^2\sin^2\theta-\alpha_3^2\right\}^{\frac{1}{2}}d\theta=n_2h$$

$$\int_0^{2\pi}p_\varphi d\varphi=2\pi\alpha_3=n_3h.$$

In the first two integrals the limits are the two roots of the radicals under the sign of integration. The first integral can be evaluated at once, with the aid of

$$\int_{r_1}^{r_2}\frac{dr}{r}\sqrt{-(r-r_1)(r-r_2)}=\frac{\pi}{2}(r_1+r_2)-\pi\sqrt{r_1r_2}.$$

We thus find,

$$-\frac{\pi se^2\sqrt{2m}}{\sqrt{\alpha_1-\omega\alpha_3}}-2\pi\alpha_2=n_1h$$

$$(\alpha_2-\alpha_3)=n_2h.$$

From the last two equations it follows that

$$\alpha_1=\frac{2\pi^2ms^2e^4}{n^2h^2}+\frac{n_3h\omega}{2\pi}$$

where

$$n=n_1+n_2+n_3.$$

The frequencies emitted in a magnetic field are therefore, by Bohr's frequency condition,

$$\nu=\frac{2\pi^2ms^2e^4}{h^3}\left(\frac{1}{n'^2}-\frac{1}{n''^2}\right)+\frac{\omega}{2\pi}\left(n'_3-n''_3\right).$$

Just as in the Stark effect the principle of correspondence may be used to show that the only transitions possible for n_3 are those that result in a change of one unit, with the radiation polarized perpendicular to the axis, and those in which n_3 does not change, when the radiation will be polarized along the axis. Moreover, the constant h does not appear in the term involving the effect of the magnetic field, and its magnitude is the same as is given by the classical theory for a vibrating electron in a magnetic field. Therefore the normal Zeeman effect is accounted for by the quantum theory as well as by the classical theory.

46. Fine Structure of Spectral Lines.—All of the previous applications of the quantum theory to physical phenomena are founded upon classical dynamics, that is, Newton's laws of motion. For velocities that are comparable with the velocity of light these laws are no longer valid, but need to be modified in accordance with the principle of relativity. Bohr[1] was the first to recognize the importance of taking relativity into account in the problem of spectral lines. Following Debye[2] the simple problem of the motion of an electron in a plane about a positive nucleus will be considered here, in order to show the kind of modifications which result from taking the principle of relativity into account.

A mass particle, or an electron, in uniform motion with velocity v, has momentum $mv\beta$ and kinetic energy $mc^2\beta$, where c is the velocity of light, and

$$\beta = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}.$$

Although the orbital motion of an electron about a nucleus is not a case of uniform motion, to a first approximation we may so regard it, if for v we take the resultant velocity,

$$v^2 = \dot{r}^2 + r^2\dot{\varphi}^2,$$

where r, φ are the polar coordinates of the electron in a plane with the nucleus at the origin. The kinetic energy of the electron may be expressed in terms of the coordinates and momenta, with the aid of,

$$p_r = m\beta\dot{r}, \quad p_\phi = m\beta r^2\dot{\varphi}$$

and there results,

$$T = mc^2\left\{\left[1 + \frac{1}{m^2c^2}\left(p_r^2 + \frac{1}{r^2}p_\phi^2\right)\right]^{\frac{1}{2}} - 1\right\}.$$

The potential energy is,

$$V = -\frac{se^2}{r},$$

where se is taken as the charge on the positive nucleus.

The kinetic energy is not now a homogeneous quadratic function of the momenta, as it is in classical dynamics; it will be assumed, however, that we may use the methods which are founded on classical dynamics, and regard the modifications resulting from the application of the principle of relativity as small perturbations. With this understanding the Hamilton-Jacobi equation may be written,

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[1] Phil. Mag. XXIX, 332, 1915.

[2] Phys. Zeit. 17, 512, 1916.

$$mc^2\left\{\left[1+\frac{1}{m^2c^2}\left(\frac{\partial W}{\partial r}\right)^2+\frac{1}{m^2c^2r^2}\left(\frac{\partial W}{\partial \varphi}\right)^2\right]^{\frac{1}{2}}-1\right\}-\frac{se^2}{r}+\alpha_1=0$$

where α_1 is the negative constant energy. By writing

$$W=\alpha_2\varphi+W_r,$$

where W_r is a function of r only, and α_2 is the second constant of integration, the variables become separable, and there results,

$$\frac{\partial W}{\partial \varphi}=\alpha_2=p_\phi$$

$$\frac{\partial W}{\partial r}=p_r=\left\{m^2c^2\left[1+\frac{\frac{se^2}{r}-\alpha_1}{mc^2}\right]^2-m^2c^2-\frac{\alpha_2^2}{r^2}\right\}^{\frac{1}{2}}.$$

The quantum conditions 22·5, 6 now give, when terms in $1/c^2$ only are kept,

$$\frac{2\pi se^2m}{\sqrt{2m\alpha_1}}-2\pi\alpha_2+\frac{\pi s^2e^4}{\alpha_2c^2}-\frac{3}{4}\frac{\pi se^2}{mc_2}\sqrt{2m\alpha_1}=n_1h$$

$$2\pi\alpha_2=n_2h.$$

Add these equations and in the correction term put

$$\sqrt{2m\alpha_1}=\frac{2\pi se^2m}{(n_1+n_2)h}$$

which is its value when relativity is not considered. We finally get for the energy, $(-\alpha_1)$ in the stationary states characterized by the two integers n_1 and n_2,

$$\epsilon_{n_1n_2}=-\frac{2\pi^2s^2e^4m}{(n_1+n_2)^2h^2}\left\{1+\frac{4\pi^2s^2e^4}{c^2}\left[\frac{1}{n_2(n_1+n_2)h^2}-\frac{3}{4(n_1+n_2)^2h^2}\right]\right\}.$$

When Bohr's frequency condition is applied, the frequencies are given by

$$\nu=\nu_0+\nu_1+\nu_2$$

where,

$$\nu_0=s^2N\left(\frac{1}{n'^2}-\frac{1}{n''^2}\right)$$

$$\nu_1=-\frac{3}{4}Ns^4\alpha^2\left(\frac{1}{n'^4}-\frac{1}{n''^4}\right)$$

$$\nu_2=Ns^4\alpha^2\left(\frac{1}{n''^3n_2''}-\frac{1}{n'^3n_2'}\right)$$

$n=n_1+n_2$. The accents and double accents refer to the final and initial states, respectively,

$$\alpha=\frac{2\pi^2e^2}{hc}$$

and the Rydberg constant, N, is

$$N=\frac{2\pi^2e^4Mm}{ch^3(M+m)}.$$

The mass of the nucleus, M, has been taken into account by the factor $M/(M+m)$.

We see from this that not only is there a small correction, ν_1, to be made to the frequency of any line due to the effect of relativity, but also a number of new components appear, which are given by the term ν_2, arising from the different ways in which the transition $n_1''n_2''\longrightarrow n_1'n_2'$ can be made for given values of n'' and n'. With the aid of Bohr's principle of correspondence, Kramers (l.c.) has calculated the relative intensities of the components of the fine structure of the first three lines of the Balmer series of hydrogen and four lines of the helium spectrum. The method of calculation is similar to that which he employed for the analogous case of the Stark effect. The results of this investigation, when compared with experiment, furnish additional evidence for the hypotheses of the quantum theory, and, of even greater interest, for the validity of the principle of relativity.

47. BAND SPECTRA.—Schwarzchild[1] has made an interesting application of the quantum theory to band spectra. Band spectra are generally regarded as having their origin in the molecules, and line spectra in the atoms. We have previously found (§37) that, regarding a molecule as a rigid body symmetrical about an axis, its energy in the stationary states is given by

$$\epsilon_n=\frac{h^2n^2}{8\pi^2A}$$

where n is any integer, and A the moment of inertia about an axis perpendicular to the axis of symmetry. Let us now suppose that electrons circulate about the molecule and disregard, as a first approximation, any mutual influence of the molecule and electrons. Then the total energy of the system in the nth stationary state will be

$$E_n+\frac{h^2n^2}{8\pi^2A}$$

where E_n is the energy of the electrons. In the mth state the energy will be

$$E_m+\frac{h^2m^2}{8\pi^2A}.$$

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[1] Berlin Akad. 1916. p. 548.

By Bohr's frequency condition,

$$\nu = \frac{E_m - E_n}{h} - \frac{hn^2}{8\pi^2 A} + \frac{hm^2}{8\pi^2 A}$$

which may be written, if n is fixed and m takes integral values:

$$\nu = a + bm^2 \qquad 47 \cdot 1$$

where

$$b = \frac{h}{8\pi^2 A}. \qquad 47 \cdot 2$$

(47·1) is Deslandre's formula for the lines in a band spectrum; from the known values of the constant b, A may be determined by (47·2). Schwarzschild gives the following results

	Wave-length of Head of Band	A
Oxygen	6277 Å.	$8.0 \; 10^{-40}$
Oxygen	6868	$10.0 \; 10^{-40}$
Oxygen	7594	$14.0 \; 10^{-40}$
Mercury	4215	$1.8 \; 10^{-40}$
Nitrogen	3883	$40.0 \; 10^{-39}$
Nitrogen	3577	$14.0 \; 10^{-40}$
Water	3200	$0.3 \; 10^{-40}$

The values of the moments of inertia of the various molecules found in this way are of about the order of magnitude to be expected. In the case of water vapor, for example, the moment of inertia of the water molecule comes out about twelve times that of a hydrogen molecule as determined from Bohr's model of the hydrogen molecule or from the behavior of the specific heat at low temperatures (§38).

CHAPTER VI

PHOTO-ELECTRIC PHENOMENA

The applications that have been made of the hypotheses of the quantum theory in the preceding chapters have all been confined to dynamical systems that are either conditionally periodic, or to systems that may be considered as conditionally periodic with small perturbations. There are, however, a number of phenomena, of which the photo-electric effect is a type, which have never received a satisfactory explanation from the standpoint of classical dynamics and electrodynamics, and which are doubtless governed by some kind of quantum principles. But when the attempt is made to describe these phenomena with the aid of a model, similar to that which has been so successful in accounting for spectral lines, we are met with the difficulty that such a model cannot be a conditionally periodic dynamical system. For in its simplest form, such a model must consist of an atom and an electron which can become wholly detached; instead of a periodic orbit, or an approximately periodic orbit, described under a central attraction, the orbit becomes hyperbolic or parabolic, and the preceding considerations are not directly applicable.

Epstein[1] has attempted to apply quantum conditions to such a model in order to account for the photo-electric effect and the emission of beta particles; in order to get definite results he was obliged, in the quantum integrals, to choose arbitrarily some of the limits of integration. His procedure has been criticized by Nicholson[2] who comes to the conclusion that a finite and determinate value of the energy cannot be obtained with the aid of the hypotheses of the quantum theory if the path of the electron about the nucleus extends to infinity. This result is not surprising; for while there is a certain element of plausibility in the quantum conditions for conditionally periodic systems as developed in §22, there is none at all in similar conditions for systems that are not of this type.

We shall now briefly mention the principal phenomena which are almost certainly governed by quantum conditions, but which are not of the kind to which the quantum conditions, previously developed, are directly applicable.

48. THE PHOTO-ELECTRIC EFFECT.—When light of frequency ν falls upon a metal it is found that electrons are emitted from the metal and

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[1] Annalen der Physik, 50, 815, 1916.

[2] Phil. Mag. XLIV, 193, 1922.

that these electrons have all velocities between zero and a maximum, determined by

$$\frac{1}{2}mv^2_m = h\nu - w. \qquad 48 \cdot 1$$

There is a definite frequency ν_0, given by $w=h\nu_0$, such that however intense the light may be, if its frequency is less than ν_0 no electrons are emitted from the metal. The evidence that the photo-electric effect is governed by some kind of quantum principles is the experimental result that the constant h in the equation has the same value that it has in all the other applications of the quantum theory. There is considerable evidence that equation 48·1 is to be interpreted in such a way that $h\nu$ measures the work that must be done to separate an electron from an atom, and w measures the work done by the electron in escaping from the surface of the metal.

The principal difficulty connected with the photo-electric effect is in accounting for the energy $h\nu$ to separate an electron from the atom. And it is this effect that has led to the view of "light-quanta." If light is of a corpuscular nature, the light corpuscles, or quanta, of frequency ν having energy $h\nu$, then the collision of an atom and one of these corpuscles provides a simple mode of explanation of the photo-electric effect. But the impossibility, in the present state of our knowledge, of reconciling such a view with the great mass of evidence arising from physical optics makes any such hypothesis very improbable.

49. Energy Levels.—It is only in the simplest case, that of a nucleus and a single electron, that the solution of the dynamical problem which must precede the application of the hypotheses of the quantum theory, can be obtained. In order to apply similar hypotheses to more complicated models, representing atoms of higher atomic weight, the conception of "energy levels," due to Bohr, has proved of great value. A model of an atom may be thought of as consisting of a nucleus with electrons arranged in successive rings or shells. These rings or shells may be considered as corresponding to definite energy levels; the transition of an electron from one level to another of less energy will, by Bohr's frequency condition, be accompanied by the emission of a train of waves of frequency ν given by

$$h\nu = \epsilon_m - \epsilon_n$$

where ϵ_m and ϵ_n are the values of the energy in the two levels. And radiation absorbed by the atom may be considered to cause a transition of an electron from a level of lower to higher energy. It is customary to denote these energy levels, proceeding outwards from the nucleus, by the letters K, L, M, $\cdots$. Thus ϵ_K denotes the energy in the level nearest to the nucleus.

50. SECONDARY BETA RAYS.—When Röntgen rays fall upon a metallic surface it is known that, in addition to radiation of other types, secondary beta particles, or electrons, are emitted from the metal, and that their velocities are comparable with the velocities of the beta particles which produced the primary Röntgen rays. This effect is thus analogous to the photo-electric effect for visible and ultra-violet light, and, like the latter, has not received an explanation with the aid of any general principles of the quantum theory. It has been shown by de Broglie that this, and other, experimental results can best be interpreted by supposing that when Röntgen rays of frequency ν fall upon a metal, radiation is absorbed by the atoms in quanta, $h\nu$, with a resulting emission of electrons having a number of different velocities, v, determined by,

$$\frac{1}{2}mv^2 = h\nu - \epsilon_K, \quad h\nu - \epsilon_L, \quad h\nu - \epsilon_M, \quad \cdots$$

where $\epsilon_K \; \epsilon_L \cdots$ are the values of the energy in the levels from which the electron comes. The large values of $h\nu$ in the case of Röntgen rays makes it unnecessary to take into account the work done by the electron in escaping from the surface of the metal. According to this view, of the energy absorbed by the atom, a part is used up in the escape of the electron from the atom, the balance of the energy being supplied to the electron as kinetic energy.

51. CHARACTERISTIC RÖNTGEN RADIATION.—When an electron is removed from one of the interior energy levels of an atom, its place may be supposed to be taken by an electron from one of the less stable outer levels. In this process of transition, according to Bohr's frequency condition, radiation of frequency ν, given by

$$h\nu = \epsilon_m - \epsilon_n$$

will be emitted. According to these ideas, this is the origin of the characteristic Röntgen radiation. If, for example, an electron is removed from the K level, a series of frequencies, the K series, will be emitted. The various members of this series are determined by the energy level from which the electron comes to replace the one removed from the K level. Actually, it is found that the various energy levels, beginning with the L level, are multiple. This is a consequence of taking into account the correction due to the principle of relativity, so that the Röntgen ray spectral lines show a fine structure; as this effect is proportional to the fourth power of the nuclear charge, it becomes very important in the more complex atoms.

In the foregoing it was assumed that the characteristic Röntgen radiation was emitted when primary Röntgen rays fall upon a metal. Now the primary Röntgen radiation is produced by the impact of rapidly moving electrons upon a metallic target, the anti-cathode in a discharge tube, and it appears that quantum principles are involved in this phenomenon also. If e is the charge of the electron, V the difference of potential between the cathode and the anti-cathode, and v the velocity of the electron on striking the anti-cathode, then

$$eV = \frac{1}{2}mv^2$$

is the energy of the electron when it reaches the anti-cathode. It has been shown by Webster that in order to excite any of the rays of a given series of characteristic rays, say the K series, it is necessary that this energy shall be equal, at least, to $h\nu_m$, where ν_m is the frequency of the ray of shortest wave-length in the series. This result is interpreted, using the conception of energy levels, by noting that in order to remove an electron from the K level of an atom an amount of energy equal to $h\nu_m$ is required.

52. Continuous Röntgen Radiation.—In addition to the characteristic Röntgen radiation, formed of series of spectral lines, it is found that there is present, however Röntgen rays are produced, a continuous radiation. It has been shown by Duane that this continuous radiation has a definite limit, on the side of the short wave-lengths, given by the relation

$$eV = h\nu_m,$$

ν_m being the frequency corresponding to the limit of this continuous radiation. This limiting frequency is independent of the nature of the metal giving rise to the radiation, and furnishes one of the best methods of determining h, the characteristic constant of the quantum theory. The existence of this continuous radiation is consistent with the view of Röntgen rays based upon classical electrodynamics, in that they result from the negative acceleration of electrons on penetrating into the metal; but the sharp limit of this radiation, determined by the quantum constant h, cannot be reconciled with the classical theory. On the other hand, the conception of energy levels in the atom, in accordance with Bohr's theory, does not readily lend itself to accounting for the existence of this continuous radiation.

53. Radiating and Ionizing Potentials.—When electrons, moving with a definite velocity v, collide with the molecules or atoms of a gas, effects of two kinds, which are governed by quantum principles, are observed. In the first place, there is an emission of spectral lines; the necessary condition for this is that the energy of the colliding electron shall be at least as great as $h\nu$, ν being the frequency of the corresponding spectral line; or the potential, V, through which the electron must fall to enable it to cause this radiation, must be at least as great as $h\nu/e$.

This potential is called the radiating potential. Secondly, the molecule or atom may become ionized, shown by the presence of positive charges; the necessary condition for this is that the difference of potential through which the electron must fall must reach a critical ionizing potential. For a given atomic model, the work which must be done to separate an electron can be calculated; the ionizing potential is thus a valuable criterion for the accuracy of the model.

54. WHITTAKER'S ATOMIC MODEL.—The phenomena which have just been briefly mentioned, in which an essential feature is the separation of an electron from an atom, cannot, as has been seen, be described with the aid of dynamical principles supplemented by the hypotheses of the quantum theory. In order to account for these phenomena it appears necessary to assume the existence of "light-quanta" if the model of an atom which has been employed is regarded as representing anything real. Now it is quite conceivable that as far as the phenomena of spectral lines, described in Chapter V, are concerned, such a model as has been employed is wholly adequate; but that for the phenomena now under consideration it is too simple. For example, the positive nucleus has been regarded merely as a particle, having a definite mass and charge, and the electron as a negatively charged particle. It may be, however, that for other phenomena such a model is wholly inadequate, and inside the atom a more complicated mechanism may be postulated which will do away with any necessity of postulating a discontinuous distribution of radiant energy.

An important contribution toward this end has been made by Whittaker[1] who has devised a model which appears to be capable of giving an account of some of the phenomena here considered. In this model, an atom contains a wheel formed of bar magnets as spokes, the like ends of all the magnets connected together at the centre, and the whole free to rotate about an axis through the centre. Consider, now, an electron in motion along the axis of rotation, taken as the x-axis. It produces a magnetic force whose value, at one of the free poles, is

$$\frac{ea\dot{x}}{r^3}$$

a being the length of each bar magnet and r the distance of the electron from the free end. The mechanical force exerted upon one of the magnetic poles is therefore in the yz-plane, perpendicular to r; and if A is the moment of inertia of the whole magnetic wheel about its axis of rotation, and M the sum of the free magnetic pole strengths

$$A\ddot{\psi} - \frac{Mea^2\dot{x}}{(a^2+x^2)^{\frac{3}{2}}} = 0$$

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[1] Proc. R. S. E., XLII, part II, 129, 1922.

where ψ is the angle that measures the rotation of the wheel. The rotation of this magnetic wheel constitutes a magnetic current in a circle of radius a, of strength $Ma\dot{\psi}$; this produces an electric force at the electron, so that the equation of motion of the electron is,

$$m\ddot{x}+\frac{Mea^2\psi}{(a^2+x^2)^{\frac{3}{2}}}=0.$$

In writing these equations of motion, for the electron and for the magnetic wheel, all radiation, demanded by the classical theory of electrodynamics, has been ignored.

The equation of motion for the magnetic wheel can be integrated at once, giving,

$$A\psi-\frac{Mex}{\sqrt{a^2+x^2}}=Me,$$

where the constant of integration has been determined by the condition that when the electron is at an infinite distance the wheel is at rest. If the two equations of motion are multipled by ψ and $\dot{x}$ respectively, added, and the resulting equation integrated, we get the energy equation,

$$\frac{1}{2}A\psi^2+\frac{1}{2}m\dot{x}^2=\frac{1}{2}mu^2,$$

where u is the velocity of projection of the electron at $x=-\infty$. The last equation expresses the principle of the conservation of energy, and shows that as the electron approaches the atom, setting the wheel in rotation, its velocity decreases. If the whole initial energy of the electron is expended in setting the wheel in rotation, the electron will arrive at a distance x from the atom, given by

$$\frac{eMx}{\sqrt{a^2+x^2}}=\sqrt{Am}\,u-eM,$$

and then return to $x=-\infty$, receiving energy back from the wheel. The electron thus experiences a true elastic collision with the atom. If the initial velocity, u, of the electron is equal to

$$u=\frac{eM}{\sqrt{Am}}$$

it just reaches the atom and has its motion reversed there; while if

$$u>\frac{eM}{\sqrt{Am}}$$

its motion is reversed on the other side of the atom. If the velocity of projection of the electron is equal to

$$u=\frac{2eM}{\sqrt{Am}}$$

the electron passes through the atom and arrives at $x = +\infty$ with no velocity. Its initial energy, U,

$$U = \frac{2e^2M^2}{A},$$

is wholly given up to the magnetic wheel, which is thus set in rotation, finally, with angular velocity ω, given by,

$$A\omega = 2eM.$$

If the initial energy of the electron is greater than U, it arrives at $x = \infty$ with velocity v, and gives up to the magnetic wheel the energy

$$\tfrac{1}{2}A\omega^2 = \tfrac{1}{2}m(u^2 - v^2)$$

which is seen to be exactly U. We therefore have a model which represents the essential feature of interchange of energy by quanta. No permanent change is produced if the energy of the electron is less than U; however much greater than U the initial energy of the electron may be, it gives up to the atom an amount of energy which is exactly equal to U. It has been shown by Baker[1] that the same relations hold when the electron is not projected along the axis of the magnetic wheel, as Whittaker assumed, but its initial velocity is inclined to the axis.

Having provided a model of an atom capable of absorbing energy from an electron in quanta, Whittaker proceeds to show how this absorbed energy may be transformed into radiation. Just as an electric current flowing in a closed circuit is equivalent to a magnetic shell, so the magnetic current, produced by the approach of the electron, may be regarded as equivalent to an electric shell, which, in turn, is equivalent to a charged condenser. If Q is the charge on one of the plates of this condenser, d the distance between the plates, and K the dielectric constant, we find, by analogy with the magnetic shell,

$$Qd = \tfrac{1}{2}KMa^2\omega.$$

Since $U = eM\omega$,

$$Qd = \frac{1}{2}\frac{Ka^2U}{e}.$$

Considered as a charged condenser, its energy is,

$$U = \frac{2Q^2d}{Ka^2} = \frac{Q^2}{2C}$$

since its capacity is $Ka^2/4d$. Eliminating d, we get,

$$e = Q.$$

Thus the electric separation produced in the atom by the colliding electron is the separation of two electronic charges, $+e$ and $-e$. The oscillatory discharge of this condenser may then be supposed to give rise to radiation of frequency

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[1] Phil. Mag. XLIV, 777, 1922.

$$\nu = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

where L is the equivalent self-induction of the circuit. It is a plausible assumption to make that in atoms of two different kinds,

$$\frac{L_1}{L_2} = \frac{C_1}{C_2}.$$

Now $e^2\sqrt{(L/C)}$ has the same dimensions, Action, in every system of units. Assume, therefore, that

$$e^2\sqrt{\frac{L}{C}} = \frac{h}{\pi}.$$

Then it will be found that

$$h\nu = U.$$

This is the relation which is known to exist between the energy absorbed by an atom from a colliding electron when it is caused to emit a single-line spectrum of frequency ν.

It is interesting to observe, as Allen[1] has shown, that the rather arbitrary relations, $e^2\sqrt{(L/C)}=h/\pi$, assumed by Whittaker, is an immediate consequence of applying the quantum condition for a system of one degree of freedom, to the atom alone. This condition is,

$$\int p\,dq = \int 2T\,dt = nh,$$

the integral being taken over one complete period of the motion. Interpreting the kinetic energy, T, as magnetic energy,

$$2T = Li^2,$$

where $i=\dot{q}$, q being the instantaneous charge on one of the plates of the condenser. Since we can write,

$$q = e\cos\frac{t}{\sqrt{LC}}$$

$$2T = \frac{e^2}{LC}\sin^2\frac{t}{\sqrt{LC}},$$

and therefore,

$$\frac{h}{\pi} = e^2\sqrt{\frac{L}{C}},$$

which is the relation assumed by Whittaker.

In order to account for the photo-electric effect with the aid of his model, Whittaker supposes that in a metal there are magnetic systems whose natural frequencies are distributed over a wide range. If light of frequency ν is incident upon the metal, energy will be continuously absorbed from it by those magnetic systems in resonance with the light, setting the magnetic wheels in rotation. Suppose that there is an electron near one of these wheels; the previous arguments need only to be reversed to show that if the energy that the wheel has absorbed from the light is equal to or greater than $U=h\nu$, the electron will be sucked through the wheel and an amount of energy equal to $h\nu$ permanently given to it by the wheel. This, as has been seen, is exactly what is needed to account for the photo-electric effect. If, on the other hand, the energy that the wheel has absorbed from the incident light is, at any instant, less than $h\nu$, no permanent change will be produced in an electron near it.

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[1] Proc. R. S. E., XLII, Part II, 213, 1922.

Whittaker finally shows the connection between his model and Bohr's frequency condition. In Bohr's model of the hydrogen atom, consisting of a positively charged nucleus and a single electron, two of whose stationary orbits have radii a_0 and a_1, let the electron fall from a position P_1 on orbit of radius a_1, to a position P_0 on orbit of radius a_0. Assume two equal and opposite charges, e, denoted E' and E'' at P_0. Instead of the electron E, at P_1, falling to P_0, the condenser whose charges are E and E'' may be supposed to be discharged, leaving the electron E' at P_0. The energy set free, on Bohr's conception of the fall of the electron is,

$$U=\frac{e^2}{2}\left(\frac{1}{a_0}-\frac{1}{a_1}\right).$$

This may be written, if C is the capacity of a spherical condenser of radii a_0 and a_1,

$$U=\frac{e^2}{2C}\cdot$$

But this is the relation previously obtained, and with $u=h\nu$, we find,

$$\nu=\frac{e^2}{2h}\left(\frac{1}{a_0}-\frac{1}{a_1}\right).$$

If, in Bohr's model, circular orbits only are considered, the radii of the various stationary orbits are connected with the quantum numbers, n, by the relation,

$$\frac{e^2}{2ha}=\frac{N}{n^2}$$

where N is the Rydberg constant. It therefore follows that the frequencies of the spectral lines are given by

$$\nu=N\left(\frac{1}{n_0{}^2}-\frac{1}{n_1{}^2}\right)$$

and this is known to give the series spectra of atomic hydrogen.

Whittaker's atomic model, regarded as a working model of an atom, appears to be much more artificial than Bohr's model in that there is not the same evidence for its principal constituent—something analogous to a magnetic wheel—that there is for the constituents of the Bohr model. But it is of very great interest and importance in indicating that it may be possible to devise a model to interpret the principles of the quantum theory. And it is important, also, from another point of view, in indicating the possibility of a magnetic interpretation of the quantum theory, which will be considered in the next chapter.

CHAPTER VII

THE QUANTUM THEORY AND MAGNETISM

55. The Magneton.—Our development of the quantum theory has proceeded from the view that the laws of classical dynamics are valid up to a certain point, but that they need to be supplemented by hypotheses peculiar to the quantum theory. This view has its origin in the fact that in the first application, by Planck, of quantum principles to deduce his radiation formula, the constant h, in that formula, has the dimensions of Action, in the principle of least action. But h has also the dimensions of angular momentum, and McLaren[1] was led to take $h/2\pi$ as the fundamental unit of angular momentum of a magneton. A magneton, on McLaren's view, is the inner limiting surface of the aether, formed like an anchor ring, with tubes of electric polarization beginning, or ending, on its surface. The whole rotates about its axis, and consequently there are tubes of magnetic induction threading its aperture. The latter give the magneton the properties of a permanent magnet; hence its name. The magnetic interpretation of the quantum theory has been developed by Allen[2] in a number of papers, and an account of this will now be given.

Consider a magneton, rotating about its axis. The tubes of electric polarization which end on the magneton rotate about the axis, giving rise to a magnetic force. It was shown by J. J. Thomson[3] that when a tube of electric polarization is in motion there is a magnetic force produced, equal to

$$H = 4\pi N v \sin \theta,$$

where N is the electric polarization, v the velocity of the tube, and θ the angle between N and v. In the case here considered, $\theta = \pi/2$, and so the magnetic energy per unit volume is

$$\frac{\mu H^2}{8\pi} = 2\pi\mu N^2 v^2.$$

If this magnetic energy is interpreted as kinetic energy, the equivalent mass per unit volume is given by

$$\frac{1}{2} m v^2 = 2\pi\mu N^2 v^2,$$

or

$$m = 4\pi\mu N^2.$$

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[1] Phil. Mag. XXVI, 800, 1913.

[2] Phil. Mag. XLI, 113, 1921; XLII, 523, 1921; Proc. R. S. E., XLI, 34, 1921; XLII, 213, 1922.

[3] Recent Researches in Electricity and Magnetism, § 12.

Let ω be the angular velocity of the magneton and r the distance of any point from the axis of rotation. Then the angular momentum per unit volume is

$$A.M. = 4\pi\mu N^2\omega r^2.$$

With

$$H = 4\pi N\omega r,$$

we have,

$$A.M. = \frac{\mu H^2}{4\pi\omega} = \frac{2}{\omega}\frac{\mu H^2}{8\pi}.$$

Therefore the whole angular momentum associated with the magneton is $\frac{2}{\omega}\times$ total magnetic energy, and is thus,

$$\frac{2}{\omega}\frac{1}{2}Li^2,$$

where L is the coefficient of self-induction of the circuit, and i is the equivalent electric current. If N_m is the magnetic induction linked with the magneton,

$$N_m = Li,$$

and if N_e is the electric charge on the magneton,

$$i = \frac{N_e\omega}{2\pi}.$$

Therefore it follows that

$$A.M. = \frac{1}{2\pi}N_mN_e.$$

Taking, now, $h/2\pi$ as a natural unit of angular momentum, for a unit magneton we shall have,

$$N_mN_e = h,$$

or, in words, the product of the electric charge on the unit magneton by the magnetic induction linked with it is equal to the constant h. And for any magneton, of n units,

$$N_mN_e = nh.$$

This view is consistent with the dynamical interpretation of the quantum theory. For if T is the kinetic energy, we have,

$$T = \frac{1}{2}Li^2 = \frac{1}{2}N_mN_e\frac{\omega}{2\pi},$$

and integrated over one period of the motion,

$$\int 2T\,dt = nh = N_mN_e$$

as before.

With $h=6.545\ 10^{-27}$, $N_e=e=4.774\ 10^{-10}$ *e.s.* units, the unit tube of magnetic induction is

$$1.371\ 10^{-17}\ \textit{e.s. units}$$

or

$$4.113\ 10^{-7}\ \textit{e.m. units}.$$

The magnetic moment, M, of the unit magneton is obtained by noting that it is equal to the strength of the equivalent magnetic shell times the area, or the current times the area. So,

$$M=\frac{N_e\omega}{2\pi}\pi a^2.$$

But for unit angular momentum,

$$ma^2\omega=\frac{h}{2\pi}$$

and therefore,

$$M=\frac{he}{4\pi m}$$

with $N_e=e$. This is exactly equal to the magnetic moment of a Bohr magneton; that is, the magnetic moment of a single electron moving in the circular orbit corresponding to the quantum number $n=1$, about a positive nucleus with charge e.

If $h/2\pi$ is regarded as a fundamental unit of angular momentum, we are led, in this way, to regard tubes of magnetic induction as discrete entities, in accordance with the views of Faraday. And the fundamental unit of magnetic induction, the unit tube, is connected with the constant h by the relation

$$N_m=h/e,$$

e being the fundamental unit of electric charge.

The electrokinetic energy associated with the unit tube is

$$\int\frac{\mu H^2}{8\pi}A\,ds$$

integrated over the length of the tube, where A is the area of the cross-section of the tube and ds is an element of its length. But

$$\mu HA=h/e$$

along its whole length, so that the energy becomes

$$\frac{h}{8\pi e}\int H\,ds.$$

But the line integral of the magnetic force around the closed tube is equal to 4π times the electric current linked with it. If the unit tube is linked with a unit magneton, the current is $e\nu$, where ν is the frequency of rotation; so the electro-kinetic energy associated with the unit tube is

$$\frac{1}{2}h\nu.$$

56. THE MAGNETON AND WHITTAKER'S ATOMIC MODEL.—Allen[1] has suggested that the magnetic wheel in Whittaker's atomic model might be replaced by two thin magnetons, near together, and having a common axis. It seems possible that a detailed investigation would show that such a result might be obtained. A model of an atom formed on this plan would have a number of obvious advantages over the magnetic wheel assumed by Whittaker. Of particular interest is the fact that such a model would help reconcile the two opposing views of atomic structure, the Bohr atom and the static atom. The Bohr atom has been remarkably successful in the phenomena connected with spectral lines, while the static atom seems to be demanded to account for the chemical properties of atoms. The force exerted by the magnetic wheel on an electron on the axis at a distance large compared with the radius of the wheel, is,

$$\frac{Mea^2\psi}{r^3}.$$

If the angular momentum of the wheel, $A\psi$, is an integral multiple of the unit of angular momentum, $h/2\pi$,

$$A\psi=\frac{nh}{2\pi},$$

and if $\psi=\omega$, $A\omega=2eM$, so that this force is

$$\frac{a^2}{2Ar^3}\left(\frac{nh}{2\pi}\right)^2.$$

Taking $A=\frac{1}{2}ma^2$, this force is exactly that postulated by Langmuir,[2] his quantum force. If this force is a repulsion, an additional attraction, e^2/r^2, from the positive nucleus will hold the electron in equilibrium at a distance,

$$r_0=\frac{n^2h^2}{4\pi^2me^2}.$$

The frequency of the vibrations about this position of equilibirum will be

$$\frac{4\pi^2me^4}{n^3h^3}$$

and this is the same as the frequency of revolution of the electron in the Bohr atom. Therefore, as far as circular orbits are concerned, results of the same kind can be obtained as from the Bohr model.

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[1] Proc. London Phys. Soc., 1922.

[2] Phys. Rev. XVIII, 104, 1921.

57. THE EXPERIMENT OF STERN AND GERLACH.—Direct evidence for the existence of the unit magnetic moment, or the magneton, is furnished by the results of an experiment by Stern and Gerlach[1], who volatilized silver in a furnace and allowed the silver atoms to escape through a slit in the wall of the furnace, traverse a non-uniform magnetic field, finally to be deposited upon a glass plate. The stream of silver atoms was limited by diaphragms so that, in the absence of a magnetic field, they were deposited in one narrow band upon the glass plate. The whole arrangement was enclosed in a good vacuum so as to avoid collisions with other molecules. The non-uniform magnetic field was furnished by a wedge-shaped pole-piece, the stream of atoms traversing a path very near, and along its edge. It was found that when the magnetic field was applied, the atoms were deposited in two narrow bands, nearly symmetrically situated with respect to the undisplaced band in the absence of a magnetic field. This result indicates that half the silver atoms were attracted towards the edge of the pole-piece and the other half repelled from it, and the displacements were of such an amount as to correspond, well within the limits of experimental errors, to each atom having a magnetic moment $\pm he/4\pi m$, the magnetic moment of the Bohr magneton. The result of this experiment may be interpreted as indicating that quantum principles are to be applied not only to the numerical value of the magnetic moment of an atom, but also to the direction in space of its axis. If the magnetic moment is regarded as due to the revolution of an electron about a nucleus, as in Bohr's model, and if the quantum condition is applied to the angle the normal to the plane of the orbit makes with a fixed direction in space, then, for a quantum number unity, corresponding to the unit magnetic moment, it may be shown that there are only two possible values of this angle, 0 and π. But in the furnace, where the atoms are formed, the fixed direction in space varies for the different atoms, being, for each atom, the direction of the local magnetic field. The experiment shows, however, that after escaping from the furnace, the fixed direction in space is the same for all the atoms, that of the applied magnetic field through which they pass. It is very difficult, on any such view of the origin of the magnetic moment, to understand how, in the very short time of flight, the atoms can all adjust themselves to this fixed direction. Einstein and Ehrenfest[2] have discussed this question and shown how unsatisfactory the interpretation of this experiment is from the standpoint of the present form of the quantum theory.

[1] Zeit. für Physik, 9, 349, 1922.

[2] Zeit. für Physik, 11, 31, 1922.

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BY

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PREFACE

President Aydelotte adequately refers, in the introductory pages of this report, to a situation in American colleges and universities which has for some time attracted the anxious attention of presidents, deans and professors. Very likely the students, too, have noticed and thought about this situation.

Of the various remedies suggested to ameliorate this situation perhaps the most interesting is the inauguration of systems of honors modelled more or less on the distinction made by English universities between honors and pass men. Many inquiries have been made of the Division of Educational Relations of the National Research Council for information as to exactly what, if anything, is being done in the United States in this direction. This report is a somewhat detailed answer to these inquiries. It is based upon material collected by the Division and turned over to President Aydelotte for digestion. Every effort has been made to bring the information here presented up to date and to make sure that it reports correctly the practice of our colleges and universities. But inevitably there are some omissions and some mistakes in the report. We should appreciate prompt correction of any of these mistakes and omissions and should be glad to have information from institutions not referred to in the report which are at the present time adopting or considering new methods of honors work.

President Aydelotte's plan in preparing the report has been to present first of all a brief description and discussion of the leading types of honors courses now being offered, laying emphasis upon the general principles underlying the work rather than upon the practice of any given institution. The different plans for honors work are classified roughly into three different types, though not too much emphasis can be laid upon this classification. Some honors work is difficult to classify, and the greater number of institutions mentioned award honors by more than one method. Nevertheless, any study of the subject will show that the different plans fall roughly into three groups:

1. Honors based on average grades in regular courses.
2. Honors based on special work undertaken in addition to the regular program.
3. Honors based on work superseding the regular requirements.

These three methods of awarding honors are discussed in general terms in the text, while in the appendix quotations from correspondence and from college and university catalogues or other official publications describe the practice of institutions in detail. Naturally there is a good deal of variation in the minuteness with which this practice is described in these extracts, but it seemed better to reprint first-hand material of this kind rather than to attempt to reduce it to a schematic second-hand statement which would probably lose in accuracy more than it gained by uniformity.

It remains for me to express the appreciation and thanks of the National Research Council and of its Division of Educational Relations to President Aydelotte for his generous expenditure of time and energy and his meticulous care in working over the material put into his hands. To this material President Aydelotte added much personal information. President Aydelotte's name gives the report a special prestige because of his well-known large personal knowledge of both English and American honors systems and his devoted efforts to introduce into American college and university practice more particular attention to the individual student.

The Division of Educational Relations of the National Research Council publishes and distributes the report with confidence that it will be a source of interest and stimulus to those who are interested in attempts to better the situation of American higher education.

VERNON KELLOGG,
Chairman, Division of Educational Relations,
National Research Council.

INTRODUCTION

The last few years have witnessed a growing dissatisfaction in the United States with the level of attainment reached in our colleges of Liberal Arts. The defects of a system under which the intellectual pace is set by the student of average ability have become more apparent since the war because of the fact that the average level of ability has been lowered by large increases in numbers. Instead of submitting to this pressure to lower standards our college faculties have become aroused to the danger, and, on the principle that the best defense is an offensive, have sought eagerly for methods not merely to maintain academic standards but to raise them.

The war and the post-war expansion of our universities and colleges are the occasion but not the cause of this movement. The defect it is designed to correct was inherent in the organization of our college work before the war, and would sooner or later have had to be corrected in any case. The whole development of our method of college teaching has centered around the needs of the average student. To meet his needs, we have evolved an elaborate and effective routine—a routine which is extraordinarily successful in achieving the aims for which it was designed. This routine makes sure that the student whose aim is merely to get a degree shall not get it without doing a certain amount of work. By the organization of our courses on the lines of secondary schools, by frequent tests and examinations, by short, clear-cut daily assignments, by a profusion of written exercises, by sharp rules dealing with cuts and absences, we have developed a system which is excellent for the average man without much thought as to whether this emphasis on a rigid daily routine may not hinder the intellectual progress of the best students and make it needlessly difficult for them to cultivate their real intellectual interests.

We train our students to take such short steps as to prevent many of them from ever getting into any long intellectual stride. They learn some tidbit of knowledge, are examined on it, and forget it, and by repeating this process a sufficient number of times are supposed to have acquired an education. We put too much responsibility on the system and too little on the individual. Our students think they are being educated; it does not occur to them that they are, or should be, educating themselves. They do not learn to plan, to co-ordinate, to organize, to deal with the content of a subject; their attention is limited to single courses, to short steps; they tend to feel that the whole duty of the undergraduate is obedient submission to a fixed academic routine.

Our faith in academic routine is shown by the fact that most of our requirements for the B.A. degree are expressed in terms of what the student should do, not in terms of what he should know. By a needless perversion of the elective system, we have come to act as if we believe in the equality of every hour of credit with every other and to make our requirements for a degree almost purely quantitative. There are, it is true, certain rules which must be observed in the acquiring of the proper quantity of credits. These rules prescribe that a student shall gain his hours in a certain specified range of subjects, so many in each, and that he shall have a certain number, larger but not too large, in some one subject of his choice. Through an almost superstitious fear of specialization, we are in danger, in the opinion of many observers, of sacrificing thoroughness. It would seem that we need some compromise which would prescribe for the undergraduate a relatively thorough grasp of some relatively broad field.

Our curriculum suffers from too much regimentation of our students. We think of them as classes and not as individuals. Our courses and departments are successful at the expense of the education of the individual student. We encourage mediocrity and the school-boy virtue of docility in the performance of daily tasks, while we penalize the student of strong intellectual interests and put too much "busy work" in the way of his mastery of the subject in which he has the keenest interest and for which he has the greatest ability. Our students do not learn to set their own tasks. We develop their obedience rather than their independence and initiative.

The love of American undergraduates for college activities (among which activities study is never by any chance included) and the belief of the American public in the educational value of these activities is due, in part at least, to the fact that in them undergraduates find more scope for their intellectual initiative than they do in their studies. In so far as this is the case, the belief in activities is justified; if the regular curriculum could offer the same opportunity for the development of independence and initiative that is now offered by clubs and teams and college journals, some of the energy which undergraduates put into these miscellaneous pursuits would go into their studies with an infinitely greater educational result.

Many attempts are being made at the present day to remedy what are felt to be these defects in undergraduate work. At Reed College and at Antioch, special curricula have been devised of a very interesting character. In a great many universities large courses are divided into sections according to the ability of the students. The practice is growing of making a certain qualitative as well as quantitative requirement for the degree; in many places high grades are given an additional quantitative weighting. A considerable number of colleges and universities offer to their more ambitious students the opportunity of doing extra work over and above the regular requirements for the A.B. degree and award the degree with honors on the basis of success in this extra work. A few institutions have gone so far as to substitute honors work for the regular program of the last two years. The common aim of all these experiments is to make our academic system less mechanical, to arouse interest in a higher grade of intellectual work, and to leave the student in that degree of freedom which is essential for the success of his more independent efforts.

PART I

HONORS BASED ON AVERAGES

The great majority of colleges and universities in the United States give what are called honors at Commencement time. These honors are ordinarily based on the average grades of the undergraduate throughout the four years of his college course, though in particular cases the average of grades is supplemented by other considerations, such as the opinions which the members of the instructing staff may hold about a given student's character and personality. Elections to Phi Beta Kappa and other honor societies are usually awarded on the same basis, as are many prizes and honor scholarships.

In a number of colleges and universities, large courses are divided into sections in order that students of high average ability may be placed in one class and those of low ability in another. In some cases, admission to particularly difficult advanced courses depends upon the attainment of a certain grade or mark in the courses which are prerequisite to the one in question. Admission to departmental clubs likewise often depends upon the attainment of a certain specified grade in the courses of the department. Meritorious work, as shown by high average grades, is rewarded in many other ways; for example, by excuse from examination, or larger privileges in regard to cuts, or exemption from the requirements of routine written work.

These distinctions are what the American undergraduate understands by college honors. In undertaking to give a brief account of what is being done in the direction of honors courses in the United States, it seems unnecessary to give any extended notice to honors of the type which has just been indicated. They are too common to need any special description or illustration. They are thoroughly well understood by American faculties and their good and bad effects are a matter of common knowledge. It is true that one bad effect of all forms of honors based on averages is not so generally understood as it might be. Distinctions which are based on average grades tend to reward intellectual docility and to punish strength of character and keen intellectual interests. The man who has an overpowering interest in one subject is likely to neglect or to hurry through a subject of a totally different type. It is not so much that he is lacking in ability to master the other subject as that he believes it not to be worth his while to work at it in the same way that he works at the subject on which he is to spend his life. The man whose interests are mainly scientific or philosophical may be impatient with the pronunciation of a foreign language or with the exercises which he has to do in composition in another tongue.

In many such cases the student is right. All subjects are not equally worth his time and attention. The result which he will get from bending all his efforts to an exceptional mastery of the knowledge in which he has the keenest interest and for which he has the greatest ability will be worth more to him both intellectually and materially than would the result of the same amount of time spent in mastering the details of a subject in which he is not interested and for which he will have little use. In considering the qualifications of a professor of Physics, we are not particularly concerned about the purity of his French pronunciation, but in awarding Commencement honors to a student of Physics the amount of attention which he has given to the pronunciation of French may be just the thing which will determine his failure or success. That is one reason why our academic honors are not a better basis than they are for predicting success in after life.

PART II

HONORS BASED ON SPECIAL WORK UNDERTAKEN IN ADDITION TO THE REGULAR PROGRAM

In a different class from the honors based on averages are those which are based on some particular task, a special course, a thesis, or a comprehensive examination, undertaken in addition to the regular program. In most cases this additional work must reach a higher standard than it would be thought fair to fix for all students. Here the determining factor is the man's work in the subject in which he is most interested and about which he knows most. In order to obtain honors he must do something which other undergraduates are not expected to do. Sometimes the ordinary program is modified to a certain extent in his favor; in other cases it is not. However that may be, the task which the student does for honors is something which may be called specifically honors work as distinct from excellence in ordinary courses. In the institutions which we are considering at this moment the honors requirement is made as an addition to the usual work for a degree and does not constitute an alternative course.

The fact that over thirty colleges and universities already offer honors work of this type indicates how widespread is the feeling that a more severe standard should be set up for exceptional students than for the rank and file. It must be repeated that practically all these institutions also award honors on the basis of average grades, and that the greater number of students who take honors take them under this plan. In many cases, the systems of honors awarded for additional work are recent and no figures are yet available to show how many students are taking advantage of the new opportunity.

There are almost as many different plans for honors based on additional work as there are institutions offering them; but a study of Appendix A will show that these different plans are, for the most part, different arrangements of the same elements. The extra work required is usually collateral reading and investigation done outside the ordinary courses of instruction. Sometimes this work is done in vacations; more often it is done in term time. The purpose is to make the student do something more than merely to meet the requirements of his courses. It is intended that he shall read between courses, that he shall rise from a mastery of separate details to some grasp of a subject as a whole. By intensive work on special topics, it is further intended to give him some experience of first-hand knowledge, some practice in dealing with original material, some training in independent investigation.

Proficiency in honors work of this character is tested sometimes by comprehensive examinations, sometimes by theses, often by both. It is usual to make as an additional requirement for honors that the student should not merely succeed in these specific honors tests but should also have maintained a high average grade in his ordinary courses.

In a certain number of institutions, special courses are offered to which only honors students are admitted. Sometimes these special courses take the place of regular courses of an equivalent number of hours; sometimes they do not. In the main, it is true to say that students who are candidates for honors are required to do substantially the same amount of work in regular courses as are the rank and file of undergraduates, and that, in addition to this, they must undertake some specific extra tasks.

This general description may be taken to apply to the whole group of institutions which have been mentioned. In Appendix A of this report will be found quotations from the catalogues of these institutions or from correspondence with their officers describing in brief form their special requirements.

PART III

HONORS BASED ON WORK SUPERSEDING THE REGULAR REQUIREMENTS

When once the principle has been accepted that candidates for the B.A. degree with honors should do some work in addition to the regular requirements, the next step in the development of the plan is a substitution of this extra work for part of the regular course. Following that, it is an easy step to cancel the old program entirely, so far as honors students are concerned, substituting for it a special honors course, based not on the quantitative credit-hour system, but more upon the English plan of individual work through a period of one or two years for a group of comprehensive examinations. A plan of this kind has been in operation at Columbia since 1911, and during the last few years somewhat similar plans have been inaugurated at Barnard, Rice Institute, Smith and Swarthmore, while a number of other institutions, among them Carleton, Hobart and Wells, are now taking steps to put such schemes into operation. Detailed descriptions of these various plans will be found in Appendix B; the purpose of this chapter is to describe in general terms their different features.

As a rule students are admitted to honors work at the beginning of the Junior year. This allows time for the completion of the ordinary required courses in English, Mathematics, History, Science and Language, which are in most institutions prescribed for all students, and time also for the individual to pursue one or two elective courses in the subjects of his preference, which will at once prepare him for honors work and be some indication of his aptitude for it. It is to be expected that, as time goes on, the requirements for admission to each honors division will specify more and more strictly the completion of certain elementary courses as preparation for it, but such requirements are rare in descriptions of honors work so far published. It may be that we shall come eventually to a series of intermediate examinations for admission to honors work, more comprehensive than ordinary course examinations, which will at once make sure that the student has covered the necessary preliminary ground and that he has the ability to do work of honors grade.

The one almost universal requirement for admission to honors work at present is a creditable record in the courses of the first two years. In some institutions the candidate must have made an average grade of at least B; in others, admission to honors work is based not upon an average grade but upon excellence in subjects of a type similar to that in which the student wishes to specialize. It seems clear that success in class work of the ordinary type is not a wholly reliable criterion of a student's ability to do honors work. In working for honors, docility in doing what one is told is of much less value than independence in laying out tasks for oneself and perseverance in carrying them through. For this reason, a certain number of students who are only moderately successful under the ordinary curriculum show special excellence in honors work, while a certain number of others who would carry off all the high marks in ordinary courses are only moderately successful in work of a more independent type. But in what is perhaps the larger number of cases, independence and ability are found together.

In most of the colleges and universities considered in this section, students who have been granted permission to read for honors are excused from all the ordinary requirements of class attendance, semester examinations, mid-term tests, and the like, and are left in freedom to spend all their time in preparation for the comprehensive examination at the end of the course. In a few cases these comprehensive examinations are held at the end of each year over that part of the work covered during that year. But it may well be questioned whether yearly examinations yield as sound results as those covering the entire two-years' period. It is noteworthy that students who have two years to spend in preparation for their examinations tend, of their own volition, to do some work in the summer vacation between, while yearly examinations tend to give the student the feeling that his year's work is done and need be thought about no longer, and that he may use his holidays for play. Our undergraduates play so much and so profitably during term time that any system which tends to make them study in the summer vacation has, on that account at least, much to be said for it.

While honors students are normally excused from the requirements of undergraduate courses, they are practically everywhere expected to make use of such courses in preparation for their examinations. Most institutions have found it wasteful and unnecessary to provide any great number of special honors lectures or classes. All of our colleges and universities have enough or too much lecturing already provided. The honors student is left in freedom to attend as many or as few of these lectures as he (or his supervisor) thinks profitable. In this respect, the advice of the supervisor or tutor will have all the force of a requirement.

While special classes or lectures are not usually found to be necessary for honors students, there is nearly everywhere provision for giving them individual supervision and instruction. In some cases this is done by the English tutorial system. In other cases, honors students in a given subject meet weekly in small seminars where they receive something like the same amount of individual attention. It is, I think, the usual experience that honors work can be well conducted only by the older and better trained men in the various departments.

In some places the honors student writes a weekly essay or report on his work by way of crystallizing his ideas in his own mind, of training his powers of expression, and, at the same time, of giving his supervisor some means of estimating his progress. It seems clear from such experience as has already been gained that honors students discuss their work among themselves to an extent unusual for American undergraduates and probably no other form of instruction has for them so much value.

The honors student has his work outlined for him not in terms of what he must do but in terms of what he must know. Instead of taking courses he studies a subject. He must (with what assistance he can get) organize his materials, set his own tasks, find out and strengthen his own weaknesses, develop his own strong points, and, in general, take the responsibility for his own salvation. This being the case, it is important that the requirements should be stated as clearly and definitely as possible. The task of phrasing a set of requirements in such form that, given to the student at the beginning of his Junior year, they will be adequate to guide his work, and, given to the examiners two years later, they will indicate with sufficient clearness the scope of the examination, is a new one for American college professors and one which they find difficult.

The importance of making this statement of requirements clear and comprehensive is self-evident. Ideally such a plan should indicate the field or periods to be covered, the large topics to be mastered, and the set books, if any, which are to be studied in detail. If the examiners in a given subject are to be not the teachers but men called in from the outside (as it is to be hoped will be more and more the case) the plan must be more reserved and conscientious than our typical college catalogue descriptions of courses or the examiners will be led to expect more knowledge from the students than they can possibly find. On the other hand, the standards of honors work should be severe and the requirements must be so stated as to stimulate the student to do his best. The first plans published tend (as might be expected) to be rather too comprehensive and too vague, which, however, is a better fault than an excess of pedantic detail. Perhaps no element in an honors course is more important for its success than the clearness of its plan.

Honors work implies unity in the field of study in which the undergraduate is to work. The most obvious way to secure this unity is to confine a given honors course to a single department. This, however, means rather narrower specialization than is usual in undergraduate work and honors courses so set up have been criticized on the ground that they do not offer a sufficiently broad training. This criticism has been met in several institutions by the device of combining two or three allied departments in a single honors course. For example, instead of awarding honors in Latin or Greek alone, the two literatures are sometimes combined into one honors course in Classics, or the Classical literatures are combined with Ancient History and Philosophy, as in *Literae Humaniores* at Oxford, making a field broad enough to cover the whole range of the ancient civilizations. In a similar way, Physics, Mathematics and Astronomy are combined in one course, or Physics and Chemistry, or Modern History, Economics and Political Science, or two modern languages, or English Literature, History and Philosophy.

There are very strong arguments for this combination of matter from several departments in one honors course. The undergraduate period is not one for narrow specialization. But the antithesis to narrow specialization need not be a course composed entirely of smatterings, but may by this plan be a kind of broad specialization which will secure breadth without sacrificing unity and thoroughness.

No question is more frequently asked by honors students than how they are to acquire a fund of miscellaneous information. The American undergraduate does this normally by taking a large variety of miscellaneous elementary courses. It is open to the honors student to do the same thing, so far as his time will allow, but it is also open to him to satisfy this curiosity by the much more economical and intelligent method of general reading. Elementary courses are not and can not be organized primarily to satisfy the needs of browsers whose main interests lie elsewhere. The beginning course in any subject is rightly planned to lay a good foundation for students who are going on with the subject. It rightly requires a much more thorough mastery of the ground work than the browser wants or needs. He can best satisfy his curiosity by reading, and if he forms the fearless habit of tackling a book on any subject about which he desires information, this habit will not be the least valuable result of his college training.

Honors work is too recent in this country to have produced much printed information on the subject of examinations. We have not as yet brought the art of examining to any very high pitch of perfection, though in this field an extraordinary number of interesting experiments are now being tried. The difficulty that has stood most in our way has been the custom of having the man who teaches a course set the examination in it and mark the papers or, at any rate, control the standards by which they are marked. In some honors courses this is still done; in other cases the candidates are examined by persons from outside the institution who have had no connection with the teaching. It seems clear that the latter method will, in the long run, make for greater independence on the part of the student and for higher standards of attainment—the two ends which honors work is designed to attain.

The amount of time allowed honors examinations varies in different places from one or two up to ten or twelve three-hour papers. Sometimes the written examination is followed by a short oral one conducted by the examiners after they have read the papers, in order to supplement the written test. In other cases, the written examination is dispensed with and a long oral held, something like the examination for the Ph.D. In a few institutions a thesis is required in the place of part or all of the examination. It is too early to say from experience just what form of examination will work best under American conditions. The preference of the writer is for ten or twelve three-hour papers, set by persons who have not been concerned with teaching the candidates, followed by an oral. In the natural sciences, it will probably be advisable to substitute a laboratory test for one or more of these papers.

Plans for honors work usually include some provision for taking care of students who use their freedom as an opportunity for idleness, and for those who, while working earnestly, turn out not to have the ability to reach an honors standard. Neither class seems so far to have caused as much difficulty as many college professors have feared. The best way to deal with both is by the exercise of care in the admission of a student to honors work. But if he slips past that test, the usual penalty for the slacker is to compel him to return to ordinary work. In some institutions, there is an arrangement for making him an allowance of general credit in accordance with his deserts. The earnest student who fails in his examinations or who does not come up to an honors standard is usually either returned to the ordinary course or, if he shows sufficient merit, given the B.A. degree without honors.

The foregoing paragraphs offer a kind of composite picture of the methods of honors work now being tried in this country. Not all of the details apply to any single honors course. The reader who desires fuller information about the practice of particular colleges or universities is referred to Appendix B or to the institutions themselves. In work of this character, the machinery is the least important part. The important things are to leave the student in freedom, to give him opportunity to develop his own independence and initiative, to provide him with a plan of work and with individual instruction which will prevent too much floundering, and to confront him at the end with a flexible but severe and independent test of the value of his work.

A large number of colleges and universities have honors systems under consideration at the present time. The actual achievement here recorded is less important than the promise implied in the widespread interest in the subject, based, as it is, upon the feeling that our educational system has hitherto devoted an undue amount of attention to the mediocre or backward students and that lock-step methods of instruction have made it difficult or impossible for students of more than usual ability or ambition to do their best work. The honors system now being introduced need not mean any curtailment of the quality of teaching enjoyed by the average student. What our best students need is not coddling, not more attention, not more teaching, but only greater freedom and more severe requirements. These two boons are promised by the general system of instruction to which this pamphlet is devoted.

APPENDIX

A

DETAILED REGULATIONS FOR HONORS BASED ON SPECIAL WORK IN ADDITION TO THE REGULAR REQUIREMENTS[1]

AMHERST COLLEGE

Final honors at graduation.—Final honors in a single department of study are awarded for special work involving collateral reading or investigation under the following conditions:

(1) The candidate must complete at least two advanced year courses in connection with one of which special work of collateral reading or investigation must be done. No student may be a candidate in more than one department except by vote of the Administration Committee.

(2) The candidates must have at graduation an average standing of not less than 80 per cent in all studies of the college course; an average standing of not less than 75 per cent in each study of Senior year, and of 90 per cent in the last year of study in the department in which the honor is sought.

(3) The proficiency of the candidate is tested by special examination or by thesis, or by both, at the end of Senior year.

. . . .

One unit is added to the total average rank of a student who takes final honors. If honors are taken in more than one department only one unit is added.—(*Catalogue*, 1923–24)

BLUFFTON COLLEGE

Departmental Honors.—

(1) A student may receive Departmental Honors in his major department only.

(2) The student, at the beginning of his Junior year, is to make application in writing to his Major Professor. This application, with the Major Professor's recommendation, is to be submitted to the Faculty. Admission to candidacy is to be by vote of the Faculty.

(3) The candidate is required to be in residence at Bluffton College during his Junior and Senior years, and to devote four full academic years to his College course.

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[1] All regulations dealing with honors based on averages have been omitted from these quotations even though in a particular institution these may be the most important honors given.

(4) In the candidate's major department he is to have no grades lower than B.

(5) The Major Professor will assign to the candidate extensive reading in the subject of his department or some large division thereof and require the preparation of a thesis showing power in the organization of material but not necessarily the ability to do original work.

(6) The candidate will be required to pass a comprehensive examination upon his Major or upon that large division thereof in which his special work was done.—(*Catalogue*, April, 1923)

BOWDOIN COLLEGE

Honors.—The award is based not primarily on the average grade attained in the six courses that constitute the major, but on the result reached in the additional work—especially that which the student has done largely on his own initiative—and on his ultimate attainment in the subject.—(*Catalogue*, November, 1923)[1]

COLLEGE OF WOOSTER

Honors.—Special honors are offered to students taking high rank in certain designated courses.

The purpose of the special honor courses is to encourage all capable and willing students to do extensive and thorough work in one or more departments of their choice. In order to be accepted as a candidate a student is regularly expected to have had two years of work in college in the field he is choosing, and at the same time it is expected that all this preliminary work be of honor grade. The candidate must pursue a specially authorized course in the field of his choice, and under the guidance of his instructors he must do a large amount of supplementary work. At the end of the year examinations are given covering the work of the entire year, part of this examination being oral and conducted by a special committee of the Faculty appointed for that purpose.

General honors.—*Summa cum laude, magna cum laude,* and *cum laude* are awarded to graduates on the general excellence of their work in this institution.—(*Catalogue*, March 15, 1923)

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[1] This brief catalogue statement is further explained in a letter from President Sills, December 19, 1923, as follows: "A student may obtain honors in a subject by passing creditably a comprehensive oral and written examination in his field of concentration or major subject, or by doing such other extra work of high grade as will secure a recommendation from his department for honors. A thesis is not necessary though sometimes demanded. A student may be recommended for honors even if his grades in the first part of his course are not high provided he has shown a real knowledge of the subject at the end of his course."

COLORADO COLLEGE

Regulations concerning the bachelor's degree with honors.—

1. A course of study leading to the bachelor's degree with honors is hereby established at Colorado College.

2. Students shall ordinarily be admitted to candidacy for this degree, under regulations hereinafter stated, at the time of registration for the junior year, but they may be admitted not later than the time of registration for the senior year, provided they can satisfy all the conditions for graduation with honors.

3. The 60 semester hours required in the junior and senior years shall, in the case of all candidates for this degree, be distributed as follows:

   (1) In the field of concentration:
      a. From 12 to 24 hours.
      b. In problems or topics which require written reports not less than 6 additional hours.
   (2) In subjects related to the field of concentration, from 18 to 30 hours.
   (3) In free election, from 12 to 24 hours.

4. Candidates for the degree with honors are required:

   (1) to present a final dissertation on some topic in the field of concentration;
   (2) to take a comprehensive final examination in the field of concentration at the end of their senior year.

—(*Report of Faculty Committee on Honors,* adopted 1923)

DENISON UNIVERSITY

Special honors.—Special Honors are awarded at graduation only, according to the following plan: Any regularly classified student who has maintained an A rank in not less than half of his work during five successive semesters of the freshman, sophomore, and first half of the junior years, and who has not fallen below C in any single study may, with the consent of the faculty, be enrolled as a student for honors in some selected department of study.

Application for such enrollment shall be made prior to the spring vacation of the junior year. If the application be granted, the professor in charge of the department selected shall assign work to the applicant substantially equal in amount to a three-hour study for the year.

The student shall be examined on this work prior to the spring vacation of the senior year. The examination shall be conducted under the direction of the professor in charge, assisted by some other member of the faculty to be appointed by the president, and in addition to the special assignment the examination shall cover, in a general way, all other work which the applicant has taken in the same department.

During the interval between enrollment for honors and examination the student must make an A rank in all work done in the department to which his special study belongs, and must not fall below B in any other department. He must also maintain unblemished record in deportment.

Work done for Special Honors must not be elementary in its character and in no case shall it be the only work done in the department in which it is taken. Advanced elective courses already offered may be utilized as honor courses, or special topics may be assigned to suit individual cases, but in no case shall work upon which Special Honors are bestowed be counted as any part of the student's requirement for his degree.—(*Report of the Committee on Rules and Regulations,* March 19, 1923)

DICKINSON COLLEGE

Honor courses.—Honor courses are open to a few high grade students of each class, who among other things:

1. Secure B grade as sophomores,
2. Average at least B for their college course,
3. Take at least 24 semester hours of work in a special chosen field,
4. Secure A grade in these 24 hours,
5. Do work outside of class equal to 8 semester hours and pass on the same with grade of A, presenting an acceptable thesis on the same, both to be done by May 15th before graduation. —(*Catalogue,* February, 1923)

HAMLINE UNIVERSITY

Departmental honors.—Special honors for distinctive scholarship in the student's major departments are given under the following conditions:

First—A candidate for this degree must have secured a passing grade in all courses since entering college and an average of at least 2.0 in all courses after the Freshman year.

Second—He must have received a grade of A in all courses after the Freshman year, which lead to the departmental honors desired.

Third—A special thesis, approved by the head of the department, must be written, and a satisfactory examination passed in the presence of a committee of the faculty appointed by the President or the Dean.

Fourth—If approved by the committee, the candidate will be recommended to the faculty as worthy of the special bachelor's diploma with distinction in the department named, and the final decision will rest with the faculty.—(*Catalogue*, January, 1923)

HARVARD COLLEGE

Honors.—General examinations over a student's whole field of concentration are now required of all seniors at Harvard except those concentrating in astronomy, biology, chemistry, geology and geography, mathematics, mathematics and education, natural history and physics. Degrees are awarded with distinction (*cum laude*), with high distinction (*magna cum laude*), and with highest distinction (*summa cum laude*) on the basis of these examinations and on the basis of the student's record throughout his four years in accordance with the rules of the several divisions of the university.

In addition to these distinctions based upon high degree of success in the ordinary work of the college, there is special provision for final honors in certain departments based upon more thorough and more individual work than that ordinarily required for the degree. The general regulations for these final honors are as follows:

"Except in the case of Mathematics, every candidate for Honors must pass, near the close of his last year, a special examination, conducted by a committee of the Faculty, to test the range and accuracy of his knowledge of the subject in which he offers himself for Honors; and must present such theses as may be called for.

"As a result of this examination he may, with the approval of his Department, Division, or Committee, be excused during his last year from final examinations in courses dealing with his particular subject.

"Students who have failed to pass with distinction any of the examinations required for Honors may, with the consent of the Faculty, make up their deficiencies by passing such additional examinations as may be required of them; but no such permission is granted for failures occurring after the beginning of the Senior year. Grades attained in courses anticipated by examination are not counted towards Honors.

"Honors, however, will be given to graduates not entitled to them at graduation who shall have complied with all the requirements for Honors within two years after graduation except in Mathematics; they may also be taken, at any time, with the degree of Master of Arts."—(*Catalogue*, 1922–23)

HAVERFORD COLLEGE

Honors.—Honors are awarded for excellence in the studies of single departments. They are never given merely for performance of routine work in courses, but a considerable amount of extra work is demanded in every case.

Honors are of three kinds:

(a) *Honorable Mention.*—To be awarded for work in a single course meeting not less than two hours per week throughout the year, plus additional work to the total amount of not less than 75 hours. Candidates for Honorable Mention must obtain the grade of A in the regular work of the course and pass creditably an examination on the additional work required. Two half-courses in the same department may be construed as a single course. Honorable Mention will be awarded for work in History 2*a,* or Economics 1*b,* provided the foregoing requirements as to grade and additional work are satisfied. History 2*a* may be counted as a full course for Preliminary Honors taken at the end of the Sophomore year.

(b) *Preliminary Honors.*—To be awarded at the end of the Sophomore, Junior, or Senior year for not less than two full courses in a single department, plus additional work to the total amount of not less than 150 hours. Candidates for Preliminary Honors must obtain a grade of at least B in all courses required for such honors and a grade of A in such of these courses as are taken in the year in which they are candidates, and must pass creditably examinations on the additional work required.

(c) *Final Honors.*—To be awarded upon graduation for work in the courses of a single department, plus additional work to the total amount of not less than 250 hours. Candidates for Final Honors must take at least four full courses in the department in which they apply for honors, at least two of these courses being in the Junior and Senior years. They must in all of these courses obtain a grade of at least B and in those taken in the Senior year a grade of A, and must pass creditably examinations on the additional work required. There are three grades of final Honors: Honors, High Honors, and Highest Honors. The requirements for High Honors are of a more exacting nature than those for Honors, and Highest Honors are reserved for very exceptional cases. Both High Honors and Highest Honors are awarded only by special vote of the Faculty.

A student who has received the prescribed grade in the regular work of a course required for honors, but who has not done the additional work required in connection with such course, may, with the consent of the professor in charge, make up his deficiency in a later year, but in the case of Final Honors all such deficiencies must be made up by the end of the Junior year.

At the time of the award of Honors there shall be added to the general average for the year of each student receiving Honors, one half of one per cent for each award of Honorable Mention, Preliminary Honors, or Final Honors. In any given year only one such addition may be made to a student's grade for work in any one subject. —(*Catalogue,* January, 1923)

ILLINOIS COLLEGE

Final honors.—Final Honors are awarded at graduation to students (a) who have made an average of 2.125 honor points in the work of the entire four years; or (b) who have made an average of 2.25 honor points in the work of the last three years; or (c) who have made an average of 2.5 honor points in the work of the last two years; provided, in cases (b) and (c), that the average in previous work is not lower than 1 honor point. No student is awarded Final Honors unless he has spent at least two consecutive years in residence at Illinois College.

Departmental honors.—Departmental Honors are awarded at graduation on the following conditions:

(a) The candidate must have a general average of at least 2 honor points.

(b) The candidate must have completed, in the department in which the award is made, the number of hours required for a major, in all of which he must have attained at least the grade of B.

(c) The candidate must have registered as a candidate for Departmental Honors with the professor of his chosen department at the time of enrollment for the first semester of his last year.

(d) Before the first day of May preceding his graduation he must have submitted all written papers required by the department and must have passed with distinction an oral examination conducted by the professor in charge with the assistance of at least one other member of the faculty. . . .

All awards of honors must be approved by the Faculty.—(*Catalogue,* March, 1923)

KNOX COLLEGE

Honors courses.—Certain departments offer Honors Courses. Registration for Honors Courses in any department is limited to juniors and seniors who are majoring in that department and who have completed with high standing such advanced courses as the departmental staff prescribes. Such students may be recommended by the department for registration in Honors Courses.

Special honors.—At the completion of certain advanced courses a student may be given Special Honors in one or more departments, upon the recommendation of the head of the department and vote of the Faculty. The names of students receiving such honors are printed in the Catalogue and announced by the President on Commencement Day.

Department honors are awarded by the departments giving Honors Courses to the students completing the Honors Courses.—(*Catalogue,* March, 1923)

LAFAYETTE COLLEGE

Degrees with distinction.—Opportunity is given for students whose majors are in certain departments to win a degree with distinction. An applicant for honors must announce his candidacy at the beginning of Junior year. In order to be eligible to such candidacy he must have secured B or higher in the courses of the Freshman and Sophomore years. He must satisfactorily complete the additional work required by his Major Department beyond that required for his Major, and at the end of his Senior year must pass a comprehensive examination upon the work done in the Department. He may be required also to present a thesis.—(*Catalogue,* February, 1923)

LAWRENCE COLLEGE

Honor courses.—Special honors are granted as an incentive to students of exceptional ability in some particular department to specialize more extensively than would be possible in the regular course. Students who have taken honor courses will be thus better prepared to enter upon professional or graduate work. Such students will receive at graduation a diploma designating them as honor graduates, and their names will be published in the catalogue as such.

Special honors are granted at graduation on the following conditions:

(1) In addition to the thirty-six hours required for the major, twelve hours must be taken in work designated as honor courses in different departments.

(2) These twelve hours must be pursued in addition to the one hundred ninety-two hours required for graduation.

(3) An average of ninety per cent must have been attained in the major and twelve hours of honor courses.—(*Catalogue,* January, 1923)

MIAMI UNIVERSITY

Graduation with honors.—

(1) Not later than the end of the sophomore year a good student may, with the consent of the Dean, enroll as a candidate for the degree of Bachelor of Arts with Honors. At the beginning of the junior year he will designate the department in which he wishes to do his honor work, and will be referred to the head of that department, whose direction as to election of courses in the honor subject and allied work shall be followed.

(2) No student shall be graduated with honors in any subject in which he has earned less than 18 hours' credit for class work. His other courses will conform to the rules for group studies.

(3) In the junior or senior year, or in both, the candidate will pursue at least one course of prescribed supplementary work in his honor subjects, in addition to his class work, and may receive for such work a credit of not more than two hours a semester, or eight hours in all. Unless this supplementary work is done in vacation, it will be counted under the rule for maximum credit in a semester. This reading will be closely supervised by the instructors concerned and tested by examinations at the close of each semester.

(4) If in any semester of the junior or senior years the student receives a grade below B in his honor subject or more than one grade of C in any other subjects, or any grade below C, he is disqualified for further candidacy for the degree with honors. At the end of his college course the candidate must pass a general examination on the whole field of his honor subject. A student disqualified for honors will retain any credit in hours earned by supplementary work.

(5) Candidates for the degree with honors will be distinguished from candidates for the ordinary degree, both in the catalogue list of students and in the arrangement of names on the commencement programs.—(*Catalogue,* March, 1923)

MIDDLEBURY COLLEGE

Special honors.—As an incentive to such students as have the ability to do more than should be required of the majority, and to promote and encourage special investigation in the various departments of the curriculum, the Faculty have established a system of honors. These are divided into two classes, Honors and High Honors, and are subject to the following regulations:

(1) The candidate must have completed major work in the department in which Honors are sought.

(2) Enough more than fifteen hours a week must have been carried in the undergraduate courses to insure to the candidate at graduation a surplus of at least six credits above the credits required for the Bachelor's degree.

(3) The requirement of the six extra credits may be met by the fulfillment of work in research or special investigation assigned by the department in which Honors are sought; or by the completion of two C courses in that department, provided that they are of that grade which would be counted for the Master's degree.

(4) The attainment of 80 per cent for Honors, or 90 per cent for High Honors, is requisite as an average rank in the courses which have been taken in completing the major work of the department, and also in the special work performed in the attainment of the six extra credits.

(5) On the completion of the six extra credits, whether attained in independent investigation or in class-room work, an examination shall be given covering as much of the work of the department which the candidate has taken as shall be deemed necessary, and, at the discretion of the instructor, a thesis may be assigned.

(6) If the additional six credits for an Honor be pursued in class-room courses, these courses must be taken in accordance with the rules regarding extra hours.

(7) Credits for Honors may be counted later toward the Master's degree.—(*Catalogue,* 1923–24)

MOUNT HOLYOKE COLLEGE

The degree with honors.—After 1923 students of high rank will receive the degree "with honor" or "with highest honor" under the following conditions:

A. All students who, at the end of their senior year, shall pass the general examination in the major subject with a grade of eighty-five per cent and who have attained an average of ninety per cent in their major work, shall receive the degree "with honor."

B. Students who at the end of their junior year have attained an average of eighty-five per cent in all their work may, at their own desire, be admitted in the senior year to study for a degree "with highest honor." In some cases students with a lower average may be accepted on the recommendation of the department of the major subject. Candidates for the degree "with highest honor" shall devote two-fifths of the work of their senior year to individual work in their major subject, under the supervision of the department. These students shall take, in place of the general examination required of all other students, a special examination set and read by a committee of three, including the head and another member of the major department. Students failing to qualify for a degree "with highest honor" may become candidates for the degree "with honor" or the ordinary degree.

C. The names of students receiving the degree "with highest honor" and "with honor" shall be printed on the Commencement program and in the catalogue, in their groups, with the major and minor subjects.—(*Courses of Instruction Bulletin,* March, 1923)

NEW YORK UNIVERSITY

Honors system.—

1. A student may become a candidate for honors in his major subject.

2. Candidacy must be announced after the end of the sophomore year and before the beginning of the junior year. The student must have attained a grade of A or B in all courses thus far taken in a department chosen for his major, and shall not have attained a grade lower than C in more than one course of his first two years.

. . . .

4. The candidate shall, in addition to passing with high grades the regular courses included in his major and two minor series, take certain extra work involving reading or other independent study. A specific statement of the reading required and of the nature and amount of other work demanded, signed by the head of the department in which honors are sought, must be filed by the candidate with the recorder at the beginning of each year of study for honors. This work will be given 3 hours per week semester credit in lieu of part of the free elective courses of the junior and senior years, and will require at least ten hours per week of the students' time.

5. No courses anticipated by admission to advanced standing may be counted in the honors series.

6. At the end of each term a preliminary written examination will be held covering the honors reading of that term. At the end of the senior year a final examination, a part of which at least shall be oral, must be taken, covering the entire ground of the honors series. It shall be given by a committee composed of the professorial members of the department concerned, with such other members of the faculty as may be designated by the head of the major department with the approval of the dean.

7. The thesis must be of a grade of A or B.

8. Honors are conferred by vote of the faculty upon the successful completion of the honors course, and when so conferred will be

printed upon the commencement program and entered upon the student's diploma.—(*Catalogue,* 1922–23)

NORTHWESTERN UNIVERSITY

Special privileges.—A student who has 90 semester-hours of credit in the College of Liberal Arts with grades averaging 2.0 or above and who has completed two-fifths of the total requirements for his major with grades averaging 2.5 or above, with the approval of his major department may be released from class attendance to the extent and under the conditions specified below:

(1) He may be released from class or laboratory attendance for not less than two or more than eight semester-hours of the 120 required for graduation. The number of hours from which any particular student is to be released shall be determined by that student's major department.

(2) A student thus released from class attendance must complete an individual program of work approved by his major department and performed under the direction of a member or members of the department. This program may consist, for example, of a course of reading or laboratory experiments or both; *and the department may prescribe such examinations, reports, or theses as it deems most suitable.* Upon the satisfactory completion of this program the student shall be credited with the number of hours of his release, this credit to be applied toward the major requirement.

(3) A student thus released shall not register for more than 17 hours, including such a program, in any semester.

(4) A candidate for this partial release from class attendance must have secured the approval of his major department not later than October 15th of his senior year. Not later than November 1st the department shall report to the Dean and the Registrar his name and the number of semester-hours of class attendance from which he is released. The Dean shall announce to the faculty the name of such candidate at the first regular meeting thereafter.

(5) Any candidate for release from class attendance may be deprived of this privilege at any time by formal action of the department under which he has the individual program of work, and the department shall promptly notify the Dean and the Registrar of the withdrawal of his privilege.

. . . .

The object of this proposal is to make some special provision for the unusually able student. The plan is designed to stimulate such a student and to reward him by giving him an opportunity to acquire a greater measure of intellectual independence and greater freedom to develop his own power of initiative than he would have in the normal routine of class work. It should also serve to establish the connection between provinces of study which are essentially connected but which are separated as a result of the increasing sub-division of the whole realm of knowledge; a well planned course of reading might properly touch the particular fields of several departments and bring out the vital relationship existing among them.—(*Report of Committee on Curriculum,* adopted by Faculty, 1923)

OBERLIN COLLEGE

Honors at graduation.—

Not less than the equivalent of ten additional semester hours in the major subject must be done, without credit, under the direction of the major adviser. This extra work may be done either in special "honors" courses, open only to candidates for honors, and to graduate students, or in additional reading in connection with regular courses, as the head of the department concerned directs.

A comprehensive examination, both oral and written, over the whole field of major study must be passed with high credit in May of the candidate's Senior year.—(*Catalogue,* January, 1923)

PRINCETON UNIVERSITY

Special honors.[1]—Special honors, determined by the standing attained in the departmental courses of the junior and senior years, are awarded only at graduation and are of three grades:

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[1] A new plan of study has recently been announced at Princeton involving for all students independent reading outside their courses and comprehensive examinations at the end of the junior and senior years. The plan affects the entire student body. Its bearing upon honors has not yet been announced. The general features of the plan are as follows:

"It is proposed that the present classification of the subjects of study in Departments be continued, with the proviso, that for purposes of election, History and Politics shall be considered separate Departments.

"It is proposed that each student at the beginning of junior year choose one of these Departments, in which he shall do the principal part of his study. The work in his Department is to be done in connection with the two courses which he will choose each term as directed by this Department, and he will do additional independent reading. The number of courses which a junior or senior is required to take is reduced to four. A candidate for a degree is required to attain an average standing in the work of his Department at least as high as the present third group, in order to be recommended for graduation.

"It is proposed that the student's work in his Department be tested chiefly by two general examinations, one of which comes at the end of junior year, and the other, which is the more important, at the end of senior year. It is expected that in these examinations several papers will be set in the work of the Department.

"It is proposed that the method of supervising the independent work of the students in a Department be determined by the Department concerned; and that the method be worked out on the basis of the personnel of the Department, with the understanding that preceptorial instruction be given not only to students of the Department, but also at least to the abler students who take a course as a free elective." (*Report of the Committee on the Course of Study,* 1923)

Highest, for an average standing of 1;

High, for an average standing of 1 to 1.5;

Honors, for an average standing of 1.7 to 2.

A Department may require a candidate for Special Honors to take in each of his last two years a course, admission to which is limited to students capable of doing work of superior excellence.

A candidate for Special Honors who is a candidate for graduation under the three-year plan may substitute one of the summer courses of his Department for one of the courses of his last year.—(*Catalogue,* January, 1923)

SIMPSON COLLEGE

Departmental honors.—Any person wishing to receive departmental honors must secure the consent of the head of the department in which he has taken his major to prepare a thesis. The subject of this thesis must be selected and the consent of the faculty secured by December 1 preceding the date of proposed graduation.

College honors.—At the close of each semester each instructor of College classes may report to the registrar the names of honor students. . . . No student can be considered for honors unless he is reported as an honor student from more than one class.—(*Catalogue,* April, 1923)

SOUTHWESTERN PRESBYTERIAN UNIVERSITY

Honor courses.—Honor courses shall be begun in the Junior year and continued through the Senior year.

A student taking the Honor Course will confine his work to specializing in two subjects only. The following shall be considered as separate subjects: Bible, biology, chemistry, economics, English and comparative literature, French, German, Greek, history and government, Latin, mathematics, philosophy, psychology, physics, sociology, Spanish.

The amount of work required for an Honor Course in any subject shall not be less than two and one-half times the amount of work required of students pursuing an ordinary third or fourth year course in that subject. The number of class room exercises required shall be at the discretion of the Professor in charge.

No student shall be classified as a candidate for Honors in any subject without the permission of the Professor in charge of that subject, and the consent of the Faculty. Only students of unusual promise will be allowed to become candidates for the Honor degree.—(*Catalogue,* March, 1923)

UNION COLLEGE

Special honors.—Special honors are also given at graduation under the following conditions: Any department may offer a course, approved by the voting faculty, leading to special honors. The head of a department may direct the choice of electives in other departments, for honor students, to the number of six hours in each year. The time of registration for honors will be determined separately for each department. The candidate for special honors must have attained in all the studies of the department in which he tries for honors a rank of not less than ninety per cent. of the maximum. The evidence that he has successfully completed the extra course prescribed for him must be submitted not later than June 1st of the senior year to the faculty, who shall decide in each case whether the work is worthy of an honor. The honors attained are stated in the diploma, and the names of the students who take honors are printed on the Commencement programme. No student may try for honors in more than two departments.—(*Catalogue,* November, 1923)

UNIVERSITY OF CALIFORNIA

Candidacy for Honors.—

Any student who is approved by his major department as a member of the honors group will, so long as he remains in the honors group, report at the beginning of each half-year to the department of his major work. The department will advise him in the choice of his studies and will specifically approve all courses taken in the department of the student's major.

After the first half of the junior year the Committee on Candidates for Honors will, upon the recommendation of the several departments, decide what students shall remain in the honors group and what students shall be promoted to the honors group. In determining these matters the committee will consider not only the student's work in the department of his major but his entire record.

The several departments have full freedom in determining the most efficacious methods for the training of candidates for honors; but no student will be required to take more than 24 units of upper division work in the department of his major. Departments may offer special honors courses in reading and research, with credit to be determined by the instructors in charge, according to the performance of the individual student, subject to such general restrictions as may be imposed by the department and by the Committee on Courses of Instruction. The work of the student in such an honors course may consist of additional work in connection with regular courses of instruction, or may be independent of such courses.

. . . .

Before Commencement, a department satisfies itself by means of a general final examination, or in such other manner as it may deem best, of the fitness of each candidate for honors at graduation. Candidates who, in the judgment of their departments, display marked superiority in their major subject receive the special distinction of highest honors. The list of students upon whom honors and highest honors are conferred, with mention of their major department, appears in the annual Commencement programme.—(*Circular of Information,* August, 1923)

UNIVERSITY OF CINCINNATI

Course in Honors for Upper-Classmen.[1]—The faculty of the College of Liberal Arts has established an honors course to be pursued by

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[1] It is rather difficult to classify the honors system at Cincinnati. The following letter of explanation from Dean Chandler, however, indicates that it is best put here:

"At this University, students who pursue the course in Honors are exempt from the usual requirements of the Junior and Senior years—namely, 64 credits, of which at least 18 must be taken in a major subject, 12 in an allied subject, and 34 are elective. An Honors student must pursue, in departments other than his major department, work amounting during the two years to between 18 and 24 credit hours. For this work he registers in the usual way, attends classes with the regular students, and is given grades according to the usual scale of A, B, C, etc. His work in his major department, however, varies according to the requirements of the individual department head. The majority of our professors seem to prefer to have their Honors students register for a sufficient amount of work in the major subject to complete a regular program of study, and for this work the students are given the usual grades. Their Honors work, then, consists in additional study and research, in the writing of a thesis, and in preparation for an examination, all pursued under the direction of members of the major department. A minority of professors, on the other hand, outline for their Honors students a complete and systematic course of study for the two years, requiring them to attend, either as auditors or as regular students, any courses in the major department that may fit in with this schedule, to do intensive study outside of class, to meet in conference with members of the staff at frequent intervals, and to write a comprehensive thesis and to pass an examination.

"In short, since all Honors students attend classes with the regular students, and since they are required to complete work in addition to the usual class work, I think that the University of Cincinnati should be placed in the second of the three groups you name in your letter. It is true that the minority of Honors students mentioned in the preceding paragraph are not given grades in the courses they attend as auditors, but at the same time they do not 'take different courses . . . from the "passed" students,' and the Honors in every case are based upon additional requirements, such as extra readings, conferences, the writing of a thesis, and the passing of a general examination."

Juniors and Seniors who are capable of more intensive study than the average, and who therefore may be relieved of ordinary academic guides and restrictions. A student who elects the course in honors shall notify the Dean of the college, at the beginning of his Junior year, of his desire to proceed in his work according to this plan. After having secured the approval of the head of the department in the subject in which he intends to take honors, such a student must register in the usual way. He shall be eligible for graduation only when the head of the department notifies the faculty that he has completed with distinction the course assigned, that he has passed a public examination, and that he has also completed creditably work in other fields amounting to between 18 and 24 credit hours. The head of the department in the student's principal subject shall report to the Registrar at the end of each semester whether or not the work the student is taking has been accomplished with distinction. The elected work in other departments will be reported by the customary grades. A candidate for a degree with honors who passes, but not with distinction, in his chosen subject at the end of the Junior year must withdraw from the course and will then receive thirty credits less the number of credits he may have failed to pass in his other fields of work. If a candidate succeeds in his Junior and passes in all of his work, but not with distinction, in his chosen subject at the end of the Senior year, he shall receive the ordinary degree of Bachelor of Arts. The diplomas of students who graduate from the honors course shall read "Extraordinary Degree of Bachelor of Arts with Honors"—or "with High Honors"—"in ———— (subject)." This distinction shall be printed and announced at Commencement, and the names of students holding such degrees shall constitute a roll of honor which shall be a public and permanent record. It is recommended that the Society of Phi Beta Kappa give preference in the choice of members to successful candidates for degrees with honors.—(*Catalogue*, July, 1923)

UNIVERSITY OF ILLINOIS

Final and Special honors.—

Special Honors are awarded at the close of the senior year. No student may receive such honors who has not completed, before the beginning of his senior year, at least twenty hours' work in the subject, or group of allied subjects, in which the honors are proposed; he must complete thirty hours' work in the same subject, or group of allied subjects, by the end of his senior year, must do such other work as the professor in charge may assign, and must prepare an acceptable thesis.

No student is eligible for special honors who, during the senior year, has received a grade of less than C in any subject. Special honors are planned for especially brilliant students who prefer to concentrate their efforts on a special course.

The Degree of Bachelor of Arts with Honors.—

For the degree with High Honors, constituting a recognition of work of exceptional merit, the student must have received the grade of A in at least three-fourths of his work offered in courses acceptable for major subject, and no grade below B in the work thus offered. He must also have attained a grade not lower than B in at least three-fourths of all the other work which he presents for graduation, exclusive of courses taken during his freshman year. Not later than five weeks before the end of the session in which he expects to complete the work for his degree he must be recommended by the department in which his major subject is chosen, and either present an acceptable thesis in his major subject or pass a satisfactory examination based upon a course of study approved by the department in which he does his major work, and pursued under the direction of an instructor selected by the student with the consent of the department.—(*Register,* March, 1923)

UNIVERSITY OF NORTH CAROLINA

Degrees with distinction.[1]—Honors of two grades, Honors and Highest Honors, will be awarded at Commencement to those candidates for the degree of Bachelor of Arts who have done distinguished work in a group of related subjects. Every candidate for Honors must register before October 15 of his Junior year with the Committee on Degrees with Distinction. . . . In connection with the regular courses, candidates are expected to do, in term-time and in vacations, a considerable amount of additional reading, or such other supplementary work as may be prescribed by the department. In every case such additional work must be closely related to the general plan of study adopted by the candidate. Students will have the guidance and assistance of special instructors, and small groups will be formed for the discussion of the reading and of the courses. This tutorial assistance will be entirely independent of the conduct of the courses, the object being to stimulate interest in reading not required as a part of the regular class-work for its culture value rather than for direct preparation for examinations.

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[1] "Your letter of March twenty-sixth, addressed to President Chase, has been referred to me as chairman of the Committee on Degrees with Distinction. The statement concerning our requirements for degrees with distinction is accurate with reference to present conditions. We are considering certain other extensions, including the possibility of establishing the difference between a pass degree and an honor degree, but no definite plan of this sort has been adopted by the faculty. It seems to me that our present system belongs mainly to your Class II, although the high average of grades in ordinary courses is also a requirement."—Edwin Greenlaw, Dean.

Besides the careful organization of the plan of study and the conferences, a thesis and a final oral examination are required. The thesis need not necessarily be a contribution to knowledge, but is designed to show the ability of the student to use his knowledge with intelligence and to interpret facts and his personal reaction upon the material studied in the courses or derived from the supplementary reading. The examination will test the candidate's knowledge of the whole field of concentrated study, and will also test his proficiency in a special topic within the general field. The aim of the entire plan of study is thus to secure, as far as possible, a conception of a field of learning as a unity; to prevent, through the various measures named above, undue emphasis on the separate courses as independent units; and to lay the foundation for right method through concentrated study in some part of the field.

Requirements for honors in language and literature.—

1. A reading knowledge of one language besides English.

2. Six courses chosen from those open to Juniors, Seniors, and Graduates. Of these, four are to be in one department of language, and literature, the other two may be (a) in another literature, or (b) in history, provided the courses are closely correlated with work in the major subject. But other combinations may be made, provided they show a definite plan.

3. In every case the major and minor groups constituting the six courses must show a definite relationship: e.g., the study of a period, such as Romanticism in France, Germany, and England, this study including history as well as literature; or the study of a type, such as tragedy, in at least two literatures; or the study of the influence of some writer, for example, Plato, Vergil, Dante, Shakespeare, Goethe, on some literature other than his own. Other means of securing correlation may also be approved.

4. The principle of correlation is further carried out through (a) the provision for lecture courses in comparative literature; (b) the fortnightly conferences at which books and reading in connection with the entire scheme of study are discussed; (c) the provision for an extensive course of reading, to be done in residence or during vacations, such reading to be ordinarily independent of prescriptions for courses and designed to give a thorough grounding in the field of learning

elected for the major subject; (d) the special seminars, at least one each year, through which provision is made for the intensive study of some subject during a limited period under the guidance of a scholar from the faculty of some other university.

5. Acquaintance with the following subjects is to be gained within or outside courses regularly elected: (a) the history of the literature chosen for the major subject, including the principal masterpieces illustrating its development; (b) some acquaintance with the political and social history of the nation whose literature is so elected; (c) some acquaintance with the history and philology of the language. The extent of the candidate's knowledge of these topics, as well as of the field or topic chosen for intensive study, will be tested by an examination to be held near the end of the Senior year.

6. An essay of some length is to be written upon some topic belonging to the field chosen for intensive study. This essay must show (a) investigation and mastery of facts; (b) power of interpretation; (c) excellence in composition and style.

7. In estimating the value of the candidate's work due regard will be paid to the grades attained by him in his regular college courses, but the matter of grades is distinctly subsidiary to other considerations, such as the extent and quality of his reading, the maturity of his thought, and his ability to use his knowledge effectively.—(*Catalogue,* March, 1923)

UNIVERSITY OF NORTH DAKOTA

The Council of the University has recently adopted a program of honors as recommended by the Committee on the Unusually Gifted Student as follows:

(1) That the unusually gifted student be encouraged to do more than the average or the normal amount of work for graduation.

(2) To encourage the gifted student to do more than the normal amount of work he be given the opportunity to earn special honors. The student may win special honors:

  (a) By doing work of superior excellence throughout his major work: these shall be known as *Departmental Honors.*

  (b) By doing original or research work of merit: these shall be known as *Research Honors.*

  (c) By completing a course of systematic readings under the direction of the Honors Committee: these shall be known as *General Honors.*

A special committee of the faculty called the Honors Committee shall have general supervision of this work and all Honors shall be approved by this committee.

. . . .

Departmental honors.—Any department offering major work may give Departmental Honors to students who fulfill the following conditions:

(a) Candidates for honors shall not have received a grade below 78 in any subject during the college course.

(b) In the department in which the honor is given an average grade of 92 must have been maintained throughout the major work and in at least one minor in another department. Such honors shall be granted by the Honors Committee on the recommendation of the head of the department.

Research honors.—Any department offering major work may permit a student majoring in that department, who shall have attained to the rank of a senior, the opportunity of doing research work. The subject of the original research work must have the approval of the head of the department and the Honors Committee, but in no case shall it exceed three hours for a semester or six hours for the year.

General honors.—A student approved by the Honors Committee may pursue a course of reading for General Honors under the direction of the Committee. Reports will be required from time to time and general discussion will be held on the works read.

The accompanying list of readings shall be required of all those seeking General Honors. (The list has not yet been approved by the Council.)

The Honors Committee shall have general supervision of this course in readings. This committee may call upon any department for someone who shall lead the discussion in that phase of the readings falling within said department.

Recommendations of students eligible to try for special honors shall have been made by the heads of departments.

From these recommendations the committee shall select such as seem to be especially qualified to try for such honors.

Any student failing to show the proper proficiency in his work shall be dropped from consideration for honors.

A student winning General Honors may be excused from not to exceed six hours of the required number of hours for graduation.

These special honors shall be announced at Commencement and the winners' names shall appear on the Commencement Program.—(*Minutes of the University Council,* June 6, 1923)

UNIVERSITY OF OREGON

Honors.—Honors granted by the University on graduation are of two kinds: Honors in General Scholarship, and Honors in a Given Subject.

Honors in a Given Subject.—

These honors shall be based upon excellence of work in connection with one or more honor courses designated by the major professor, considered in connection with general excellence in the subject of the department. The standard of requirements is intended to be so high that the proportion of students attaining honors in a department will be small, and in more than one department exceedingly small. These honors are intended to be the highest honors conferred by the University upon the candidate receiving the Bachelor's or Master's degree, and equivalent to a recommendation for a scholarship or fellowship in a graduate school.

Seniors may be admitted to honor standing upon registration in the first term; juniors upon registration in any term. Honor courses shall be listed on the regular registration card. Candidates for honors shall have been registered for such honors during at least three terms previous to the honor examinations.—(*Correspondence with Registrar,* October 24, 1923)

UNIVERSITY OF VERMONT

Honors.—Honors may be awarded at graduation for general high standing in scholarship, and also for conspicuous attainment in a particular department, as provided below.

Honors may be granted by the Faculty of any of the Academic Colleges in any academic department of instruction in the University under the following conditions: The candidate must have taken with credit the equivalent of six three-hour courses (*i.e.,* eighteen lecture "hours" or "periods" extending through the year) in the subject offered, or in such cognate subjects as may have been designated or accepted by the head of the department in which honors are sought. He must have passed satisfactorily a special examination in such additional work as may have been accepted or assigned by the instructor; or have presented a satisfactory thesis on a subject previously approved; or have fulfilled both these conditions, as the instructor in charge of the department may determine.

. . . .

The Honors awarded at graduation will be indicated on the Commencement program, and the graduate who wins Honors for general high standing may have the words *cum laude,* or *magna cum laude,* inscribed on his diploma, the particular designation to be determined by vote of the Senate.—(*Catalogue,* March, 1923)

UNIVERSITY OF VIRGINIA

Final and special honors.—

1. Any student who holds the Certificate of Intermediate Honors (awarded for high average grades in the courses of the first two years), or any student who has credit for not less than 33 session-hours, including all required A courses, passed in this college or elsewhere, and who satisfies a Special Committee on Honors of his capacity for superior work, may become a candidate for Final Honors.

2. In order to receive Final Honors the candidate must obtain, without failure in any course, the baccalaureate degree not later than the end of his second year of candidacy.

3. In addition to completing the requirements for the baccalaureate degree the candidate for Final Honors will, during the final examination period of the year in which he applies for his baccalaureate degree, take a special comprehensive examination, oral and written, for Final Honors in the group of subjects selected by him from the list of such groups published by the special committee on honors-subjects.

4. Graduates who pass the honors examinations in any group of honors-subjects shall be rated according to relative standing with first honors, with second honors, or passed with honors, and the baccalaureate diploma given such graduate shall indicate his standing and the honors-subjects in which it was attained. [This recommendation not approved: see note below.][1]

5. The candidate for Final Honors must, in his application for candidacy, designate one of the following Honors Groups in which he wishes to be examined for Final Honors:

Group I. Greek, Latin, and Ancient History.
Group II. Modern Languages.

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[1] Extract from letter of October 18, 1923, from Professor W. H. Faulkner, Chairman of the Committee.

"The report of the Special Committee on Honors which I sent to you on April 24th, was passed by the Academic Faculty here on May 7th without any changes in the general principle. Two details were slightly changed. Somewhat to my regret the Faculty removed the regulation as to classification under Final Honors and provided that the only Honors degree should be the B.S. or the B.A. with Final Honors in a specific subject, thus doing away with the distinction awarded for first or second place. The Faculty also made a ruling by which it is possible for a student who passes in all the required courses for the B.S. or B.A. degree in three years to continue as a candidate for Honors if enrolled in a professional department or in the Graduate School here, and have the Final Honors added to his degree one year after graduation."

Group III. English.
Group IV. Mathematics.
Group V. Natural Science.
Group VI. History, Economics, and Government.
Group VII. Philosophy.

—(*Extract from Report of the Special Committee appointed by President Alderman for the purpose of investigating the Stimulation of Scholarship and the Recognition of Scholarly Achievement in Undergraduate Students of the College,* 1923)

WESLEYAN UNIVERSITY

Senior honors.—Any senior of normal standing may become a candidate by applying not later than the 10th of October of his Senior year, at which time an outline of proposed study or investigation in connection with the courses which he is pursuing for his major study, or for his concentration group, shall be presented for approval to the professor in charge of the major study, and by him to the committee on departmental honors. The amount of work shall approximate that of a three-hour year course.

. . . .

Upon recommendation of the professor, and with the approval of the administration committee, a student satisfactorily pursuing work for Senior honors may, during the second semester, receive an increased allowance of absences from college exercises, except in the case of announced written recitations and examinations, subject to revocation in case of abuse of the privilege.

Upon recommendation of the instructors in the department concerned, candidates for Senior honors will be examined, both in the general field of their major study and in the departmental honor work, by the committee on departmental honors and such others as they may wish to associate with themselves. In the award of honors, a thesis, report, or other evidence of proficiency may also be considered.—(*Catalogue,* 1923–24)

WILLIAMS COLLEGE

(Extracts from letters from Dean George Edwin Howes to Dr. Vernon Kellogg)

(*May 31, 1922.*) " . . . We are working, though not as fast as we should like, toward the establishment of an honors course. We have, however, so-called pro-seminar courses in English, History, Government, Economics and Philosophy. Only those men, with some exceptions, are admitted to these subjects who are registered in the Department involved. Consequently a higher grade of work can be expected. In the Freshman class in Latin for many years we have had, in the second semester, an honor division, composed of those men whose record for the first half year has been high. We have been very well pleased with the results of this separation of the good students from those of lesser ability. In some years there has been a similar separation of students in the case of other Departments, but the Latin alone, I think, has carried this practice into effect year after year. . . .''

(*July 3, 1922.*) '' . . . The pro-seminar courses in History, Government, Economics, and Philosophy are just finishing their first year, and the honor course in English begins with the coming year. I have heard the members of the Departments involved say that their students are doing much more work than in the ordinary courses. It is of a more individual nature and of a higher grade. This is made possible because fewer of the mediocre and weaker men are probably majoring in those subjects because of the extra work involved. I am sure I am justified in saying that the establishment of the pro-seminar courses has been a distinct success. How soon we shall come over to a real honor course system I cannot tell. . . .''

YALE COLLEGE

Honors Courses

Sophomores whose scholarship standing during Freshman year and the first term of Sophomore year has been 80 or higher may become candidates for honors in one of the several departments of study. Sophomores whose average is below 80 but who have no deficiencies may also, with the permission of the department concerned and with the approval of the Dean, become candidates for honors. In order to continue eligible for honors, the student must maintain the average of 80 for the remainder of Sophomore year, and throughout Junior and Senior years in all studies outside of the honors courses.

The work for honors amounts, in general, to twelve hours divided between Junior and Senior years. This work is offered partly in regular courses and partly in special courses to which only candidates for honors will be admitted. These special courses frequently involve individual reading or research by the student under the guidance of the instructor, in place of the ordinary classroom work. The requirements for honors courses are stated below, preceding the detailed announcement of the courses of the respective departments in which they are offered.

At the end of Junior year, candidates for honors must pass an examination upon the whole of their honors work for the year, and a preliminary (Junior) honors list will be published, based largely upon the results of this examination. Final honors will be awarded in a similar way upon the basis of an examination at the end of Senior year.

An honors student in Junior year who does not continue work for honors in Senior year must satisfy the usual requirements for a degree.—(*General Catalogue,* May, 1923)

APPENDIX
B

DETAILED REGULATIONS FOR HONORS COURSES SUPERSEDING THE ORDINARY REQUIREMENTS FOR THE DEGREE

BARNARD COLLEGE

Requirements for honors.—The course at Barnard has been described by the Chairman of the Faculty Committee on Honors in the following words:

"Exceptionally well equipped students with pronounced interest in any subject may be allowed to substitute for the regular prescribed curriculum a special course of study in that and closely related subjects. Entrance into this course is optional with those students who are eligible.

"*Eligibility.* Students may be admitted to this course: rarely, and only in cases of exceptional maturity and promise, at entrance; usually, as a result of conspicuous ability in college work, at the beginning of the Sophomore or Junior year; provided they meet in September such tests as the Committee on Instruction may determine.

"*Supervision—Prescription—Exemption.*—The appropriate department then takes charge of the student's work and, subject to the approval of the Faculty, arranges the course to be pursued for a degree with honors. A sound reading knowledge of French and German is required before graduation, except in the departments of modern foreign languages, which may substitute other modern languages at their discretion. Previous to the Senior year, the manner of testing proficiency rests with the department in charge. In the Senior year all regular examinations are omitted, and at the end of the year every honors student must pass a comprehensive examination in her subject as a whole. Honors students are exempt from the technical requirement of 120 points, from regular class attendance and from the usual system of grading, but not from the customary supervision of the Department of Physical Education.

"To this may be added that applications of honors students are given first consideration by the Committee on Scholarships and that such students will have all possible privileges in the libraries of the University.

"Each honors student is thus free to arrange her own course with the advice and acquiescence of the department in charge of her career in college. In order to give some form and also a degree of uniformity to the plan as conceived by the various departments there will appear in the new announcement at the head of each departmental section, a brief definition of the material that will be assumed in the final comprehensive examination. The following is the example on which all the rest were modelled:

"Honors Course, Chemistry. The comprehensive honors examination assumes (a) knowledge of inorganic, organic, physical and analytical Chemistry; (b) a more thorough acquaintance with one of these divisions and of a special problem in this; (c) the history and present trend of Chemistry; (d) Mathematics, Physics, French and German.

"These statements are designed to focus the efforts of departments as well as students, rather than to introduce rigidity of requirement.

"The administrative details of the course as a whole are in the hands of a committee which is ancillary to the Committee on Instruction. The most exacting task of this committee is the selection of students who are to be allowed to qualify for transference from the regular to the honors course. The process this year has been as follows: shortly after the opening of the second term, each department as such was requested to send to the committee the names of outstanding Sophomores, preferably, or very exceptional Freshmen. Twenty-seven names were submitted. The records to date were then obtained from the office and the committee met to consider the eligibility of the students first in the light of departmental recommendation and second from the point of view of general ability and industry as reflected in the formal grading. Eight students, all Sophomores, were selected and asked to make known their intention in the matter to the Registrar, after due reflection and consideration. They will next obtain from the departments in which they propose to work, instructions and assignments of reading for the qualifying examinations in September. This summer preparation and the subsequent examination are intended to test the strength of purpose of the student, her power to work without the usual stimuli, and her ability to organize and assimilate unaided.

"Once admitted to the honors course the experience of students has been different in different departments. The numbers have been so small, twenty in all at present, that procedure has been very informal. Doubtless repetition will result in the following of some recognized mode of treatment in each case. Generally speaking the science departments have had their students go into existing courses, often graduate, in their own and allied subjects and have assigned in addition special practical problems as an introduction to the methods of investigation. The departments of Psychology and Economics have done more with the latter type of training than others have thought compatible with thorough grounding in their own fields. In History and the languages, ancient and modern, the work has been confined to acquisition of already available knowledge and method.''

CARLETON COLLEGE

Graduation honors.—Graduation honors may be taken in a department, or in a group of departments, known as a ''Field of Concentration.'' The method known as the ''Candidates-for-Honors Plan'' was adopted by the faculty March 13, 1922. The purpose of the plan is to secure to students having the requisite ability free opportunity to do work that is superior in point of individual initiative, independence, sustained effort and final scope; to make possible during the latter half of one's college course an integration and a cumulative grasp of some one body of related truth such as would not be accomplished by a series of disconnected courses; to give recognition to those who succeed in this undertaking by the award of honors when the final degrees are conferred, and in this way to enlarge the Department Honors plan formerly in use.

Method of admission to candidacy.—Any student who has finished the first semester of his sophomore year and who has not entered upon his senior year may apply to the Secretary of the Faculty for admission to candidacy for honors by filling in the form provided for that purpose and by securing the written recommendation of at least one member of the faculty. His application shall be acted upon by the faculty and, if accepted, his special treatment as an honors candidate shall begin with the semester immediately following that in which he has been admitted. An honors candidate may be demoted by vote of the faculty upon recommendation of the instructors in his field of concentration.

Treatment of honors candidate.—It shall be the policy of the faculty to comply as fully as is possible, with the needs of the honors candidate in the work which he has projected. Special requirements for each candidate as such shall be determined by the departments in which specialized work is to be done. (Such opportunities for extra scholarship might consist of readings in addition to class requirements or of work carried on independent of class sessions and in conference with the instructor from time to time, or of special investigations, reports and essays.) Honors candidates shall take a final, general examination in the field in which they concentrate and this examination shall determine their final honor standing. This standing shall be indicated when degrees are conferred by appending the phrase, "With Honors in ———— " (mentioning the Concentration Field or Department).

Separate examination in specific courses taken in the final semester of the senior year and lying within the field of concentration shall be waived for honors candidates. When any honors candidate produces a piece of work of outstanding merit the college will publish the same upon recommendation of one or more departments. For honors candidates who have entered Carleton subsequent to their freshman year those requirements with respect to major and minors, which they have failed to meet in their preceding college work and which if taken would interfere with logical procedure in their fields of concentration, may be waived at the request of the departments in which the student is concentrating when such request has been granted by a majority vote of the faculty.

In the graduating semester of his senior year the honors candidate shall be granted not more than three hours credit for a final thesis satisfactory to the department or departments in which he has been concentrating. In view of the free nature of some of the work which the honors candidate will be doing the instructor or instructors within his field of concentration may assume direct responsibility for his non-attendance upon their classes without reporting his absence to the deans. For honors candidates no extra charge, beyond the regular tuition fee, is made for work in excess of the ordinary seventeen-hour maximum.

Fields of concentration.—The following Fields of Concentration are offered, but other groupings, if advantageous to the individual students, may be made upon the recommendation of the departments in which specialized work is to be done.

THE FIELD OF CLASSICS, including Art, Bible, Greek, Hebrew, History, Latin, Music, Semitic Literature; THE FIELD OF EDUCATION, including Biology, Economics and Business Administration, Education, Philosophy, Sociology; THE FIELD OF MODERN LANGUAGE, including English, French, German, Italian, Spanish; THE FIELD OF ENGLISH LITERATURE, including Bible, Biography, History, Modern Language, Public Speaking; THE FIELD OF NATURAL SCIENCE, including Astronomy, Biology, Chemistry, Geology, Mathematics, Physics; THE FIELD OF SOCIAL SCIENCE, including Economics, Education, History, Philosophy, Political Science, Sociology.—(*Catalogue,* March, 1923)

COLUMBIA UNIVERSITY

General and special honors.—Students not exercising professional option may become candidates for General Honors. At least two years before graduation the student shall consult with the Dean or with a Director of Honors Work regarding the desirability of his declaring his candidacy for Honors.

If the candidacy is accepted he shall (a) pursue a course of reading in common with other candidates for Honors, and (b) pursue a course of independent study in some field of knowledge which he may choose.

All such candidates for honors will meet with the Honors Directors Wednesday evening each week for a discussion of their common reading, will submit at least one written report involving individual work in their chosen subject in each of the four terms of their Honors course, and will finally be examined in the entire field of their Honors work.

Students exercising professional option, who complete their studies with high distinction may, on the recommendation of the Committee on Instruction, be awarded General Honors, without pursuing the course in reading mentioned in (a) above.

Students who are not candidates for General Honors but who complete with high distinction a sequence of studies in a single department, which sequence has been approved for the purpose by the Committee on Instruction, may on receiving their degree be awarded Honors in the subject of their sequential work.—(*Catalogue,* September 8, 1923)

The following extract from a letter from Dean Herbert E. Hawkes indicates how far the separation goes between the work of honors and of pass students:

"In response to your letter of April 21 addressed to Dean Woodbridge, I would say that our honors students do take an entirely different course so far as their honors work is concerned, from the other students. That is to say, General Honors 1–2 and 3–4 are not taken by any other students in the University. Their other courses are the same as those taken by the rank and file of undergraduates."

HOBART COLLEGE

Honors plan.—Henceforth students at Hobart having exceptional mentality, initiative and qualifications for leadership will be given a wide range of freedom in their studies and upon passing satisfactory comprehensive examinations in the fields of study which they select will be given a special honor degree. As outlined by President Bartlett, the plan is as follows:

The conditions of admission to candidacy for this course shall be:

(1) The completion of the freshman and sophomore years under

the ordinary conditions with an exceptionally high grade. During these years the hour and credit system is to be followed, but as far as possible, the more capable men will be placed in sections by themselves.

(2) Careful consideration by the heads of departments of the personal characteristics of each applicant, with the object of choosing not merely men who receive high grades, but men who are fitted in other ways for leadership.

When admitted to candidacy the student, with the approval of the heads of departments in his chosen field, for example, languages, natural sciences or the history-economics-philosophy group, shall select a course of study equivalent to three years of ordinary college work. These courses will be approved by the full committee of heads of departments in order to insure sufficient breadth of study. The student then, so far as his advisers approve, shall be freed from all restrictions as to hours and credits, attendance at classroom exercises and ordinary examinations. At stated times he shall meet his professors in intimate, personal conferences, but his entire work will be under their supervision.

At the end of two years a comprehensive and general examination will be given the student covering the entire field of his study. If this is successfully passed it is proposed that he be given a special honor bachelor's degree, with a definite statement on the diploma that he has completed the equivalent of five years of college work with an official certificate stating the character and amount of the work completed during the last two years. His diploma will also state that he is prepared for taking up advanced research work which will lead to the doctor's degree.—(*School and Society*, June 9, 1923)

JOHNS HOPKINS UNIVERSITY

Honors students.—

"The Johns Hopkins University has recently taken action in the matter of encouraging students of exceptional ability by giving them the privilege of attending certain graduate courses during their final year. The Johns Hopkins University has always kept its graduate and undergraduate work perfectly distinct.

"The resolution recently adopted by the Board of Collegiate Studies is as follows:

> 'A student whose work has been of high standards, and whose request for the privilege is approved by a committee, to be appointed by the President, may, if he has complied

> with the specific requirements for the A.B. degree other than the number of points and the completion of a major, spend his final year in such graduate work as the committee and the head of the graduate department concerned shall approve. At the close of this year and after passing a comprehensive examination before a committee of the department or departments in which his work has been done, he shall receive the A.B. degree.'

"These students will be regarded as honor students and given greater liberty than is accorded undergraduates generally. They will also be expected to rely more on their own resources and to do work of an intensive character in one department or in closely allied departments."—(*Extract from a letter from Dean John H. Latané to Dr. Vernon Kellogg,* June 16, 1922; confirmed, January 10, 1924)

RICE INSTITUTE

Honors courses.—The third and fourth year honors courses are intended for students who wish to specialize in particular branches of knowledge with a view to research work or teaching or later professional studies.

In view of these special objects the requirements in such courses will be more severe than in the general courses in the same subjects. For this reason it is recommended that students exercise due caution and seek advice before electing to take an honors course. Only those students who have shown in their first and second years that they are especially well qualified will be permitted to take an honors course. A student proposing to take an honors course must satisfy the department concerned that he is qualified to proceed with the study of that subject. He will be required to take the lectures and practical work provided for honors students in that subject during each of the two years and in addition certain courses in allied subjects.

In 1923–24 honors courses will be available as follows: (1) Pure and applied mathematics, (2) theoretical and experimental physics, (3) modern languages and literatures, (4) biology, (5) chemistry, (6) economics and mathematics, (7) English.

The following programme in honors courses in physics may be taken as typical of such courses:

Third year, five subjects: (1) mathematics, (2) physics 300, (3) physics 310, (4) physics 400, (5) physics 430, (6) one other subject.

Fourth year, five subjects: (1) mathematics, (2) physics 400, (3) physics 420, (4) physics 500, (5) physics 410.

A student who wishes to become a candidate for a degree with honors should report his candidacy at the beginning of his Junior year and renew his application at the beginning of his Senior year.

The degree of B.A. with honors will be awarded at the end of the fourth year to students who have completed an honors course. Candidates for honors who fail may be excused such part of a general course as may be equivalent to the work they have done. Candidates for honors who are not making satisfactory progress may be required to discontinue their honors course and may be excused such part of a general course as may be equivalent to the work they have done. —(*Catalogue,* 1923–1924)

The following extract from a letter to Dr. Kellogg from Dean Robert G. Caldwell, April 26, 1923, furnishes additional information:

"Our honors students take different courses and do different work from the 'passed' students. The essential principle of our plan is to treat honors students in their last two years very much as ordinary graduate students. Candidates for honors are considered by the same committee which also considers candidates for advanced degrees."

SMITH COLLEGE

Special honors.—By the provision of Special Honors, Smith College is giving to students of outstanding ability opportunities which cannot be realized in the ordinary work of the classroom. The rate of progress aimed at in college courses is determined by a rough averaging of the capacity of all the students in them, with the result that, while this rate is barely maintained by the weakest students, it is very far from keeping the ablest employed. These latter, who should contribute most to the intellectual life of the college, are liable, on the contrary, either to fall into habits of intellectual loafing, or to occupy their too-abundant leisure by a disproportionate amount of non-academic activities. The objection to a uniform pace for all abilities applies also to a uniform method of instruction. Frequent recitations and lectures may be the best means of keeping some students intellectually awake; for others, they are unnecessary and wasteful. After a good student has acquired habits of study and keen intellectual interests, she needs leisure for thinking and large quantities of solid reading rather than hours a day of classroom work. Furthermore, after the large range of subjects required by our curriculum in the first two years, she is ready for a more intensive application to some chosen field, so that at the end of her college course she may carry away not merely a great variety of scraps of knowledge, but power and method for the mastery of one field and its correlated parts. Too often college students regard the subjects covered in "courses" as things apart; they neatly pigeon-hole the information so derived, and, after the examinations are over, they tie up each bundle of facts and put it away, seldom realizing the relation such facts may have to others acquired subsequently. In the work for Special Honors each candidate, excused from "courses" and from the examinations which terminate them, is given every opportunity to view the various subdivisions of her work as one connected whole. Her study should therefore lead both to a deeper and to a more comprehensive understanding of her subject than is normally gained by the student who follows the usual curriculum.

It is not the aim of the Faculty to turn the system of Special Honors into a school for training in research. The candidate for these Honors is rather given as deep an insight as is possible into the knowledge already existing in her chosen field; on graduation she will then be thoroughly familiar with its problems, and ready, if she so desire, to begin research on her own account. The advantage of this deeper knowledge is clear, not only for the future investigator and writer, but also for those who expect to teach.

Recognizing these principles, the Faculty approved in 1921 a scheme by which at the end of the Sophomore year, students having a general average of at least B, or, in exceptional cases, students whose work in the department of their choice is highly recommended as showing unusual ability, though their general average may not be so good, are permitted to apply for candidacy for Honors in a special field. By "field" is not necessarily meant one subject, as subjects are usually understood in college; for instance, Honors are obtainable in Classics, not in Latin or in Greek alone; or a combination may be made of modern languages; or of History as major subject with Government, or *vice versa*; and in other ways studies intimately allied may be connected. Application should be made through the office of the Registrar before April 15 of the Sophomore year by the students who, having obtained the necessary average in their first three semesters, desire to become candidates for Special Honors; but students whose average reaches B only at the end of their Sophomore year will still be eligible, and may, with the approval of the department concerned, apply before the beginning of their Junior year.

Those students whose application has been approved by the Committee in charge of Special Honors and by the department in which their chief study is to be followed, are relieved during the last two years of the routine of class attendance and course examinations. Each student comes under the guidance of a "General Director" of her work, who plans with her a series of eight units of study in her chosen field, two units being equivalent to the full work of one semester. Six of these units, the work of the Junior year and the first semester of the Senior year, are distributed among the subdivisions of her chosen study; two subdivisions are usually followed simultaneously. These subdivisions may coincide with single units of study and change each semester, or two units may be devoted to one subdivision. The work is carried on in the various subdivisions under the guidance of "special instructors," by means of suggested readings, written reports, and by conferences, weekly or fortnightly, in which these reports are criticized and instruction given for the preparation of others. The instruction is therefore planned for each individual student; but, should several candidates elect to follow the same work with the same instructor, a small group may be combined in a little seminar, or class for report and discussion. Each candidate will be allowed, as far as is practicable, to choose her "special instructors." Honor students may in any semester be advised or required to attend such courses or parts of courses as seem advantageous for the pursuit of the selected studies; but these students are not thereby obliged to fulfil the class requirements or to take the final examination in such courses. Candidates for Special Honors are exempted from all requirements exacted in the case of other students during the Junior and Senior years, with the exception of the requirement in Group VIII (Philosophy and Biblical Literature). Students, therefore, who expect to be candidates for Special Honors should, if possible, complete the requirement in Group VIII in the Sophomore year.

The two units of the last semester of the Senior year are to be devoted to the writing of a long paper on some subject chosen within the student's field, and to a general review preparatory to an extensive examination covering the whole range of study of the last two years. The paper, in typewritten form, will be placed in the Library after acceptance.

Honors are awarded in two grades: Honors and Highest Honors, according to the quality of the work done. In the event of a student's failing to be awarded either grade at the end of her Senior year, she may yet be granted a pass degree if her work is of sufficient merit. A student who on grounds of health or other serious cause finds it impossible to continue her Honors work may petition to withdraw, resuming her position as a candidate for the degree without Honors, or with such Departmental Honors as may be open to her. Her petition will be granted if approved by the department in charge of her work, and by the Committee on Special Honors.

The scheme of Special Honors does not supersede the different methods of obtaining Honors at present in force in college; and any student who wishes to follow the present course of study is eligible for Departmental or General Honors as heretofore.

It may be noted that, in its restriction of the system of individual guidance to a small picked group, the scheme of Special Honors differs from the tutorial system adopted in recent years by some American universities. On the other hand, while it resembles the Honor Schools of the English universities by giving the student a comprehensive view of her field in uninterrupted and intensive study of its various parts, it yet differs materially from these Schools in requirements and in organization, for it demands that the first two years of a student's work be devoted to a wide range of prescribed subjects, and it is organized to meet the needs of each individual candidate for Honors. The same curriculum of study is not necessarily devised for all students in one department, and the same examinations are not necessarily set for all; both study and final examinations are based on the programme drawn up separately with each student by the General Director of her course.—(*Report of Committee on Special Honors,* 1923)

SWARTHMORE COLLEGE

Honors courses.—Honors work at Swarthmore College is described in the report of the President for the academic year 1921–22, as follows:

"The theory underlying our Honors work is that the highest standard of intellectual attainment cannot be reached in our colleges if, as at present, the pace is set by the average. Our more ambitious students can keep up the required pace without being pushed to anything like the limit of their abilities. Under the present system, they find it impossible to work harder or more effectively even if they should wish. Our academic machinery, which is excellent for stimulating students who are only mildly interested in their work and for directing those who have no great amount of intellectual initiative, is a hindrance to the better students, restricts their efforts, limits their independence, treats them as boys and girls, when they deserve to be treated as men and women.

"It is our practice at Swarthmore, while keeping up in every respect the care and attention given to the majority, to provide, through our Honors Courses, an opportunity for more ambitious students to work harder and more independently, reaching for the A.B. degree a distinctly higher standard of attainment than would be possible for the whole body of students. We are able to allow these Honors students more freedom from requirements, more opportunity to follow their own interests, and we test their work by a system of examinations far more comprehensive and far more severe than those required in ordinary undergraduate courses.

"Students are allowed to volunteer to read for honors at the end of their Sophomore year, admission to the status of Honors student depending upon the quality of their work in the first two years. In the consideration of this record, special aptitude is considered of more importance than a mere high average of grades all round. Honors students are excused from the ordinary examinations and course requirements. Instead, they are expected to spend two years in mastering a certain definitely outlined field of knowledge over which they are examined at the end of their two years' work. Their instruction is mainly individual, and a large part of the work is done independently by their own reading. It is open to Honors students to attend as many or as few of the regular classes of the College as they desire; they are guided in this respect by the advice of the Chairman of the division in which they are reading. The comprehensive examinations at the end of their course consist of from ten to twelve three-hour papers followed by an oral examination. These tests are conducted not by the persons who have had charge of the preparation of the candidates but by professors from other institutions. On the basis of these examinations, Honors students are given the degree of Bachelor of Arts with Honors of the first or second or third class as their merits may deserve. Candidates whose work is not of a high enough quality to entitle them to any of these classes may be given the ordinary B.A. degree without Honors."

WELLS COLLEGE

Special honor courses.—Honor courses have been erected by the departments of Biblical History and Literature, Classics, Economics and Sociology, English, History, History of Art, Mathematics, and Romance Languages and Literature. Honor Courses in other departments may be announced later.

The general rules governing all Honor Courses are as follows:

1. Honor Courses may be erected entirely within the field of one department or in combined fields of several departments, provided that in the latter case the course shall always provide at least as much specialization within some one department as is prescribed in the standard course of that department.

2. To be admitted to an honor course a student must have shown unusual ability in her previous college work and especially in the department in which she wishes to read for honors. She must further secure the recommendation of the department or departments offering the course and the approval of the Committee on Choice of Studies.

3. Students will be admitted to honor courses at the end of their sophomore year, and should apply for admission before May first of that year. They shall be informed of the result of their application before the end of the year and if admitted to the honor course shall immediately come under the direction of the department or departments concerned.

4. At the beginning of every academic year a general statement of the work planned for each honor student shall be filed by the department concerned with the Committee on Choice of Studies, and at the end of every academic year a general statement of the work accomplished by each honor student during the year shall be filed by the department with the registrar.

5. An honor student who fails to maintain a satisfactory standing in her work may at any time be transferred to the standard course on the recommendation of the department of the honor course concerned and the approval of the Committee on Choice of Studies.

6. Every honor student shall be required at the end of her senior year to submit to a general examination or examinations on all the subjects in which she has done honor work.

7. Honors shall be of three grades, summa cum laude, magna cum laude, and cum laude.

8. Honor students failing to attain honors in the final examination may be recommended for a degree without honors by the department in which the honor course was taken.

9. Honor Courses shall differ from standard courses both in the quantity and the quality of the work required.

10. The scheme of Honor Courses adopted at this time shall be regarded as experimental and shall be subjected to reexamination at the end of four years.

11. In any case where the work of an honor student in one department involves to a great extent the subject matter of another department, it is recommended that a member of the latter department be called into conference with student and tutor.—(*Catalogue*, January, 1923)

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FOURTH ANNUAL MEETING, APRIL, 1923
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GENERAL ADDRESSES

THE ORGANIZATION AND AIMS OF THE AMERICAN GEOPHYSICAL UNION[1]

LOUIS A. BAUER, *Chairman*

I. HISTORICAL

This is the fourth annual meeting of the American Geophysical Union, as such, although on June 24-25, 1919, there was held a preliminary meeting of a tentatively organized "American Section of the proposed International Geophysical Union," then in process of formation under the auspices of the International Research Council, which, as will be recalled, was established in 1918, when the world conflict was drawing to a close. However, the concept of our Union may trace its origin even prior to the formation of the International Research Council and of the present International Geodetic and Geophysical Union. For soon after the establishment of our National Research Council in 1916, the chairman, Dr. Hale, requested the writer to prepare a memorandum respecting provision to be made for certain geophysical sciences. To quote from this memorandum but one sentence germane to our topic today—"It would appear that great opportunities are before us to make our influence felt in the world-sciences, by which are meant those subjects of research which embrace the Earth, or even the Universe, within their legitimate provinces." In subsequent correspondence with Dr. Hale it was suggested that the term "geophysics" be understood to include geodesy, geological physics, meteorology, terrestrial magnetism, terrestrial electricity, seismology, tides and oceanography; and it was recommended that there be formed a special committee of the Council, or a subcommittee of physics dealing specially with "world physical sciences."

To stimulate further interest in the subject, at the joint meeting in 1916 of the American Physical Society and the Section of Physics of the American Association for the Advancement of Science the speaker presented a paper entitled, "Our part in the Advancement of World Physical Sciences."

It will be seen from these references that the general idea was the formation of a body similar in its organization and aims to those embodied in the statutes of the present International Union and, in particular, of our own Union. The chief desire as expressed in these early documents was to provide a meeting-ground for those working, more or less independently, on geophysical subjects having points of contact and common interests. This was accomplished later in Great Britain by the appointment in July 1917 of a special committee on geophysics of the British Association for the Advancement of Science.[3]

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[1] As presented at the opening general session of the annual meeting of the American Geophysical Union at Washington on April 17, 1923, with some slight additions. The review was prepared in compliance with a request made at the closing session of the annual meeting of the American Geophysical Union in 1922, and in answer to inquiries from members as to the precise status of the Union, especially with regard to its origin and development and its present relations to the National Research Council and to the International Geodetic and Geophysical Union.

The council of our National Academy, however, was not yet prepared to recognize the need of a special body for geophysics alone, and considered that the desired object could be attained by merging geophysics with geography, and accordingly four of the present members of the American Geophysical Union became members of the Geography Committee, under the chairmanship of Professor William Morris Davis. But soon after geophysics received a more distinctive recognition, being specifically mentioned in the title of one of the most important divisions of the war organization of the National Research Council, namely, the "Division of Physics, Mathematics, Astronomy and Geophysics." It was the writer's privilege to serve as a member of that division, under the chairmanship of Dr. Robert A. Millikan, throughout our participation in the war.

Then came the close of the war and with it a reorganization of the National Research Council on a peace basis. One of the live questions was how best to secure representative membership for the various divisions: Physical Sciences, Chemistry, Geology, etc. The solution of this problem was comparatively easy for the sciences which had national societies. Thus the Division of Physical Sciences would be recruited from time to time by representatives recommended to the National Research Council by the American Physical Society, the American Astronomical Society, the American Mathematical Society, etc. But how were the geophysicists to be represented? For seismology, there was the Seismological Society of America, and for meteorology, there was at that time in process of formation what has since become the American Meteorological Society. But there were no distinct national societies for geodesy, terrestrial magnetism and atmospheric electricity, volcanology, oceanography, and geophysical-chemistry. To be sure, quite a goodly number of geophysicists were members of the national societies devoted to physics, astronomy, mathematics, geology, and geography. Hence, to a limited degree, representation on a Geophysics Committee of the National Research Council could be obtained from certain existing societies, and the number of representatives thus secured could be supplemented by the National Research Council through choosing members at large, interested in branches of geophysics not distinctly recognized by the existing national societies.

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[3] The British Geophysics Committee had its first meeting in November 1917, in the rooms of the Royal Astronomical Society at Burlington House, London. Since then there have been "Geophysical Meetings" about once a month during each year from about October to May. The topics discussed have covered every branch of geophysics and the leading men of science in Great Britain, interested directly or indirectly in geophysics, have participated. Abstracts of the papers and discussions are published in the British monthly journal, *The Observatory*. Some of the papers have been published in the Geophysical Supplement of the *Monthly Notices of the Royal Astronomical Society*. Three of these "Supplements" have thus far appeared, namely for March 1922, January 1923, and May 1923.

Finally, in February 1919, Dr. Hale, as Chairman of the National Research Council, appointed a small committee to consider the question of the organization of an "American Section of the proposed International Geophysical Union," and the chairman of the committee, Dr. R. S. Woodward made a report, which as it is brief, yet full of interest at the present time, we shall quote here in its entirety:

"MARCH 4, 1919.

"To the Chairman of the National Research Council:

"Your Committee appointed to consider the question of a logical and practicable organization of the proposed American Section of the International Geophysical Union respectfully submits the following report:

"The earth is at once the subject and the object of many sciences. Of these the most important are Astronomy, Geodesy, Geology, Meteorology, Seismology, Terrestrial Magnetism, Terrestrial Electricity, Tides, and Volcanology.

"While each of these sciences is more or less distinct in itself, they are closely related to one another, and progress in any one of them may be expected to depend to a great extent on the general progress attained in the others. Each of these sciences has its devotees and its experts, and the number of these in the aggregate is now very large. Hence in any scheme of effective organization it is essential to secure groupings of these various subdivisions of geophysics in order that the number of groups may not be too unwieldly in the transaction of business essential to such organizations. But it should be distinctly understood that in recommending a limited number of groups for purposes of administration it is not desired to discourage relations of closest reciprocity between the devotees to the various sciences included in the groups. On the contrary, it is the opinion of your Committee that progress in the future is most likely to result from active cultivation of the borderlands that now serve to distinguish, but only indefinitely, the several fields of geophysics.

"It should be understood also that the groupings recommended are to be regarded as provisional and subject to such changes as future experience may suggest. It is recognized also that the groupings here recommended may not be the most appropriate for all countries or possibly for an international organization, since much regard should be given in all such matters to historical precedents and to the circumstances presented at any epoch by individual investigators, and especially by governmental organizations, of any country.

"With these reservations the Committee recommends that the following groups of subjects should be recognized in the organization of the American Section of the International Geophysical Union:

"Group 1. Geodesy. This group may be assumed to deal with questions concerning the size, the shape and the mechanical properties of the earth.

"Group 2. Seismology and Volcanology.

"Group 3. Meteorology and Mareology, including especially all questions presented by the mechanical properties of the atmosphere and the oceans.

"Group 4. Terrestrial Magnetism and Terrestrial Electricity. This group is intended to deal with the magnetic and the electric properties of the earth, including its atmosphere.

"The Committee recommends that initially the designation of members to constitute the proposed Geophysical Section be made by the National Academy of Sciences. It is further recommended that in making such designations regard be had to the desirability of securing representatives from the following Government Bureaus:

Bureau of Fisheries
Bureau of Mines
Bureau of Standards
Coast and Geodetic Survey
Hydrographic Office, U.S.N.
Geological Survey
Weather Bureau

"Similarly, the Committee suggest that representatives also may be fitly chosen from the following national societies:

American Astronomical Society
American Mathematical Society
American Physical Society
Geological Society of America
Seismological Society of America

"The Committee further recommends that in order to promote research and discovery in geophysical science in general, steps be taken by the American Section of the International Geophysical Union toward the formation of a new society to be called the American Geophysical Society.

(Signed) "R. S. Woodward,
"For the Committee."

At a meeting of the Executive Committee of the Division of Physical Sciences, March 10, 1919, the organization and initial membership of an American section on Geophysics were approved, and soon after were also approved by the National Academy of Sciences. The exact designation of subjects to be included in the various groups was deferred, however, until after the Brussels meeting in July 1919 of the International Union. The question of the establishment of a geophysical society was also referred for settlement to the Geophysical Section after its permanent formation. Messrs. William Bowie, Louis A. Bauer and C. F. Marvin were made members of the Division of Physical Sciences to represent Geophysics.

Four meetings of the Provisional Executive Committee of the Geophysical Section were held in May and June 1919 and on June 24 and 25 the Section was tentatively organized, Dr. William Bowie acting as chairman. Permanent organization was deferred until after the Brussels meeting. The Section accepted the proposal to be the Geophysics Committee of the Division of Physical Sciences, and decided to concern itself with national as well as international problems in geophysics. At this meeting delegates were appointed to the Brussels meeting and resolutions were passed respecting various matters pertaining to the organization and sections of the proposed International Union. The delegates were also constituted a committee to draft a plan for permanent organization of the American Section after the Brussels meeting.

On November 13, 1919, the secretary of the tentative American Section transmitted to the members "Proposals for the Permanent Organization and Statutes of the American Geophysical Union," prepared by a sub-committee, after giving very careful consideration to the following points: (1) The structure and Statutes of the International Research Council, and of the International Geodetic and Geophysical Union, definitely formed at Brussels in July 1919; (2) the present relations of the existing "American Section of the proposed International Geophysical Union" to the National Research Council, and the relations considered desirable in future; and (3) the views expresseed by present members, especially at an informal conference of the Committee of the Brussels Geophysical Delegates with members of the "American Section" who reside in or near Washington. The said conference was held on October 31, 1919, at the office of the National Research Council. Furthermore, in drafting the "proposals," the sub-committee had the benefit of extended conference with Dr. C. E. Mendenhall, at that time Chairman of the Council's Division of Physical Sciences, and of Dr. Henry S. Washington, both of whom were present at Brussels.

The secretary, Dr. Harry O. Wood, further stated in his transmitting letter:

> "The present status of the 'American Section' is that of a *committee* of the Division of Physical Sciences of the National Research Council. This status it will retain until it is discharged as such, or until it is superseded by the adoption, as of some definite date, of proposals for permanent organization which change its status. In this connection it has seemed best to the committee on organization to recommend that the proposed 'American Geophysical Union' be made a Committee of the Council, as a whole, and that the 'Union' be so constituted as to *unite and co-ordinate* in our country the interests of physics, chemistry and geology in geophysical questions.
>
> "The name 'American Geophysical Union' was settled upon as the result of careful consideration and evolution of various preliminary drafts. Thus, instead

of 'Union' it was originally proposed to use the term 'Committee,' and later 'Commission'; however, as progress was made in drawing up the proposals, the need of a more appropriate term became evident. Thus, the 'American Geophysical Union,' as proposed, is a union of two distinct committees, namely, the American 'National Committee' of the International Geodetic and Geophysical Union, and the Committee on Geophysics of the National Research Council. In the absence of an American Geophysical Society in the United States, the proposed 'Union' would seek to bring together and co-ordinate all societies, bodies, and institutions in the United States interested in geophysics."

The "Proposals," having received the unanimous approval of the members composing the former "American Section," were approved by the Executive Board of the National Research Council on December 20, 1919. On February 14, 1920, the American Geophysical Union was made a committee of the Executive Board of the Council, and the first meeting of the Union was held on April 23, 1920. Dr. William Bowie, who had been acting chairman since 1919, was elected chairman of the Union for the statutory two-year period 1920-1922.

II. ORGANIZATION

These introductory remarks will suffice to show the initial difficulties encountered and how, in consequence, there was gradually developed the "American Geophysical Union," which has the unique distinction of being like no other body of the National Research Council. During 1919 and 1920, we were attached to the Division of Physical Sciences as the National Geophysics Committee, just as are the national committees for astronomy, mathematics and radio-telegraphy. However, we proved ourselves too lusty an infant for in membership we outweighed the entire Division. Then again the Council soon found that Geophysics was of so wide a scope that its points of contact were not simply with one division, but with several. And so finally in 1920 we were made one of the Technical Committees under the direct parentage of the Executive Board of the Council. But our woes did not end there, for at that time we had no representative voice on the Board and as the membership of the Board changed from year to year, we suffered the natural consequences resulting from variable parentage. Just as our parent towards the end of his year's membership on the Board would become truly sympathetic with our aims and alive to our needs, he would be succeeded by a new parent and the process of enlisting interest had to be begun all over again.

Fortunately, although our Union, unlike other national bodies and other sciences, still has no direct representation on the Council's Executive Board, it has certain ex-officio members on its Executive Committee; namely, the chairmen of the following divisions of the

Council: Foreign Relations, Physical Sciences, Chemistry and Chemical Technology, Geology and Geography, and Biology and Agriculture. We must depend upon these ex-officio members, who also, in general, vary from year to year, for representation at the meetings of the Executive Board of the Council. During previous years it was only rarely, for one reason or another, that one of the ex-officio members was present at the monthly meetings of our Executive Committee. During the present fiscal year, however, there has been an exceptionally faithful attendance on the part of some of the ex-officio members.

We also, in our entirety, proved too "bulky" as a committee of the Executive Board, especially when it came to the matter of the possible moral obligations involved to provide traveling expenses for every member of the Union coming to the annual meetings. Hence, in the spring of 1921, the relation of the American Geophysical Union to the Council was changed so that the Union as a whole was replaced as the Committee on Geophysics of the Executive Board of the National Research Council by the Executive Committee of the Union. The Union, as a whole, however, continues to be the American National Committee of the International Geodetic and Geophysical Union.[3] Although not every member of our Union is recognized now as a member of the National Research Council, it is the Union as a whole which nominates to the Council the members of the Executive Committee of the Union.

In other words, the American Geophysical Union is now virtually on the same footing as existing national societies, which choose and recommend to the National Research Council their representatives for the various divisions and committees of the Council. While we still recognize the sponsorship of the Council, in that we submit to it for approval any amendments to our statutes or additions to the membership, it is not believed that the Council would interpose any objections if we thought it desirable to form ourselves into a national geophysical society. As has been shown, in many respects we are already essentially such.

According to action taken at the annual meeting in 1922,[4] our permanent membership was limited to 75, subject to future change as circumstances required; there are at present 65 members, widely distributed geographically, even Hawaii being represented.

Whether the time is ripe to constitute ourselves into an "American Geophysical Society," or whether we should retain the present name

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[3] See Sixth Annual Report of the National Research Council, p. 6; and minutes of the Executive Committee of the American Geophysical Union, May 7, 1923.

[4] Also confirmed at the annual meeting in 1923.

"American Geophysical Union" and the present organization as indicative of a union or confederation of geophysical sciences, and alter or expand our organization and membership, is a matter for discussion. If we follow the type of organization as represented by a society, we may readily include in the membership citizens of other countries than the United States. In accordance with a recent action taken by the Executive Board of the National Research Council, "membership in the Council shall be limited to citizens of the United States. This, however, shall not be construed as applying to membership on committees appointed by or acting under the Council whose members are not necessarily members of the Council; provided that members not citizens of the United States shall in no case form a majority of any such committee.[5] The majority of our Executive Committee and of chairmen of sections must accordingly be citizens of the United States.

III. OBJECTS

Quoting from the statutes[6] as adopted in 1919 and amended to May 1923:

> *Preamble.*—"The American Geophysical Union is hereby established by the National Research Council. It supersedes the 'American Section of the proposed International Geophysical Union.' The American Geophysical Union is the American National Committee of the International Geodetic and Geophysical Union and the Executive Committee of the American Geophysical Union is the Committee on Geophysics of the National Research Council."
>
> *Objects.*—(a) To assist in carrying out the objects of the International Geodetic and Geophysical Union, which are: To promote the study of problems concerned with the figure and physics of the Earth; to initiate and co-ordinate researches which depend upon international co-operation, and to provide for their scientific discussion and publication; and to facilitate special researches, such as the comparison of instruments used in different countries.
>
> (b) To promote and co-ordinate the study of the various branches of geodesy and geophysics in the United States of America and its outlying territories."

A review of the work and functions performed by the Union, since its existence, would show that the objects just stated have steadily been kept in mind and that we have taken our part, as fully as circumstances permitted, in the promotion of international and national projects pertaining to geophysics, in general. From time to time we have furnished information solicited by the National Research Council and International Geodetic and Geophysical Union, and have reviewed proposals submitted to us by or through the Council.

The American Geophysical Union may justly lay claim to having taken a leading part in the initial recognition, at the organization meeting of the International Geodetic and Geophysical Union at Brussels in 1919, of the chief branches of geophysics, and in the establishment of sections pertaining thereto. Prior to the formulation of resolutions passed at the spring meeting in 1919 of our preliminary organization, the International Research Council at first favored the adoption merely of sections of geodesy, meteorology and seismology by the proposed International Geophysical Union. The American delegates, however, acting in accordance with the conclusions reached at the preliminary meetings in Washington, of the "American Section" and on the basis of the information at hand, advocated the immediate establishment of the following sections: Geodesy, Seismology, Meteorology, Terrestrial Magnetism and Electricity, Volcanology, and Oceanography. And the American view finally prevailed at Brussels as the result of various preliminary meetings of our delegates with those from other countries represented. It was a matter of keen regret to the American delegates, as well as to those of other countries, that the briefer and more logical designation, "International Geophysical Union," as originally proposed, was not finally adopted by the International Research Council, in deference to the wishes of one country.

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[5] Action of Executive Board of the National Research Council, February 13, 1923.

[6] See pages 24 to 26.

Five members of our Union are at present officers of the International Geodetic and Geophysical Union, viz: Messrs. Bauer, Bowie, Littlehales, Marvin and Washington, and, accordingly they are also members of the Division of Foreign Relations of our National Research Council. Furthermore various members of our Union are members of committees of the sections of the International Union.

In the matter of arousing renewed interest in the scope of geophysical sciences, and in the unusual opportunities for important and fruitful research, our Union has already done much, through its meetings and publications, though more remains to be done. The observational and investigational work in the various branches of geophysics in the United States is being largely done at present by governmental bureaus and certain research institutions, though notable work is to be credited also to investigators connected with some of our universities. It is believed, however, that the endeavor should be made to enlist the help of still more university men, by presenting in as concrete a form as possible definite problems awaiting experimental and theoretical investigation. During the war we were fortunate in the number of university men who found opportunity for their activities and participation in war investigational work pertaining to various branches of geophysics, notably meteorology. But as soon as the war ended their interests waned and it behooves us to find new ways of re-stimulating them.

IV. FUNDS

The funds required for the maintenance of the American Geophysical Union, especially in connection with the annual meetings, averaging about $1,000 per annum, are provided by the National Research Council. The Council, through its Division of Foreign Relations, made provision in part for the payment of the expenses of some of the delegates to the Rome 1922 meeting of the International Geodetic and Geophysical Union. During the first year of the existence of The International Geodetic and Geophysical Union, the annual dues for the United States were paid by the National Research Council from special funds received for the payment of annual dues of the established international unions. In the following years, however, the annual dues have been paid by Congressional appropriation in the Diplomatic and Consular bill of the State Department, there being included in this appropriation also the dues for the International Research Council and unions established by it. The Council has also borne the expense of publication of papers and reports by the Union.

V. CHARACTER OF ANNUAL MEETINGS

The character and purpose of our annual meetings has naturally varied from year to year in accordance with circumstances. The meetings of 1920, 1921, were largely concerned, as is the present meeting, with submission and discussion of special reports and papers. The meeting of last year was chiefly devoted to the discussion and preparation of reports relating to the Rome meeting of the International Geodetic and Geophysical Union in May 1922.

Our programs of this year for each of the sections are replete with interest, and while the Committee on Meetings has laudably endeavored to arrange the meetings of the several sections so as to avoid conflicting interests as much as possible, one might, nevertheless, wish that it were possible to have all papers and reports presented at general meetings of the entire Union. There might be held, however, brief sessions of each section for the transaction of strictly business matters and for the presentation of purely technical reports and papers. We may be in danger of losing the chief advantage of our annual gatherings if we do not find it possible to make provision in future for more general sessions to which all papers, or at least the chief ones, would be presented and then afford ample opportunity for discussions, conferences, or symposiums, on mooted questions, not infrequently of interest and importance to several of the sections, and possibly to all of them.

VI. SECTIONS AND COMMITTEES

Perhaps some consideration may have to be given in the near future as to how best to avoid the dangers arising from possible over-organization and too intense sub-division into groups and committees. One of the chief objects of our Union from the very beginning has been, as already stated, to provide a common meeting-ground, or clearing-house, for our varied aims and researches, with the minimum amount of machinery or organization.

Not all national committees have as yet formed themselves into sections corresponding in general to those of the International Geodetic and Geophysical Union, as Belgium, Canada, France, Japan, Spain, and the United States have done. However, national committees in most countries are composed of less than twenty-five persons, so that splitting up into sections might not be advantageous. The status of the American Geophysical Union, as already explained, is somewhat different from that of other national committees. For, in the first place, in order to have adequate representatives of the greatly varied interests in geophysics over the entire extent of the United States and outlying territories, it has been necessary to have in our Union about three times the average membership of national committees. In the next place, we have endeavored to unite in our Union the objects of both a national committee and a national geophysical society.

It is of interest to note the provisions for geophysics made by progressive Japan. The National Research Council of Japan consists of about 92 members, grouped into the following 9 divisions: General Affairs, Astronomy, Geophysics (10 members), Chemistry, Physics, Geology and Geography, Biology and Agriculture, Medical Sciences, and Engineering. In addition to this Division of Geophysics, which is on equal footing with the other divisions, there is a "National Committee for Geodesy and Geophysics," consisting of the 10 National Research Council members of the Division of Geophysics, and 13 co-opted members, who are not members of the Council. The chairman and vice-chairman of both the Division of Geophysics and of the National Committee are, respectively, Professor A. Tanakadate and Professor K. Nakamura.[7]

In conclusion, permit me to advert to a question raised in our pioneer days, as to whether an organization like ours, composed of so many different groups and devoted to such a variety of world-embracing subjects, which to some appeared but slightly related, could find sufficient points of interest to avoid disintegration and splitting up into several organizations. But our Union has held together and promises still to hold together.

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[7] National Research Council of Japan, Proceedings, No. 1, March, 1922, pp. 16–21.

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ORGANIZATION AND AIMS OF THE NATIONAL GEODETIC AND GEOPHYSICAL COMMITTEE OF CANADA[1]

NOEL OGILVIE, *Superintendent, Geodetic Survey of Canada*

Dr. E. Deville, Chairman of the National Committee of Canada of the International Union of Geodesy and Geophysics, received a letter dated April 4, 1923, from Dr. Louis A. Bauer, Chairman of the American Geophysical Union, advising him that arrangements had been made to allow time at this meeting for the presentation of an address on "The Organization and Aims of the National Geodetic and Geophysical Committee of Canada." This honour Dr. Deville has kindly conferred upon me and it gives me great pleasure to be with you and to present this address on behalf of the National Committee of Canada.

The National Committee of Canada, of the International Union of Geodesy and Geophysics, was formed by the Honourary Advisory Council for Scientific and Industrial Research, and I received a letter from Dr. A. S. Macallum, the Administrative Chairman, dated the 23d of April, 1920, reading as follows:

> "I have the honour to inform you that you have been appointed a member, ex efficio, of the National Committee for Canada of the International Union of Geodesy and Geophysics, the other members of which are as follows:
>
> The Director of the Dominion Observatory, ex officio
> The Surveyor-General, ex officio
> The Director of the Meteorological Service, ex officio
> The Director of the Geological Survey, ex officio
> The Superintendent of Tidal and Current Surveys, ex officio
> The Chief Hydrographer of the Naval Service, Department, ex officio
> Four representatives of The Royal Society of Canada.
>
> "The appointment of the representatives of The Royal Society of Canada will probably be made at the meeting to be held in the third week of May.
>
> "As soon as these representatives are appointed a meeting of the Committee will be called by Dr. Deville, who has been appointed Convener."

At a meeting held on May 20, 1920, the following four representatives of The Royal Society of Canada were appointed and their names added to the list of members of the National Committee of Canada of the International Union of Geodesy and Geophysics:

Professor L. B. Stewart, University of Toronto
The Seismologist, Dominion Observatory
C. A. French, Magnetician, Dominion Observatory
Professor R. W. Brock, University of British Columbia.

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[1] Presented at the Annual Meeting of the American Geophysical Union in Washington, April 17, 1923.

Dr. Deville was elected Convener of this Committee and his first duties were the preparation of the By-Laws and Regulations governing the work.

A preliminary and organizing meeting of the National Committee of Canada was held at the Geographic Board rooms, Ottawa, on Tuesday, May 25, 1920. Dr. Deville was made Chairman of the National Committee and Mr. C. A. French was made Secretary.

At this meeting Dr. Deville gave a detailed account of the genesis of the International Research Council, the General Assembly of which was held at Brussels during July 1919, and the subsequent formation of the International Union of Geodesy and Geophysics, reading the essential extracts from the constitution and statutes of the Council and of the Union, pointing out especially that the Union was divided into the six following sections: Geodesy, Seismology, Meteorology, Terrestrial Magnetism and Electricity, Physical Oceanography, and Volcanology, and that each of these sections has its own president and secretary, and that each may publish reports, etc.; in short, that each is an entity in itself. It was stated that Canada represented two units in the Union, its population being between five and ten millions, the maximum units being eight for population over twenty millions; and furthermore, that the Research Council had paid the first annual contribution of 5200 francs for Canada to the International Union.

Dr. Klotz amplified the remarks of the Chairman by giving an account of what had been done in the United States in connection with a similar national committee of which he had been supplied with the minutes; in connection with the National Committee there had been formed a permanent American Geophysical Union which, in co-operation with the United States National Research Council, would take part in the International Union.

Before the adjournment of the meeting, Dr. Deville and Dr. Klotz were appointed a committee to draft the necessary Constitution and By-laws for the National Committee of Canada, for subsequent submission to the Committee and then for the approval of the Research Council.

The preliminary draft of the Aims of the National Committee was prepared by the above-mentioned Committee and at a meeting held on May 19, 1921, the Statutes were amended as follows:

STATUTES

The National Committee of Canada constituted by the Advisory Council of Scientific and Industrial Research for the International Union of Geodesy and Geophysics.

ARTICLE 1

Objects

(a) To assist in carrying out the objects of the International Geodetic and Geophysical Union, which are: To promote the study of problems relating to the shape and physics of the earth; to initiate and organize the conduct of researches which depend on co-operation between different countries, and to provide for their scientific discussion and publication; to facilitate particular researches such as the comparison of instruments used in different countries.

(b) To promote and co-ordinate the study of the various branches of geodesy and geophysics in Canada.

(c) To formulate problems encountered in investigational work requiring solution, for presentation through the National Committee to the International Union.

(d) To seek the assistance of others not members of the National Committee in matters pertaining to any of the sections of the Union.

(e) To publish, if found desirable, the activities of the National Committee in order to further research in Canada in the various fields of the Committee of the Union.

ARTICLE 2

Membership

The membership of the National Committee shall consist of the eleven originally provided by the Research Council, and such other active investigators in the above fields who have been proposed to the National Committee, and whose election requires a two-thirds affirmative vote by ballot of the members of the National Committee. Additional members may be elected without calling a meeting of the Committee.

ARTICLE 3

Sections

The National Committee shall be divided into Sections corresponding to those of the International Union, viz: Geodesy, Seismology, Meteorology, Terrestrial Magnetism and Electricity, Physical Oceanography, and Volcanology. Each section may appoint its own Chairman and Secretary.

The members shall designate the Section, or Sections, to which they desire to belong.

ARTICLE 4

Officers

The officers of the National Committee shall be a Chairman and a Secretary to serve until the conclusion of the next General Assembly of the International Union and until their successors have been duly elected.

ARTICLE 5

Appointments

In case of absence of the Chairman at any meeting, the meeting shall appoint a temporary Chairman for such meeting.

ARTICLE 6

Executive Committee

The Executive Committee shall consist of the Chairman and Secretary, and the Chairmen of Sections, if such have been appointed.

The Chairman and Secretary of the National Committee shall be the corresponding officers of the Executive Committee.

ARTICLE 7

Delegates

The National Committee shall nominate a delegate or delegates to the International Union of Geodesy and Geophysics at least six months before the next meeting of the General Assembly.

ARTICLE 8

Amendment

Amendments to the Statutes may be made at a meeting called for that purpose provided notice of such amendment has been sent to the Secretary at least one month before the above meeting.

The question of additional membership and the personnel of the various Sections was taken up at the meeting held on May 10, 1921. It was decided to increase the membership of the National Committee as provided for in Article 2 of the Constitution; also, in order to facilitate the work of the Committee and Sections, to appoint the Chairman and Secretary of each Section. Each new member was elected and assigned to a Section as that particular Section was under consideration, it being assumed that the action of the Committee in assigning the member to the Section would meet with his approval. The members constituting the the different Sections, also the officers, as chosen by the members present, are as follows:

Geodesy

Chairman, Dr. E. Deville, Chairman, National Committee, Ottawa.
Secretary, Noel J. Ogilvie, Superintendent of the Geodetic Survey of Canada, Ottawa.
Professor L. B. Stewart, University of Toronto, Toronto.
W. M. Tobey, Senior Geodetic Engineer, Geodetic Survey of Canada, Ottawa.

Seismology

Chairman, E. A. Hodgson, Seismologist, Dominion Observatory, Ottawa.
Secretary, F. Napier Denison, Meteorological Service, Victoria.
Dr. Otto Klotz, Director, Dominion Observatory, Ottawa.

Meteorology

Chairman, Sir Frederick Stupart, Director, Meteorological Service, Toronto.
Secretary, J. Patterson, Meteorological Service, Toronto.

Terrestrial Magnetism and Electricity

Chairman, Dr. Otto Klotz, Director, Dominion Observatory, Ottawa.
Secretary, C. A. French, Magnetician, Dominion Observatory, Ottawa.
W. J. Stewart, Chief Hydrographer, Department of Marine and Fisheries, Ottawa.
W. E. W. Jackson, Magnetician, Meteorological Service, Toronto.

Physical Oceanography

Chairman, Dr. W. Bell Dawson, Superintendent of Tidal and Current Surveys, Department of Marine and Fisheries, Ottawa.
Secretary, W. J. Stewart, Chief Hydrographer, Department of Marine and Fisheries, Ottawa.

Volcanology

Chairman, Dr. William D. McInnes, Director of Victoria Memorial Museum, Ottawa.
Secretary, Dr. W. H. Collins, Director of the Geological Survey, Ottawa.
Professor R. W. Brock, University of British Columbia.

The membership of the National Committee of Canada of the International Union of Geodesy and Geophysics, including the eleven original members, is at present twenty-one.

The fees for Canada due to the International Union of Geodesy and Geophysics are at present charged to the appropriation of the Geodetic Survey of Canada.

In conclusion I may say that the aims and organization of all the Sections of the National Committee of Canada, namely, Geodesy, Seismology, Meteorology, Terrestrial Magnetism and Electricity, Physical Oceanography and Volcanology, are first to co-operate with other investigators in the British Empire, and to unify and co-ordinate all results with those of the United States and Mexico, so as to build up one complete whole over North America, and secondly, to co-operate as much as possible with the whole of Europe, etc., in the obtaining of uniform results and methods which can be used for scientific purposes.

STATUTES AND BY-LAWS OF THE AMERICAN GEOPHYSICAL UNION

(As amended to May, 1923.)

PREAMBLE

The American Geophysical Union is hereby established by the National Research Council. It supersedes the "American Section of the proposed International Geophysical Union." The American Geophysical Union is the American National Committee of the International Geodetic and Geophysical Union and the Executive Committee of the American Geophysical Union is the Committee on Geophysics of the National Research Council. The American Geophysical Union is constituted as follows:

ARTICLE 1

Objects

(a) To assist in carrying out the objects of the International Geodetic and Geophysical Union, which are: To promote the study of problems concerned with the figure and physics of the Earth; to initiate and co-ordinate researches which depend upon international co-operation, and to provide for their scientific discussion and publication; and to facilitate special researches, such as the comparison of instruments used in different countries.

(b) To promote and co-ordinate the study of the various branches of geodesy and geophysics in the United States of America and its outlying territories.

ARTICLE 2

Membership

The membership of the American Geophysical Union shall be as follows:

(a) Those who composed the membership of the "American Section of the proposed International Geophysical Union" on July 1, 1919.

(b) Those who may be appointed by the Executive Board of the National Research Council upon nominations submitted by the Executive Committee of the American Geophysical Union.

(c) The Chairman of the Division of Foreign Relations, the Chairman of the Division of Physical Sciences, the Chairman of the Division of Chemistry and Chemical Technology, the Chairman of the Division of Geology and Geography, and the Chairman of the Division of Biology and Agriculture, of the National Research Council, members *ex officio*.

(d) The American officers of the International Geodetic and Geophysical Union and of its Sections, members *ex officio*.

ARTICLE 3

Sections

The American Geophysical Union shall be divided into Sections, in accordance with the organization of the International Geodetic and Geophysical Union, as follows: (a) Geodesy; (b) Seismology; (c) Meteorology; (d) Terrestrial Magnetism and Electricity; (e) Oceanography; (f) Volcanology; and, in addition, (g) Geophysical Chemistry.

The members shall designate the Section, or Sections, to which they will belong. The *functions of the Sections,* in their respective fields, shall be the promotion of the objects of the American Geophysical Union.

ARTICLE 4

Officers of the American Geophysical Union

The *Officers of the American Geophysical Union* shall consist of a chairman and a vice-chairman, elected to serve for a period of two years, and not to be eligible for immediate re-election to their respective offices; and a secretary, to serve for a period of three years.

ARTICLE 5

Officers of Sections

The *Officers of the Sections* of the American Geophysical Union shall consist for each Section of a chairman, a vice-chairman, and a secretary, whose periods of office shall correspond to those of the Union. The Chairman of each Section shall have the power to appoint, with the right to vote, a representative of the Section to such meetings of the Executive Committee of the Union as he may be unable to attend.

ARTICLE 6

Executive Committee

The *Executive Committee* shall consist of the chairman, the vice-chairman, and the secretary of the American Geophysical Union, and of one representative from each of the sections of the Union, who shall be the chairman thereof or a member appointed by him; and, in addition, *ex officio,* the chairman of each of the following Divisions of the National Research Council: Foreign Relations, Physical Sciences, Chemistry and Chemical Technology, Geology and Geography, and Biology and Agriculture. Those present shall constitute a quorum. This Committee shall have charge of all administrative matters of the American Geophysical Union, and shall be the representative of the Union in its relations with the National Research Council and with the International Geodetic and Geophysical Union. The Executive Committee, in consultation with the Sections, shall nominate to the Executive Board of the National Research Council delegates, or representatives, to the meetings of the International Geodetic and Geophysical Union and of its Sections, in accordance with the regulations fixed by the International Research Council for the voting power of the United States of America.

ARTICLE 7

Annual Meetings

The American Geophysical Union and its Sections shall meet annually at such time and place as may be chosen by the Executive Committee of the Union. The officers of the Union and of the Sections shall be elected at this annual meeting.

ARTICLE 8

Special Meetings

Special Meetings of the American Geophysical Union, or of its Sections, may be called when necessary by the respective chairmen.

ARTICLE 9

By-Laws

The American Geophysical Union is authorized to enact such by-laws as may be necessary for the performance of its functions and duties.

ARTICLE 10

Amendments

Amendments to the Statutes of the American Geophysical Union shall be subject to the approval of the National Research Council.

BY-LAWS

The fiscal year of the American Geophysical Union shall conform to the fiscal year of the National Research Council.

Standing committees on Statutes and By-Laws, on Membership, on Meetings, and on Budget, shall be appointed for the ensuing fiscal year by the Chairman-elect of the Union prior to the beginning of each year.

All committees of the American Geophysical Union shall terminate at the end of the fiscal year for which they may be appointed.

Meetings of the Union shall be called at least thirty days in advance, and those present at the meeting shall constitute a quorum.

Meetings of the Executive Committee shall be called by the Chairman, and those present shall constitute a quorum.

Final action shall not be taken by the Executive Committee on matters affecting any Section in the absence of the Chairman of such Section, or his representative.

No mail ballot shall be closed until at least thirty days after the date of issue.

After thirty days' notice has been given, amendments to the Statutes (all of which are subject to the approval of the National Research Council) may be voted on either by mail or at a regular meeting of the Union.

Amendments to the By-Laws may be proposed either by mail or at a regular meeting; when voted on by mail they require for adoption a majority vote of the members; when voted on at a regular meeting they require for adoption a three-fourths vote of the members present.

Any amendment to the Statutes or By-Laws proposed by at least five members shall be submitted for vote to the members of the Union by the Executive Committee.

A member of the American Geophysical Union may be dropped from the Union by a three-fourths vote of the members of the Executive Committee.

Statutes and By-Laws adopted by a Section shall not conflict with the provisions of the Statutes and By-Laws of the Union.

Matters pertaining to a Section or Sections, and not pertaining to the Union as a whole, shall be acted upon by the Section or Sections concerned.

REPORT OF MEETING OF THE INTERNATIONAL GEODETIC AND GEOPHYSICAL UNION AND ITS SECTIONS HELD AT ROME, MAY 2–10, 1922, PREPARED BY DELEGATES FROM THE UNITED STATES

GENERAL CONFERENCE OF THE INTERNATIONAL GEODETIC AND GEOPHYSICAL UNION

WILLIAM BOWIE

The first general conference of the International Astronomical Union and of the International Geodetic and Geophysical Union, since their establishment at Brussels in July 1919, was held at Rome, Italy, May 2 to 10, 1922. The meetings of the two Unions and of their various Sections were held in the Reale Accademia dei Lincei. About three hundred delegates and guests, representing about twenty countries, attended the various meetings.

The first day was occupied by the registration of delegates and the opening of the conference, which was held at the Campodoglio, at which were present the King of Italy and other officials. On the second day there were meetings of the two Unions at which various committees were appointed to consider certain questions of administration and policy. On the last day of the conference the Unions also met to receive reports of committees and to conclude the work of the conference. From May 4 to 9, 1922, inclusive, there were held meetings of the several sections of the International Geodetic and Geophysical Union and of the Committees of the Astronomical Union.

The principal matters coming before the Union as a whole were finance and changes in the statutes and organization of the Union. There was complete harmony among the delegates as to the report of the Committee on Finance, dealing with the apportionment among the sections of the quotas of the nations adhering to the Union.

On the question of changing the statutes some decided differences of opinion arose, especially on the question of whether the designation "Section" should be changed to that of "Association" or to some other term. It was the opinion of the American delegates that the term Section does not carry as much authority nor give quite the idea of importance of the several groups of scientists coordinated into the International Geodetic and Geophysical Union. The conference decided that the matter of changing the designation of the Sections should be left to the action of the International Research Council which was to hold a meeting in July 1922.

It was decided, with very little opposition, to recommend to the

International Research Council that the countries adhering to the Reduced Geodetic Association should be allowed to join the Section of Geodesy only, without adhering to the other Sections of the Union. It was also decided that a new Section called Section of Scientific Hydrology, should be added to the Union.

Although there were many advantages in having the two Unions meet at the same time and place it was the feeling of many delegates that the time was entirely too short to accomplish all that was desired by the Sections of the International Geodetic and Geophysical Union and by the various Committees of the International Astronomical Union. Many of the delegates were accredited to both Unions and, necessarily, two or more meetings in which they were interested were frequently held at the same time.

It was decided by the officials of the International Geodetic and Geophysical Union that they would bring before the Union the question of having a meeting in 1924, in order that, thereafter, the three-year interval would bring the meeting of the two Unions in different years. At the close of the meeting of the International Geodetic and Geophysical Union it was decided that the next meeting should be held in 1924 at Madrid, Spain. Representatives of Spain had kindly invited the Union to meet in that city.

As usual, in international scientific conferences social features played a prominent part. The most important of those at the Rome meeting were the inaugural ceremony, at the Campodoglio, mentioned in the early part of this report, the reception at the same place by the municipality of Rome, on the evening of May 4, the visit to the Palatino at the invitation of the Secretary of Antiquities and Fine Arts on the afternoon of May 8 and the visit to the Vatican and the audience with the Pope, on May 10. There were a number of special excursions during and after the meeting, especially one to a number of volcanoes after the close of the conference. At the closing session of the Union appropriate resolutions were adopted, expressing the appreciation of the delegates to those individuals and organizations who by their efforts had made the conference so successful and pleasant.

The following table shows the officers of the International Geodetic and Geophysical Union and of its Sections:

Officers of the Union

President: Ch. Lallemand
Vice Presidents: Presidents of the Sections
Secretary General: H. G. Lyons

Officers of the Sections

Section	President	Vice President	Secretary
Geodesy	W. Bowie	R. Gautier	G. Perrier
Seismology	H. H. Turner	E. Oddone	E. Rothé
Meteorology	Napier Shaw	C. F. Marvin E. Delcambre	F. Eredia
Terrestrial Magnetism and Electricity	C. Chree	L. Palazzo	Louis A. Bauer
Oceanography	Prince of Monaco[1]	J. Parry (Tidal Commission) Odon de Buen (Atlantic Commission) G. W. Littlehales (Pacific Commission)	G. Magrini
Volcanology	A. Lacroix	H. S. Washington	A. Malladra G. Platania
Scientific Hydrology	E. B. H. Wade	A. Wallén	G. Magrini

U. S. Coast and Geodetic Survey,
Washington, D. C.

REPORT OF THE SECTION OF GEODESY

WILLIAM BOWIE

As is well known, the geodesists of the world have maintained for a number of years an association called the International Geodetic Association. At its last meeting at Hamburg, Germany, in 1912, the fiftieth anniversary of the Association was celebrated. That Association had held triennial conventions for the purpose of bringing together representative geodesists from the twenty or more adhering countries.

That the International Geodetic Association and its conferences were of great benefit to the science of geodesy is indicated by the fact that practically all of the geodetic work done throughout the world since the organization of the Association has been of such a high standard that it has not had to be duplicated. These beneficial results have been largely due to the personal contact of the geodesists with each other at the triennial conventions and to the efforts of committees and rapporteurs dealing with the various branches of geodesy, and of the officials of the association.

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[1] The President died soon after the meeting.

The Section of Geodesy of the International Geodetic and Geophysical Union, which was created at Brussels in 1919, was designed to take the place of the old International Geodetic Association which had been greatly reduced as a result of the war. The old Association had been perpetuated in 1917 by seven neutral countries of which the United States was one; the others were Norway, Sweden, Denmark, Holland, Switzerland and Spain. After the United States entered the war, the Association was composed only of the other six countries mentioned. Too much credit cannot be given to those countries for preventing a break in international co-operation in geodetic affairs during the war. Especial credit is due to the president of the reduced Association, Dr. Raoul Gautier, and the late Dr. H. G. van de Sande Bakhuyzen, the Secretary.

As a result of correspondence of the President and Secretary of the Section of Geodesy with the officials of the reduced Geodetic Association, each one of the countries forming that Association had representation at the Rome meeting in May, 1922.

The Section of Geodesy held almost continuous sessions from the third day of the conference to and including the next to the last day. The first two days and the last day of the conference were devoted to the opening exercises and the meetings of the International Astronomical Union and the International Geodetic and Geophysical Union.

An Agenda had been prepared for the meeting of the Section of Geodesy which, with much explanatory material showing what had been done by the officials of the Section since the organization of the Section at Brussels, was furnished in printed form to the delegates at Rome. The Agenda was very strictly adhered to in the proceedings of the Section, and all that had been projected was accomplished, although there was much crowding of events owing to the limited time available.

At the first session of the Section, a number of committees were appointed, which dealt with questions involving cooperation of geodesists in two or more countries. There was also provided a committee, which should meet with a committee of the International Astronomical Union, to consider the question of the international variation of latitude work. Another committee was appointed to deal with the finances of the Section.

The meetings of the Section of Geodesy were well attended with from 25 to 50 persons present. Committee meetings which, in some cases, had to be held during the sessions, cut down the attendance at various times; besides, some of the geodesists were also representatives of their countries to the International Astronomical Union and to other sections of the Geodetic and Geophysical Union and, necessarily, they were not able to devote all of their time to the Geodetic Section.

The statutes and by-laws of the International Geodetic and Geophysical Union were adopted for the guidance of the Section of Geodesy.

One of the most serious questions which came before the Section was the character of its organization but, after much thought and unselfish effort on the part of a committee appointed on the statutes and organization, a plan was drawn up which was submitted to the Section and adopted without modification. This plan provides that there shall be a president, vice-president and secretary who shall form the Bureau of the Section. These officers together with four others elected by the Section at a general conference will constitute an Executive Committee to carry out the will of the Section, as expressed in its acts at the general conferences, and also to carry out the will of the Permanent Commission.

It was provided that there should be a Permanent Commission of the Section of Geodesy, which should be composed of a single representative of each of the states adhering to the Section. This Commission would have as its principal function to act for the Section on legislative matters which might appear to be necessary during the interval between two general conferences.

The joint committee of the Section of Geodesy and of the International Astronomical Union, on the variation of latitude work, reported that unanimous agreement had been reached, that the latitude work should be conducted by the Section of Geodesy but that there should be a joint committee of the Section of Geodesy and of the Astronomical Union, to consider the various scientific phases of the subject. It was also provided that the Astronomical Union should make contribution towards the reduction, discussion and publication of the results.

The writer of this report made the statement at one of the sessions of the Section of Geodesy that he was authorized to propose that, if the property of the reduced Geodetic Association, at the two variation of latitude stations in the United States, Ukiah, California, and Gaithersburg, Maryland, were transferred to the U. S. Coast and Geodetic Survey, every effort would be made to secure the authority of Congress for the Coast and Geodetic Survey to continue the observations in this country without expense to any international body, provided that the stations in Japan and Italy should be maintained without international expense by those two countries. The representatives of Japan and Italy agreed to this proposition provided, of course, their countries would confirm their action. Since the Rome meeting the Director of the U. S. Coast and Geodetic Survey has been in communication with the Secretary of the Section of Geodesy, making efforts to put into effect the plan proposed at Rome.

At the last session of the Section of Geodesy, reports were received from the various committees which had been appointed at the beginning of the conference, and such action was taken on the reports as seemed desirable. It is not necessary to refer to the Committee reports, except in so far as has already been done in the early part of this statement for there will be given a complete report of the proceedings at Rome in the *Comptes-Rendus,* now being prepared by the Secretary.

The Section of Geodesy elected as its vice president Raoul Gautier who since the death of General Bassot of France, in January, 1917, has been president of the reduced Geodetic Association. The four members elected to the Executive Committee are H. J. Heuvelink, G. P. Lenox-Conyngham, Paul Stroobant, N. Vacchelli.

William Bowie of the United States will continue as President of the International Section and G. Perrier of France will continue as Secretary for another term.

It was provided that there should not be any Central Bureau, such as had existed in the old International Geodetic Association. To replace the Central Bureau, provision was made for rapporteurs for the several branches of geodesy. The rapporteur for a subject would have as his duties the collection of information from the geodesists of the world and the furnishing of information and advice to such as might desire it from the officials of the Section, and the making of a report at the general conventions on the status of the subject assigned him and the progress which has been made since the preceding conference.

The following rapporteurs were appointed at the Rome meeting: Bases and Triangulation, G. Perrier; Precise Leveling, Ch. Lallemand; Geodetic Astronomy (latitude, longitude, azimuth), to be designated by Ordnance Survey; Deviation of Vertical, H. DeGraaff Hunter; Intensity of Gravity, E. Soler; Isostasy, W. Bowie; Projections, H. Roussilhe; Earth Tides, H. G. Gale; Stability of the Earth as a Timepiece, E. W. Brown.

U. S. Coast and Geodetic Survey,
Washington, D. C.

REPORT OF THE SECTION OF SEISMOLOGY

HARRY FIELDING REID

The meeting of the Union took place in Rome, May 2-10, 1922, at the headquarters of the Reale Academia dei Lincei.

The Section of Seismology, which was not organized in Brussels in 1919, on account of possible complications with the International Seismological Association, was organized on May 4. Professor H. H. Turner, of Oxford, England, was elected President; Signor Emilio Oddone, of Rome, Italy, Vice-President; and M. Rothe, of Strasbourg, France, Secretary and Director of the Central Bureau, which it was decided to maintain at Strasbourg.

The section is to receive 27,000 francs annually from the Union, a sum far inferior to the income of the former International Seismological Association (about 51,000 marks). Its activities will therefore be much restricted. To continue the cataloguing of important earthquakes recorded by seismographs, 10,000 francs annually were voted to Professor Turner, who will have to supplement this sum (by probably an equal amount) from other sources. The remainder of the income, 17,000 francs, was voted to the expenses of the Central Bureau: they will be used to publish the proceedings of the meetings, to disseminate seismological information by wireless, to carry on the general work of collaboration of the Central Bureau, and, if possible, to publish some memoirs; but the activities of the Section will necessarily be more restricted than were those of the old Association.

Several committees were appointed: (1) to work out a practical system of rapid transmission of seismological information (2) to consider the transmission of earth waves due to explosions (3) to consider a means of expressing the usual scales of earthquake intensities in dynamic units (4) to study the cause of microseisms (5) to study the depths of earthquake foci.

Three seismological papers were read and discussed. M. Somville (Belgium) gave a simplified method of determining the constants of a Galitzin seismograph, Signor Lo Surdo (Italy) explained an easy method of determining experimentally the intensity of earthquake vibrations in dynamical units, and of correlating this intensity with the Rossi-Forel, the Mercalli, or any other similar scale; and Mr. Reid (United States) showed that the friction of seismographs registering mechanically could be taken into account satisfactorily more accurately and more simply than by Galitzin's theory.

An interesting excursion was made by the Section to Rocca di Papa, the principal Italian seismological observatory. A luncheon was given by the Mayor of the town, and Professor Agammenone, in charge of the observatory, received us most cordially and explained the instruments and the work he is carrying on.

Johns Hopkins University,
Baltimore, Maryland.

REPORT ON THE DISSOLUTION MEETING OF THE INTERNATIONAL SEISMOLOGICAL ASSOCIATION, STRASBOURG, APRIL 24-25, 1922

HARRY FIELDING REID

The delegates met at the Institute de Physique du Globe, Strasbourg, April 24-25, 1922. The following allied nations were represented by delegates: Belgium, Brazil, Canada, France, Great Britain, Japan, Norway, Portugal, Spain, Switzerland, United States.

Dr. Hecker (Germany), former director of the Central Bureau of the Association, and Professor de Kovesligethy (Hungary), former secretary were present. Several other states of Central Europe were represented by proxy.

On account of the death of Prince Galitzin, the president of the Association, and the illness of Professor Lecointe, the vice-president, Sir Arthur Schuster, a former president, was elected to preside.

It was decided that the International Seismological Association came to an end on March 31, 1916, that being the end of the period determined by the original convention. It seems that some States have not paid their quota to that date, whereas others have paid their quota for some succeeding years. It was decided to ask all States in arrears to March 31, 1916, to make up their arrears; and to return to all States which have paid quota beyond that date the sums so paid. If the member States agree to this plan and act accordingly there will remain in the treasury of the Association about 18,400 francs. (It is not certain that all arrears can be collected.) It was decided to pay to the former secretary, Professor de Kovesligethy the sum of 5000 francs for the very considerable work devolving on him in closing up the affairs of the Association, and to use the remainder for printing and distributing the proceedings of the Dissolution Meeting, and in meeting other small expenses incident to the dissolution.

It was decided to transfer to the Seismological Section of the International Geodetic and Geophysical Union, the library and scientific documents, and other properties of the Association. The "other properties" are unimportant.

The decisions of the meeting were adopted unanimously. In order to make these decisions effective, it is necessary that the member States ratify them. I think them eminently fair and practical, and I strongly hope that our State Department will ratify them. They offer a simple solution of what might have been a difficult matter. Our country has paid its quota to the Association only to March 31, 1915, and therefore according to its undertaking when it joined the Association, and in accordance with the action of the Dissolution Meeting, it still owes one year's quota, April 1, 1915 to March 31, 1916. The amount is 4000 francs, which, at the present rate of exchange is but little over three hundred dollars. Our annual quota before the war was 3200 marks, or about $800.

Johns Hopkins University,
Baltimore, Maryland.

REPORT OF THE SECTION OF METEOROLOGY

HERBERT H. KIMBALL

Meetings were held daily, except Sunday, from May 4 to 9, inclusive, and in addition a joint meeting with Section (D), Terrestrial Magnetism and Electricity, was held on May 5. Sir Napier Shaw of England presided at all the meetings. In the absence of Professor Marvin, the Secretary, Dr. Kimball, the delegate from the United States, was asked to serve as Secretary pro tem; and Professors Platania and Eredia, delegates from Italy, were designated Assistant Secretaries.

In all, thirty-three delegates were in attendance, representing most of the allied nations, and a few that were neutral, during the world war.

The Section particularly concerned itself with two subjects: (1) its relations with the International Meteorological Committee, and the International Conference of Directors of Meteorological and Magnetic Institutions, and (2) plans for international cooperation on certain research problems.

With reference to (1) it was agreed that all questions of a routine or an administrative character should be referred for consideration to the International Meteorological Committee, the Section confining its activities to scientific problems the solution of which calls for international cooperation. Furthermore, the International Conference of Directors of Meteorological and Magnetic Institutions, which is to meet in Holland in 1923, was asked to consider if there is a duplication of work in its functions and those of the Union.

With reference to (2) resolutions were adopted providing for certain researches, the procedure for the carrying out of which will be outlined later by the Bureau of the Section. These researches include the following:

(1) The determination of the quantities of hydrogen in the atmosphere in different localities and at different heights.

(2) The exploration of the upper atmosphere by means of sounding balloons in regions which are of especial importance from the point of view of the general circulation of the atmosphere, and which have not at the present time an effective meteorological organization for carrying on work of this kind.

(3) The examination of the dust in the atmosphere in different countries.

(4) A report on our knowledge of the radiation from the sun, the soil, the sea, and the air, from the point of view of the general circulation of the atmosphere.

(5) A report from the several countries affiliated with the Union on the methods they employ in weather forecasting.

(6) A study of the details of convection in the atmosphere.

(7) Commendation of the work on solar radiation carried on by the Astronomical Observatory of Naples, and by the Astrophysical Observatory of the Smithsonian Institution of Washington.

The budget of the section was approved as follows:

General expenses	5,000 francs
Exploration of the air by sounding balloons	20,000 francs
Exploration of the stratosphere by pilot balloons	5,000 francs
Examination of the dust of the atmosphere	2,500 francs
Total	32,500 francs
Allotment	32,542 francs

The Bureau of the Section is constituted as follows:

President,	Sir Napier Shaw,	England.
Vice President,	Professor C. F. Marvin,	United States.
Vice President,	Colonel Delcambre,	France.
Secretary,	Professor F. Eredia,	Italy.

The Executive Committee consists of the Bureau, and the following:

Dr. G. C. Simpson,	England.
M. Alex Wallen,	Sweden.
Professor F. Gamba,	Italy.

U. S. Weather Bureau,
Washington, D. C.

REPORT OF THE SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY[1]

Louis A. Bauer

A well-attended meeting of the Section of Terrestrial Magnetism and Electricity of the International Geodetic and Geophysical Union was held at Rome from May 4–9, 1922. During the seven sessions, twelve different countries were represented by thirty-three delegates and guests, viz:

J. M. Baldwin, G. F. Dodwell, E. F. Pigot (Australia); J. Jaumotte (Belgium); H. Morize (Brazil); J. Bosler, H. Deslandres, E. Delcambre, E. Mathias, Ch. Maurain (France); C. Chree, A. L. Cortie, W. C. Parkinson, Arthur Schuster, Napier Shaw, G. C. Simpson (Great Britain); A. Alessio, F. Eredia, L. Palazzo, D. Pacini, G. Platania, A. Pochettino, G. B. Rizzo, C. Somigliana (Italy); K. Nakamura (Japan); J. Krassowski (Poland); J. Galbis, L. Rodés (Spain); D. Stenquist, A. Wallen (Sweden); L. A. Bauer, G. W. Littlehales (United States); E. Deville (Canada).

Professor Tanakadate, who was unable to be present at Rome, had requested to be relieved of the presidency of the Section, because of his inability to attend to the duties. The resignation was regretfully accepted and Dr. Charles Chree, who as vice-president had presided at all sessions, was elected, at the closing session on May 9, president, and Professor Luigi Palazzo, vice-president; both of these officers, according to the statutes, serve for two terms. The secretary and director of the Central Bureau, Dr. Louis A. Bauer, continues in office until the next meeting, which will be at Madrid, Spain, about September 1924. Directors J. Jaumotte (Belgium) and Ch. Maurain (France), and Professor A. Tanakadate (Japan), in addition to the three officers of the Section, were constituted the Executive Committee. It was agreed that administrative matters should be left to the Bureau, consisting of the officers of the Section.

Since the meeting at Brussels in July 1919, when the International Section of Terrestrial Magnetism and Electricity was established, nearly three years have elapsed. While the organization of the work of the Section, because of the post-war conditions, could not proceed as rapidly as it was hoped, nevertheless definite progress has been made regarding which the Agenda (Ordre du Jour) for the present meeting were at least one indication.[2] Perhaps for the first time we have had presented in so concrete a form the salient questions, both practical and theoretical, pertaining to the magnetic and electric states of our Earth and its atmosphere. It was not to be expected, nor indeed, was it desirable, that definite decisions on all the questions should be reached at that meeting. However, it must be a source of gratification that by the united action of the National Committees the crucial questions and problems have received a definite formulation.

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[1] The full report has appeared in a volume published by the International Section and is entitled "Transactions, Section of Terrestrial Magnetism and Electricity, Rome, 1922"; it is designated as "Bulletin No. 3." A general report, published in the September 1922 issue of *Terrestrial Magnetism and Electricity*, was reprinted as "Bulletin No. 2," and a copy was sent to each member of the American Geophysical Union.

Reports were presented showing the status of magnetic and electric work in the various countries represented, and containing the opinions of National Committees, leading organizations, and investigators on items of the Agenda. There were, furthermore, reports from committees constituted at the Brussels meetings, as also reports and letters expressing the views of some, whose countries either were not officially represented at Rome (Greece, New Zealand), or did not yet belong to the International Geodetic and Geophysical Union. Among the latter there were letters from E. van Everdingen (Holland); G. Reinius, D. Stenquist, and V. Carlheim-Gyllensköld (Sweden); C. Störmer (Norway); G. Melander (Finland); and C. Ryder (Denmark).

On the basis of all information on hand and the ensuing discussions on items of the Agenda, twenty Resolutions[3] were passed. The Executive Committee was empowered to formulate more definite recommendations on some of the mooted questions of procedure, especially at magnetic observatories, as soon as further information has been received by the secretary from all service and observatories engaged in magnetic or electric work, in response to a questionnaire sent out.

Five committees were appointed:[4] (1) Committee on Magnetic Surveys and International Comparisons of Instruments; (2) Committee on Observational Work in Atmospheric Electricity to report on Objects, Instruments, and Methods; (3) Committee on Measures of Magnetic Characterization of Days; (4) Committee on Best Methods, Instruments and Compilations for Polar Light Observations; and (5) Committee to Consider and Report on Best Methods and Instruments for Earth-Current Observations. The provisional organization of these committees was undertaken. The Executive Committee was empowered to add to the membership of committees, as additional countries join the Union, and to form any additional committees deemed necessary to put into effect the Resolutions. Provision has also been made for consideration of questions concerning the relationships between solar and the Earth's magnetic and electric phenomena in the following manner: (1) A committee on solar radiation, under the chairmanship of Dr. George E. Hale, of which Dr. Bauer is a member, was formed by the International Astronomical Union; (2) The International Research Council at its Brussels meeting in July 1922 has decided to assist in initiating studies of the correlations between solar and terrestrial phenomena, this being a subject of the kind in which several unions would be interested.

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[2] See *Terr. Mag.*, vol. 26, pp. 151–152, 1921 for English text; the French text will be found in the reprinted general report (Bulletin No. 2).

[3] See reprinted general report (Bulletin No. 2), or "Transactions" (Bulletin No. 3, pp. 174–177).

[4] The representatives of the American Geophysical Union on the international committees are: (1) Louis A. Bauer, chairman, and N. H. Heck; (2) W. F. G. Swann; (3) R. L. Faris; (4) J. A. Fleming; (5) S. J. Mauchly, secretary.

The balance, 30,892 francs, of the accumulated funds in hand, arising from the contributions, 1919–1922, of the adhering countries at the rate of 400 francs per contributing unit, was made available to the International Section. This is in addition to the sum of about 1,500 francs which had been supplied in 1921 for incidental expenses. There will, furthermore, be available during the period of 1922–1924, from the contributions of adhering countries, annually an amount at the rate of 320 francs per contributing unit. It is estimated that the annual amount from this source will be about 22,000 francs. Hence the International Section will have available for its various purposes during the period 1922–1924, in all about 75,000 francs,[6] which is subject to increase as additional countries join the International Geodetic and Geophysical Union. With the aid of the funds thus available it is hoped that matters of international concern and benefit may be energetically pursued, as, for example, frequent intercomparisons of magnetic standards, the chief expense of which during the past sixteen years has been borne by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. The Secretary and Prof. E. Mathias, as alternate, were made members of the Finance Committee of the Union.

The Executive Committee was authorized to incur the necessary expense for the publication, in the most suitable form, of the minutes and proceedings of the Rome meeting and of the various reports received, as well as for the issue of any additional publications which may be found desirable and which the available funds may permit. It is estimated that the volume of proceedings will approximate 200 octavo pages, and that it may be distributed in 1923.[5] By means of bulletins, issued from time to time, it is hoped that prompt information may be given regarding matters of international interest, progress on mooted questions, action taken and recommendations by the Executive Committee, and other committees of the Section, and National Committees, latest values of the magnetic and electric elements at observatories, and so forth.

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[6] Approximately, according to present rate of exchange, about 4,000 United States dollars.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

REPORT OF THE SECTION OF PHYSICAL OCEANOGRAPHY[1]

G. W. LITTLEHALES

At the first assembly of the Section, which took place on Wednesday, May 3, 1922, confirmation was given to the recommendations of the Executive Committee for the appointment of three commissions to be called, respectively, the Commission of the Atlantic, the Commission of the Pacific, and the Commission of the Mediterranean.

The proposal of the Executive Committee that, in addition to the president or chairman of the Tidal Commission, the presidents or chairmen of these three commissions should be vice-presidents of the Section was adopted.

Your delegate was elected Vice-President of the Section and President of the Commission of the Pacific, and was continued in membership in the Commission of the Atlantic and its sub-commission and also in the Tidal Commission. Steps have been taken to organize the Commission of the Pacific in harmony with the Committee on Pacific Investigations of the Division of Foreign Relations of the American National Research Council.

The Commission of the Mediterranean was continued as constituted in November, 1919.

Indorsement was given to the proposal of Senator Volterra, for the publication in the Bulletin of the Section the *Comptes Rendus* of transactions in physical oceanography throughout the world.

The proposal of Professor Magrini, for the formation of a new section in the International Union of Geodesy and Geophysics to be called the Section of Continental Hydrography was referred to a special committee of which the American delegate to the Section of Physical Oceanography was a member. The proposal was opposed under instructions from the American delegation. This opposition occasioned the modification of the proposal of Professor Magrini into a new proposal for the creation of a Section of Scientific Hydrology, which was adopted by the Committee of the Union appointed to consider and report upon the original proposal.

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[1] A detailed report of the proceedings of the International Section of Physical Oceanography at its meeting in Rome May 2–10, 1922, is soon to be published by the Secretary of this Section, in connection with the reports of the other Sections of the International Union of Geodesy and Geophysics and with the report of the proceedings of the Union itself. Accordingly the present statement touches only upon such transactions as are entitled to anticipatory mention.

The committee appointed to prepare an International Glossary of Nomenclature and Terms used in Hydrography and Oceanography reported an advanced stage of progress.

A new committee was appointed to examine into existing international conventions which might aid researches in physical oceanography.

The International Hydrographic Bureau was brought into connection with the International Section of Physical Oceanography by the election, as a Vice-President of the Section and Chairman of the Tidal Commission, of Sir John Parry, President of the Directing Committee of the International Hydrographic Bureau. This proceeding followed the resignation of Professor Lamb as Vice-President of the Section and Chairman of the Tidal Commission.

The Tidal Commission reported in favor of introducing, into the title-note concerning tides on nautical charts, a table of harmonic constants for the computation of the time of tide instead of the usual statement of the Establishment of the Port or High Water at Full and Change, especially in these regions where declinational tides are predominant.

At the final meeting of the Section on May 10, in connection with the passage of the budget for the ensuing fiscal year, provision was enacted for the annual expenditure of 5,000 francs in each of the ensuing five years for the preparation and printing of the International Manual of Oceanography.

U. S. Hydrographic Office,
Washington, D. C.

REPORT OF THE SECTION OF VOLCANOLOGY

H. S. Washington

I have the honor to report as follows on the meeting of the Section of Volcanology held at Rome, Italy, May 2–10, 1922. It will be pertinent to state that I have not yet received any copy of the minutes of the Proceedings made by the Second Secretary, Prof. Gaetano Platania, of Catania. I understand that he met with a somewhat serious accident, which has evidently delayed him. I must consequently rely on my memory and on some scanty notes made by me. The present report must, therefore, be considered as subject to correction in the future.

The first meeting of the Section of Volcanology was held at the rooms of the Accademia dei Lincei, in the Corsini Palace, at Rome, on May 4. Those present were: Messrs. Lacroix (France); Malladra, Millosevich, Nasini, Novarese, Oddone, Gaetano Platania, and Somigliana (Italy); Iwawate (Japan); and Washington (United States). Prof. Lacroix was elected Chairman of the Section for a term of three years, and Dr. Washington was elected Vice-Chairman for a like period. Dr. Malladra was elected First Secretary to fill out two years more of his term, and Prof. Gaetano Platania was elected Second Secretary. In the absence of Prof. Malladra after the first day, Prof. Platania acted as Secretary during the whole meeting.

A general discussion of the agenda which had been proposed for consideration took place. These agenda were:

1. International co-operation in the sending out of news and information regarding new eruptions.
2. International co-operation on the obtaining of data regarding submarine eruptions and earthquakes, and the examination of ships' log books for records of them.
3. The subject of deep borings at volcanoes. During the discussion of this topic an interesting account of the Larderello Works in the development of power from subterranean heat and steam was given by Prof. Nasini and one of the engineers of the company.
4. International Catalogue of Volcanoes.
5. Study of thermal gradients.

Other matters discussed were: The publication of a Bulletin of Volcanology; what should be understood by the term "active volcano"; an international bibliography of the active volcanoes of each country; the establishment of a central volcanological library. Further consideration of these matters was laid over until the next meeting.

The next and last meeting of the Section took place on May 9, the same persons being present, except Dr. Malladra. Discussion of the topics mentioned above took place; it was finally decided that the formulation of definite proposals and decisions be left to a Committee composed of Messrs. Lacroix, Malladra, and Washington, who were to meet later at Vesuvius. Announcement was made by the chairman that the annual income of the Section would be about 20,000 francs, derived from about 70 units of the International Geodetic and Geophysical Union, each unit being 260 francs. After the usual votes of thanks the meeting then adjourned, until the next meeting to be held in Madrid, Spain, in 1924.

After the meeting an excursion was made to the active volcanoes of Italy by Prof. and Mme. Lacroix, Prof. and Signora Gaetano Platania, Prof. Oddone, Prof. Iwawate, Dr. Washington, and several others. The trip lasted from May 12 to 22 and included an excursion around Etna by rail and motor car, the ascent of the volcano and the spending of the night at the top. The Aeolian Islands were visited, by the courtesy of the Ministry of Marine, who placed a torpedo boat at our disposal. A night was spent on Lipari, the volcano of Vulcano was visited, and a night circuit gave a splendid view of the activity of Stromboli, and the ruins of Messina were seen. On the return the crater of Vesuvius and the Observatory were visited by several members of the party.

Messrs. Lacroix and Washington met Prof. Malladra at the Observatory of Vesuvius on May 21, and held the Committee meeting; at this meeting Prof. H. F. Reid was also present by invitation. It was agreed that:

1. The Central Bureau of the Section of Volcanology should be at the Vesuvius Observatory, with an office in Naples, to be in charge of Prof. Malladra, as First Secretary; a subsidiary bureau to be at Catania, under the charge of Prof. Gaetano Platania.

2. It was recommended that, if it can be arranged, a Bulletin des Publications Volcanologiques be established, devoted especially to the recording of volcanic eruptions and related occurrences, including hot springs, mofetti, volcanic earthquakes, the petrology and mineralogy of volcanoes, etc. It was understood that by the term "active volcanoes" those in eruption in Quaternary and Recent times be meant.

3. The publication of an International Catalogue of Active Volcanoes was recommended. It was suggested that a committee from the volcanologists of each country should take charge of the cataloging of the volcanoes of its own territory, as well as of the bibliography, as suggested in the next topic. The details of what is to be included in the catalogue were discussed, and it was agreed that each member of the present committee should make some trials as to the best procedure.

4. The publication of a Bibliography of Volcanoes was recommended, a committee from each country to deal with its own volcanoes. To determine the best form, etc., Prof. Malladra undertook to prepare a bibliography of Vesuvius, and the other two members also agreed to undertake bibliographies of some special volcanoes, such as those of Reunion or Pelee by Lacroix, and those of Hawaii by Washington.

5. It was recommended that suggestions be made to the marine authorities of the various countries as to the advisability and scientific value of issuing instructions or requests to naval and merchant marine officers and to cable companies to report on the occurrence of submarine eruptions, volcanic eruptions in little-visited islands and coastal districts, and such phenomena. It was also recommended that attempts

be made to collate the log books of the various national navies and merchant marines for such data, if possible by some official. All such data should be reported to the central bureau at Naples.

6. The international communication of news of volcanic eruptions and the construction of a code for this purpose was discussed, but no definite action was taken.

7. It was recommended that central volcanological libraries be established at the Vesuvius Observatory, at the Geodynamic Observatory at Catania, and at the Volcano Observatory at Kilauea. To these, authors are requested to send copies of all their publications (past, present, and to come).

The meeting then adjourned.

Geophysical Laboratory,
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Washington, D. C.

STATUS OF PROBLEMS IN GEOPHYSICS

RECENT PROGRESS IN GEODESY

JOHN F. HAYFORD

Let us consider the present status and scope of geodesy and look at its problems with a view to making the best progress possible in the near future. The recent past to be brought under review should not be limited to a single year, but to a few years. Similarly our attempt to look into the future should not be limited to the one year until we meet here again.

In recent time there has been a continually increasing application of the results of geodetic surveys to the control of other less accurate surveying and to mapping. There has been a steady improvement in instruments and methods in spite of the fact that these were already very highly developed. Geodesy is once more internationally organized. The available store of fundamental data of geodesy is being increased rapidly. In North America the extension of the area over which there is continuous triangulation all reduced to one datum has been, and is now, very rapid. Geodesy is developing many contacts with other fields of science, by discussion and by action. The discussions of isostasy have been given new life by the realization on the part of many men that isostasy brings new tests to their hypotheses in other fields of thought than geodesy and that their observations in other fields shed light upon the probable mechanism of isostatic readjustment.

This summary statement gives a general view. Let us now consider the separate items of the statement with more detail in order that we may see the matters more clearly, though in doing so we lose some of the better perspective of the compact summary.

With each passing year the statistics of routine operations show that the accurately determined elevations, lengths, directions, and positions given by geodetic surveys, are being used more and more extensively for the control of other surveys. This is the practical application of geodesy. Geodetically determined elevations, lengths, directions, and positions are used to a steadily increasing extent in the control of topographic surveys, which culminate in topographic maps. Recently, also, there has been a strong tendency to use geodetic control to an increasing extent for city surveys which culminate in cadastral surveys and in engineering estimates of proposed projects as well as in maps. Extensive practical applications of the results of any science tend greatly to bring a strong financial support to that science which in turn tends to invigorate the abstract, apparently non-practical, part of the science. That has happened in North America in the past few years.

Geodesy is an old well-cultivated field of science. The instruments and methods of geodesy were developed to a very high degree of perfection before the year 1900. Yet in the decade which is just past, there has been a steady improvement in instruments and methods. Much of this improvement has been intended primarily to decrease the cost and to increase the speed of the observations; as, for example, the improvements in signal lights used on triangulation, in the use of invar tapes in base measurement, improvements in the details in precise leveling, and the introduction of apparatus to record wireless time-signals in observations of gravity and in longitude determinations. Such improvements contribute strongly to an ultimate increase in the accuracy of the final conclusions of geodesy. By merely increasing the *number* of deflections of the vertical and of gravity determinations of a given degree of accuracy, the accuracy and reliability of the conclusions reached are much increased. With increase in the amount of precise leveling there goes normally an increase in the number of closed circuits and a decrease in their size, and this in turn leads to a considerable increase in accuracy of the final results even though there is no change in accuracy in the observing on the separate lines of the net. The application of wireless signals to longitude determinations has made it possible to secure determinations of the highest degree of accuracy in regions, such as the greater part of Alaska for example, in which, formerly, one was forced to use the crude methods of transportation of chronometers or even of moon-culminations.

The crying demand that geodesy furnish a reasonably accurate and reliable method and instrument for determining gravity at sea has apparently produced no progress in the past year. This is probably the greatest opportunity now open for an epoch-making step in geodesy.

With the past year, the International Geodetic and Geophysical Union has become a going concern and its Section of Geodesy, which met for the first time in May 1922, focuses the thoughts of at least eighteen countries so effectively as to make it a worthy successor of the old International Geodetic Association. Once more, after a lapse of a decade, geodesy has the incalculable benefits of international co-operation.

The rapid increase in the United States in the fundamental observational data of geodesy, gravity determinations and observed deflections of the vertical, was brought before this same audience one year ago in a brief review of the statement then about to be taken by Dr. William Bowie to the meeting of the International Geodetic and Geophysical Union. Similar recent rapid progress has been made in Canada. My impression is that there are now available in North America new geodetic data in the form of available gravity determinations and deflections of the vertical equivalent to at least one-half of all of such data as used in the latest comprehensive geodetic discussions. It seems clear that in the near future a new comprehensive discussion would be opportune.

The most important recent development in geodesy has been the great increase in North America of the area over which there is continuous triangulation all reduced to one datum. That area in the "Supplementary investigation in 1909 of the figure of the earth and isostasy" extended, in 1909, to the four corners of the United States. But within the extreme limits mentioned, there were many large continuous blank regions which have since been supplied with triangulation. Since that time the larger of these blank regions have been greatly reduced in size by new triangulation extending through them. The most notable new pieces of triangulation, from this point of view, have been (1) a new arc of triangulation and traverse lines which is, in effect, continuous from the southern Atlantic Coast westward through Texas to a connection with the old triangulation in southern California; (2) a new triangulation in the vicinity of the 104th meridian from Pecos, Texas, to the Canadian border in North Dakota and (3) an oblique triangulation northwestward from Utah to Oregon with a branch southwestward to the vicinity of Mt. Shasta in California. Before 1900 any one of these three would certainly have been heralded to the world by a special publication and individually recognized as a worthy contribution to progress. Now, in the midst of the much more rapid advance of geodetic operations, they have attracted little attention. But of even more importance than this filling in of the gaps in the already occupied area, are extensions of that area of continuous triangulation down to latitude 17° in Mexico by the government of that country, to the northeast into Canada in the provinces of Ontario and Quebec, and the great extension to the northwest through Canada and Alaska which it is expected will become continuous from the United States to Skagway, Alaska, in the present year, 1923. In due time, in the near future, that extension may reasonably be expected to go far to the northward and westward, to the mouth of the Mackenzie River in Canada and far into the main portion of Alaska. To appreciate the great increase in the accuracy of the determination of the figure and size of the earth which is gained by such extensions of continuous triangulation on one datum, one must realize that said accuracy increases much more rapidly than the linear dimensions of the area. I estimate that in those cases in which the extension is both to new latitudes and new longitudes the accuracy varies at least as much as the cube of the linear dimensions of the area. It appears, therefore, that North America, through the cordial co-operation of the United States, Canada, and Mexico, will certainly, for a whole generation yet to come, have a stronger hold on geodesy, a better determination of the figure and size of the earth, and a better grip on the closely related problems than Europe and its connected continents can hope to have.

On the other hand, in Europe, together with the two continents with which it is connected by land, Africa and Asia, there is the ultimate possibility of a much larger continuous net of triangulation on one datum than can be secured on the western hemisphere. One of the auspicious signs of new progress in geodesy is the fact that at the 1922 meeting of the Section of Geodesy of the International Geodetic and Geophysical Union resolutions were passed which contemplate the development of great areas of triangulation reduced to one geodetic datum, regardless of international boundaries.

In the past decade, geodesy has developed many contacts with other fields of science. The following list of illustrations of the multiplicity and importance of these contacts is intended to make the preceding statement more concrete, though it is not intended to be exhaustive. Bowie, whose field is geodesy, has found himself forced, in recent discussions, to deal largely with geologic and physiographic aspects of the mechanics of isostasy. Barrell, a geologist, wrote the most extensive and vigorous discussion of isostasy which has appeared in the decade. Lawson, a geologist, has contributed to the understanding of the mechanics of motion in the earth's crust, especially in the form of overthrusts. Such an understanding is now evidently equally desirable to geodesy and geology since it has become clear that isostatic readjustment necessarily involves large horizontal transfers of material. Washington has shown that the densities of the rock in various portions of the earth's surface as computed from their chemical constitution are in general accord with their relative densities as inferred from the doctrine of isostasy. Nansen, in his recent book on "The Strandflat and Isostasy," finds in isostasy the solution of a great physiographic problem. The re-triangulation now in progress in California is an application of geodetic methods of thought and of observation to seismology and to studies of slow deformations of the earth's crust whether accompanied by earthquakes or not. The recent measurement of a very long base in California and its transfer to nearby mountain peaks was a loan of geodetic methods and apparatus to a physicist who is bent on a redetermination of the velocity of light. I understand that the loan has resulted in a raising of the attainable limit of accuracy of precise base measurement. All these matters are indicative of much more numerous and more intimate contacts than ever before between geodesy on the one hand and geology, geophysics, geochemistry, physiography, and seismology on the other hand. The geodesist no longer works in a field surrounded by a high wall; the wall is disappearing. He finds at many points that the limits of his proper field are so indefinite that he cannot be sure whether he is outside or inside. He finds the workers in adjacent fields coming in to work and talk with him and enticing him out to work and talk with them to a far greater extent than in any earlier decade.

The discussions of isostasy have been given new life by these new contacts, by the new keenness of realization of mutual interests in overlapping fields of thought.

Having in mind that experience indicates that to make the maximum progress in a given field of thought it is desirable to keep in good contact with adjacent fields and even make excursions into them, to keep all factors entering into a problem continuously in mind, and that it is also important to conquer the key problems first, I am led to make the following suggestions:

In the near future, possibly without waiting for a new determination of the figure and size of the earth, the accumulated geodetic data of North America, old and new, in the two forms (1) deflections of the vertical and (2) anomalies of gravity, on a uniform geodetic datum in each of the two cases, should be published, together with a comprehensive discussion by the geodesists in such form as will enable those in other fields of thought to continue the discussion to good advantage from their respective points of view. It is taken for granted that the data will be published both without and with corrections for topography and isostatic compensation computed on a uniform basis.

In the geodesist's discussions as published, especial attention should be given to the character and the rapidity of change of the deflections of the vertical, and of gravity anomalies, as one progresses horizontally from station to station. Is the isostatic compensation regional or local? What is the effective depth of isostatic compensation? To what extent does the isostatic readjustment lag behind the addition of loads at the surface by deposition, as in river deltas, or lag behind the subtraction of loads in regions of long continued erosion? To what extent are anomalies in the direction and intensity of gravity attributable to surface geology even though the isostatic compensation may be substantially complete and perfect? It seems to me that the answers to all these questions are contained in the deflections of the vertical and the anomalies of gravity. The answers may be more reliably and accurately obtained the larger the area over which the deflections and anomalies extend on a uniform datum, and the greater the variety of the conditions at the stations. Experience indicates also that it is especially important to have some groups of stations with a very dense distribution, even in regions where there is believed to be little local disturbance. The road to progress seems to lie in the publication of the new large collection of deflections and anomalies as proposed, and in a keen analysis of the collection.

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STATUS AND NEEDS OF SEISMOLOGY

W. J. HUMPHREYS

STATUS

The present status of seisomology, at least in respect to popular interest and instrumental equipment, reminds one strongly of the proverbial man who locked his stable door after the horse was stolen. Perhaps, too, if the door had no lock at the time of the theft, in his excitement the owner of the horse bought the first lock he could find, then soon tired of the necessary routine of turning the key, and again became neglectful before the next thief came along.

Whenever any region is visited by a severe earthquake, everyone in the disturbed territory who has a dead seismograph under his charge is immediately galvanized into temporarily running it again, while many others in innocent excitement clamor loudly "Oh, for a seismograph, a seismograph," and various inventors come forward with marvelous designs. But all this voluble enthusiasm commonly is of short duration, while those who got their new toys soon find them less interesting and more exacting than they had imagined, and in the end avoid them as the wise man avoids the deadly upas tree.

And so it has come about that there are scores of seismographs all over the world that are of no value whatever—poorly designed, badly installed, and inadequately operated, or, better, perhaps, not operated at all, but just awaiting the time when someone will have the courage to consign them to their proper place—the junk heap.

But the situation is not wholly bad, for there are a few seismological stations at which valuable records are continuously obtained, although, in great measure, their equipments are more or less experimental and radically different, and the data thus secured all but as variously treated.

There has been, and there still is, much wasted effort in seismology; nevertheless all these independent explorations in many directions, these inevitable pioneer efforts, have served, as nothing else could, to make clear what some of the fundamental problems of seismology are, and how we should go about getting the data essential to their solution.

NEEDS

We know that seismology requires, as do few other sciences, international co-operation, or, at least, agreement among seismologists, the world over, unitedly to attempt the solutions of certain specific problems, and, therefore, to have comparable equipments and to assemble their records in much the same way.

Theory.—But first of all, now that the preliminary stage of this new science is over, we need, to guide us, a searching mathematical analysis of what to expect, as to velocity and period, of each of the several kinds of seismic waves, under this and that reasonable hypothesis in regard to the structure of the earth. We want to know, for instance, the details of the travel of the waves through the sedimentary layer, the granitic layer, and basaltic layer; and the combination effects of these layers on reflections, and on the so-called long-period or surface waves.

To push these problems, as they should be pushed, much beyond the stage to which they have already been advanced would demand exceptional ability in setting up and solving the equations applicable to elastic solids. The extraordinary difficulty of this problem, or, rather, host of problems, would leave us in despair of any marked progress in seismology, but for the fact that therein lies its appeal to the mathematical physicist. He loves the next-to-impossible problems, and here he has them in abundance.

Design of apparatus.—When the elastician has told us what this and that reasonable hypothesis predicts, instruments must then be designed to test these predictions, and to gather data that will enable the theorist to improve his hypothesis. Here, too, much is demanded—rigid mathematical analysis, ingenious designing, and skillful construction; for neglect of any one of the three means failure.

Location of stations.—The next thing to do, if we are to have an adequate study of seismology, is to determine where the instruments are to be located. The difficulties here are only practical, but, for all that, quite real. Certain regional problems of the highest interest to geologists and engineers may indeed present no serious difficulties in this respect. They can be provided for within the territory of a single government, and financed by a single institution or organization. In short, for this class of problems, co-operation and continuity, each essential to success, are easy to obtain. But that other class, the teleseismic problems, requires a distribution of instruments in various countries and, after that, a continuity of operation, and a unity of purpose, difficult to inaugurate, and all but impossible to maintain.

Publication of data.—What particular seismological data shall we publish? Where shall we publish it? When shall we publish it? All these, and more besides, are important and pressing questions that cannot be rightly answered except by international agreement.

In short then, seismology is well established as a science, and blessed with enough difficult, unsolved problems to guarantee it a future full of healthful struggle and exciting interest.

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STATUS, SCOPE, AND PRESENT-DAY PROBLEMS OF METEOROLOGY

C. F. Marvin

Little is to be gained in this brief account of meteorology and its problems by attempting to state with any fullness the status and scope of the science.

The domain of meteorology is the entire atmosphere, and no boundaries of nations, continents, or oceans of any kind delimit its scope or problems.

For more than half a century the great nations of the earth have given meteorology a national status, either by the entire maintenance of national weather services in some form or other, or by subsidies or contributions in support thereof. At the present time, nearly all civilized countries of the world maintain more or less completely organized services.

The Southern Hemisphere and portions of the tropical regions of the globe are but poorly represented by observing stations, not merely because of the small proportion of land to water areas, but because of imperfectly organized services.

The contributions to meteorology from private observatories and special institutions were invaluable in the past, but the place and work of these are now almost completely occupied by the numerous national services.

The essentially international aspect of the science was early recognized, and harmonious co-operation among the nations has been secured through the so-called International Meteorological Congress, which had its origin in a conference held at Leipzig in 1872 and which has continued to function in a highly satisfactory manner up to the present time. Although its regular sessions were interrupted by the war, the organization was reconstituted at a conference held in Paris in 1919 by invitation of the French Government.

This international organization consists of so-called Congresses which are held at infrequent intervals. Delegates to a Congress include representatives of all the meteorological services and of the principal private observatories of the world. The permanent working body of the organization is the International Meteorological Committee, which at the present time consists of not more than twenty members, each of whom is a director of some national service. The work of the committee is specialized by the organization of so-called Commissions, which are in the nature of sub-committees charged with special fields of work.

At the meeting of the International Committee held in London in September, 1921, there were twelve commissions of which two were newly created in 1921. The names of a few of these will illustrate the nature of the organizations: Terrestrial Magnetism and Atmospheric Electricity; Maritime Meteorology and Storm Warnings; Meteorology and Agriculture, etc., etc.

The International Geophysical and Geodetic Union, through its section on meteorology, is a new and additional agency by which effective international co-operation, so essential to the future progress of meteorology, can be carried forward.

The International Meteorological Congress will hold its next meeting during the second week of September of this year at Utrecht, pursuant to the invitation of the Director of the Meteorological Institute of Holland, Professor Van Everdingen. The relations of the work of the International Meteorological Committee and that of the Section on Meteorology of the International Geodetic and Geophysical Union is an important topic on the agenda of the meeting.

As lack of time prevents a fuller consideration of these highly important matters, I must take up next some problems of meteorology.

PRESENT-DAY PROBLEMS OF METEOROLOGY

All the phenomena of meteorology are naturally confined to conditions in the earth's atmosphere, and man's first observations and problems were encountered in dealing with terrestrial conditions. It was soon known, however, that the heat reaching the earth from the sun was the ultimate cause of all earthly weather and climate.

Problem 1.—The sun and its radiant energy impressed upon the earth is the first great problem of meteorology. How much do we really know, and of how much are we ignorant, concerning solar radiant energy and its geophysical effects?

Conservative meteorologists have hitherto found the effects of solar *thermal* energy either constant in amount or slightly variable as it reaches the earth fully adequate to explain practically all the phenomena involved in atmospheric circulation. On the other hand, a few fanciful thinkers, little restrained by a grave paucity of both facts and theory, boldly announce that such solar phenomena as sunspots, very minute and as yet scarcely proved fluctuations from day to day in the intensities of thermal radiations of the sun, exercise important controls upon weather phenomena, growth of vegetation, etc. With even slighter basis of observed facts and more fanciful flights of imagination, a few boldly acclaim that climatic changes occur under the control of solar electric and magnetic emanations.

Here is a genuine problem of fundamental importance.

The meteorologists necessarily must depend upon astronomers and solar physicists for the unequivocal facts concerning the sun and its radiant emanations, and upon the magneticians for terrestrial data concerning phenomena in their domain. America may point with much pride to three great institutions which are unsurpassed elsewhere in the whole world and which are steadily supplying the solar and magnetic data needed in this problem. I refer, of course, to the Mt. Wilson Observatory and the Department of Terrestrial Magnetism, both of the Carnegie Institution of Washington, and to the Astrophysical Observatory of the Smithsonian Institution. Systematic observations from these sources as yet cover barely a score of years, about two sunspot periods, a length of time entirely too short upon which to establish obscure and unknown relations and to evaluate effects which are very certainly of small magnitude.

Finally, on the side of the facts in this problem we have what may be a great superabundance of meteorological data, because these began many years before the needed solar observations. On the other hand, meteorological data are bunched into certain geographic localities and are sadly lacking in others.

The solution of this great problem of solar emanations and our weather and climate is a study of statistical analysis and correlation. It is largely an arm chair problem of the analyst, not a laboratory job of experimentation.

The Weather Bureau, through the careful work of its technical staff, is *slowly, surely and sanely* advancing the science of meteorology with reference to this difficult problem of solar emanations and terrestrial weather and climate. *But everything depends upon intelligent interpretation of data and results.* This is a subject which cannot be discussed in the brief time now alloted.

Problem 2.—Passing from the sun and its emanations to our own atmosphere, we choose for our second problem—*The General Circulation of the Atmosphere.*

Meteorologists of the past, and many of the present day, accept without question the standard text-book versions of the primary wind systems of the globe. Must we not pause to ask ourselves if we are building upon a perfectly sound foundation of theory and fact? The basic theories of atmospheric circulation were first put upon a firm footing by the mathematical productions of Wm. Ferrel. His contributions are the more remarkable because they really preceded, by a decade or more, the general world-wide organization of numerous meteorological stations and the systematic collection of statistical data. Ferrel's equations were checked against the merest fragment of observed facts, but his general theories continue to hold an unquestioned place in meteorological science.

Let me state a few basic assumptions which underlie these theories and let us see if errors are not likely to be introduced. The old canal theory of interchange of warm and cold air between the equator and the poles calls for an over-and-under type of flow which on a stationary earth should take place along meridians. On a rotating globe, however, the northward moving warm currents must be turned to flow eastwardly while the southward moving currents are diverted to a westward course by the influence of the earth's rotation. There have been many doubts expressed as to the reality of this over-and-under type of flow, as a major feature of the general circulation, and both cloud observations and daily weather maps for the Northern Hemisphere at least seem to justify the conviction which many hold that thermal interchanges between the tropics and the poles are effected by flow and counterflow at the same levels as at the earth's surface; rather than the over-and-under interchange of the theory.

Furthermore, Ferrel has assumed that the Northern and the Southern Hemispheres constitute a balanced pair of atmospheres much as if there might be an impermeable wall or plane of separation extending indefinitely outward from the equator. At least, his theory takes no account of the very great annual transfer of air masses back and forth across the equator which we know must take place and is so exemplified in the historic systems of monsoon winds of the Indian Ocean.

The deepest thinkers among meteorologists believe a re-survey of world-wide meteorological conditions is called for to establish a more comprehensive theory of the general circulation of the air that will take better account of the winds of the globe as they actually appear on weather maps from day to day.

The solution of this problem requires (1) the prompt (telegraphic, if possible) collection of daily or twice-daily weather observations and reports sufficient to construct accurate *daily* weather maps for the entire globe or on separate polar hemispheres; (2) the study of such data by the meteorologist, the mathematician and the physicist, with a view to the *intelligent* interpretation thereof and the formulation of improved laws and equations of motions.

Obviously, this problem is of such magnitude that the collection of the necessary data and the issue of appropriate world weather maps, embracing and depicting not only surface conditions but, in addition, cloud observations at intermediate and high levels, including free air sounding, require international co-operation and co-ordination of effort of the most earnest and cordial character.

For a period of about six months during 1914 the United States Weather Bureau published its daily weather map of the Northern Hemisphere. It has not been practicable to resume this issue, which the war brought to an immediate termination, although the Bureau continues to prepare the manuscript maps which serve a highly useful purpose in its daily forecasts.

It is believed the time is near at hand when proposals for international co-operation may be made that some great nation be entrusted with the duty and responsibility of the issue of a truly world weather map based upon a suitable number of station reports, the total expense thereof to be equably apportioned among those in cooperation. The Weather Bureau contemplates proposals of this character, and the National Research Council, through the International Geophysical Union and Foreign Relations Committee, can undoubtedly be of great assistance in promoting results.

Problem 3.—I will assign to the third place in my list a group of problems all very closely related to Problem 2, but at the same time capable of separate work and effort. Time permits of presenting these only by title, so to speak:

(a) The ultimate cause of origin and sequence of the cyclones and anticyclones of extratropical latitudes.

(b) The expression *Centers of Action* has been current among meteorologists for a score or more of years. Are all in accord as to the physical reality of the thing connoted by the name? Where is the locus and what are the confines of these indefinite entities?

(c) What makes West Indies hurricanes, typhoons, and like tropical cyclones? How little more we know than the destruction wrought by these mysteries of tropical meteorology.

(d) *Ocean Meteorology and Marine Forecasting.* Full and detailed observations over the vast ocean expanses of the globe are almost impossible of attainment, but the near future promises most important extensions in this field of effort. The subject can merely be mentioned here and some discussion of it will be taken up at another time by the Chairman of Section (c) who has recently made some special study of the problem.

Problem 4.—I include within this problem what we have come to call *Agricultural Meteorology,* which deals with the influence of weather on vegetation, particularly its influence upon animal and vegetable food and textile supplies of the nations and the world.

The greatest present obstacle to progress in this most important economic study is the great inaccuracy in data expressing acreage and yield, and no more important step for real progress can be made than that those agencies of the Government and the nations of the world should inaugurate at once plans for procuring really *accurate* figures on acreage and production of the big staple crops of the world.

The meteorologist engaged in the study of weather and crop relations is constantly confronted with the internal evidence of gross inaccuracies and lack of homogeneity in acreage and yield values.

Problem 5.—Meteorology in Aid of Aeronautics has become a new duty and responsibility devolving upon national services of the major nations. The nature and scope of the problem is obvious from the title. Its solution calls for many more observations by means of kites, pilot and sounding balloons than have been available in the past, also, a co-operation between the personnel of meteorology and aeronautics, in order that each may better understand the problems, needs and services of the other.

Problem 6.—Professional forecasting of the weather is a primary duty imposed upon national weather services generally. Without exception, almost no service attempts to make forecasts for long periods in advance, that is, for months, seasons, etc. I do not believe such forecasts are scientifically impossible, but no sound method is yet known by which they can be made with a sufficient measure of success.

The meteorological problem involved in long-range weather predictions is the problem of what are the *laws of the sequence of weather conditions.* This problem embraces, as one of its parts, the very popular question of *weather and climatic periodicities.*

Many claims have been advanced, and about the same number of skepticisms have been entertained, but as a fact very little is really known, either pro or con, and the subject is one needing careful and systematic study. One serious obstacle is the shortness of nearly all records of data. Little can really be done with long-time changes like sunspot periods and others of still greater length. On the other hand, if short periods of days, weeks, months, or a few years in length, actually exist and persist, abundance of data are available for their disclosure. Almost everything depends upon the use of adequate methods.

Problem 7.—Meteorological Education. Many problems in meteorology cannot be considered here for lack of time, but this brief category would be incomplete without mentioning the subject of meteorological education in the schools, and especially advanced education in the universities of the nation.

There is pressing need for higher training in dynamical and physical meteorology as a basis for the profession of weather forecasting of the highest type. The public, agriculture, commerce, and the industries are coming more and more to seek, utilize and demand advance information concerning the weather. The Weather Bureau is finding it increasingly difficult to recruit and maintain its staff of professional forecasters adequate for the increasing demands incident to nothing more than the normal growth of the nation. The great universities should make professional meteorology and forecasting a major course in the physical departments of education. The possibilities of future occupation for students in this field may not seem to be attractive under conditions as they now exist, but I am certain that if a reasonable supply of material and talent of this kind were once to become available it would soon find reasonable opportunities of employment.

U. S. Weather Bureau,
Washington, D. C.

STATUS, SCOPE, AND PROBLEMS OF THE SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY

W. F. G. SWANN

The work of the student of the earth's magnetic and electric phenomena naturally divides itself into two parts, first the study of the main phenomena themselves, and second the study of their causes and of such subsidiary phenomena as are bound up with them. As regards the main phenomena, we have on the magnetic side the study of the magnitude and distribution of the earth's magnetic field over the earth's surface, and its diurnal, annual, and secular variations. On the electrical side, we have the study of the magnitude and distribution of the earth's potential-gradient, of the atmospheric ionization and conductivity, and the diurnal, annual, and secular variations of these quantities. We also have a class of phenomena which are to some extent distinct, since they are associated with abnormal rather than with normal conditions in the atmosphere. I refer to phenomena associated with the thunderstorm and with the electrification of rain.

Imperfect as is our knowledge of the origin of the earth's magnetic and electric fields, however, we know enough about them to know that they are closely bound up with certain less obvious phenomena in such a way as to make the study of the latter a necessary complement to that of the other.

We have reason to believe that the primary agencies concerned in the diurnal variation of terrestrial magnetism are currents induced in the upper atmosphere on account of the tidal motion of the atmosphere across the earth's lines of force. The magnitudes of these currents depend upon the conductivity to be assigned to the upper atmosphere, so that all phenomena likely to be associated with the production of such conductivity come under the natural scope of our study. We believe the aurora to be caused by the entry into our atmosphere of corpuscular rays of some kind from the sun, and to such rays we also look in part for an explanation of the conductivity of the upper atmosphere. We naturally expect, as indeed we find, a close connection between auroral phenomena and those of terrestrial magnetism. The closeness of the relation is increased by the known connection between these phenomena and magnetic storms. Thus, the study of the aurora, and of all matters concerned with it, such as the nature of the radiations responsible for it, the nature and composition of the upper atmosphere in which the luminiscence occurs, the theoretical aspects of the bending of the corpuscular rays by the earth's magnetic field, all these fall within the scope of our interest. The close connection between these phenomena and the sun's activity as exemplified in such matters as sun spots brings the sun itself within the field of our studies. Again, the existence of a state of high conductivity in the upper atmosphere has important consequences in respect of the transmission of wireless waves, so that the study of the transmission of such waves around the earth, and of the fading of signals become of importance to us; and, while speaking of wireless telegraphy, I must not omit to mention the matter of atmospheric strays which are evidence of some kind of electric disturbance, whose elucidation would be of great interest to the student of the earth's electrical behavior.

The study of earth-currents interests us, for two reasons; first in relation to their connection with auroral phenomena, and with the variations of the earth's magnetic field, and second, in so far as they may exist in part independently of these varying phenomena, and may be bound up with the earth's permanent magnetic field.

Any phenomena associated with a departure from the normal state of affairs is likely to form a clue in binding together the different elements of our study, particularly if that phenomenon is likely to have a direct influence on some of the main agencies concerned. The measurement of atmospheric and electric elements during an eclipse of the sun is consequently of considerable importance.

Turning to the purely atmospheric-electric side, our interest extends not only to the fields which I have mentioned, but also to various matters concerned with more or less pure physics, to the ionization which is known to occur in hermetically sealed vessels, and which may be attributed to a radiation of cosmical origin, or may arise from more local causes. The nature of the ionization produced by high speed corpuscles, and the penetrating power of such corpuscles are of fundamental importance in connection with theories of the earth's charge. The amount and distribution of radio-active material over the earth's surface, and in the atmosphere has an important bearing on the atmospheric conductivity at low altitudes, and the student of atmospheric electricity must draw very largely upon the information of the meteorologist in connection with air currents, and the various other meteorological features which so greatly influence the nature of atmospheric electric phenomena at low altitudes.

Finally, I must emphasize the importance to the cosmical physicist, of a close scrutiny of the facts and principles of pure physics. The situation in this regard is enhanced, moreover, from the fact that in dealing with a body of the earth's size we find that phenomena which would be quite imperceptible in laboratory experiments assume a degree of importance which is at times almost fantastic. Thus, currents started in wire circuits in our laboratories and left to themselves will not die out immediately. The inertia of the electric current will insure that they persist for a short time, but for a time so short that, in general, delicate means are necessary to register it at all. Currents started in a copper sphere of the earth's size, and left to themselves will take 10 million years to decay to one-third of their initial values as Lamb has shown. A current density of the order one hundred millionth of an ampere does not produce very startling effects in most laboratory experiments, but, a current density of this order in the body of the earth will give rise to a magnetic field equal to the whole of the earth's magnetic field. So the story goes, and he who would seek the origins of the earth's magnetic and electric phenomena must be prepared to peer into all sorts of possibilities and even to concern himself with the structure of matter and the problems which lie in the domain of the pure physicist. Moreover, in doing this, he must adopt a view point which is in some cases more fine grained than that of the pure physicist, for he must be prepared to seize upon the most minute possibilities in existing theory, and upon possibilities of modifying existing theories to extents which, while negligible for the purpose of the pure physicist may be for him the solution of some great problem of cosmical scale. A student of the subject must possess a rapid intuition, and a keen sense for orders of magnitude that he may see quickly what is likely to be important and what trivial. He must possess mathematical power to enable him to trace the consequences of the facts to their logical conclusions; but frequently, a mathematical problem chosen *ad hoc* leads to difficulties which no man can surmount, so he must possess a certain subtlety of vision, which will enable him to avoid the more difficult paths to his goal, and cause him to formulate his problems in such a way that while they lead to the information desired, they are possible of solution. Finally, he must possess a sound knowledge of the principles of physics, and above all of their logical connection with each other. He must not merely know the laws, but he must understand the structure; for, he will have to tamper with structure. He may have to remove a prop here and there, or modify the scaffolding occasionally, and woe betide him who tampers with a structure with which he is not familiar. He will assuredly produce many results, but alas for some of their consequences.

It seems that the time has come when the student of terrestrial magnetism will have to familiarize himself more and more with that great revolution in our ways of thinking exemplified in Einstein's General Theory of Relativity. Already we hear that Einstein has been successful in including terrestrial magnetism in his general scheme. Every student of the subject has felt that this step must come sooner or later; for, so fundamental is the change which the theory of relativity makes, not, of course, in the facts, but in our way of describing them, that to develop a theory of relativity for gravitation and leave terrestrial magnetism out would be to talk in two different languages in two different branches of physics. It may always be convenient for most practical purposes to speak in terms of magnetic fields and the like, just as it will always remain convenient for most purposes to speak in terms of the Newtonian theory of gravitation; but it is important that one who interests himself in the fundamentals of the subject, and seeks what he may choose to call the causes of the phenomena which he studies shall be fully conversant with those modern developments in thought in which the ideas of cause and effect exhibit themselves in such altered form.

I now turn to suggestions for problems which it would seem worth while to attack. First let me refer to a few pertaining more particularly to atmospheric electricity.

The earth's charge is leaving it at such a rate that 90 per cent of it would be gone in ten minutes were there no means of replenishing the loss, and there is a very important class of theory which attributes the replenishment to high speed corpuscles shot into the earth either from outside of our atmosphere or from the atmosphere itself. There are two great difficulties in such assumptions. The first arises from failure to detect any charging effect due to the absorption of these corpuscles by an insulated mass of metal, and the second arises from the fact that since, as far as our laboratory experiments go, a corpuscle moving with a velocity comparable with that of light should produce 40 pairs of ions in each centimeter of its path, and since the replenishment of the earth's charge requires the influx of 1,500 corpuscles per square centimeter per second, we should expect these to produce about 40,000 pairs of ions per c.c. instead of only about one ten thousandth of this amount as experiment shows to be the case. I have recently tried to remove the second of these difficulties by showing that corpuscles with velocities sufficiently near to the velocity of light should fail to ionize, but the first difficulty remains; and, while it is not unsurmountable in the case of certain types of corpuscular theory, it would be a great comfort if we could measure such a charging effect. I sought such an effect eight years ago, and more recently the effect has been sought independently by Von Schweidler, but in both cases without success. On the other hand, in view of the fact that auroral phenomena, which are attributable to corpuscular radiations from the sun, are concentrated so sharply in certain zones of the earth's surface, it would seem of the greatest importance that a search for this charging effect should be sought in those zones.

The second problem I have to suggest has to do with certain consequences which should result from the existence or non-existence of the supposed region of high conductivity in the upper atmosphere. I can illustrate the matter in this way. Suppose that there is no such condition as this layer and suppose that we have rain falling from a cloud to the earth, the rain being as it usually is, positively charged. The positive charge will not spread itself uniformly over the earth, because it will be held bound in the immediate vicinity of the cloud. In other words, the effect of a rain storm on the potential gradient would be practically local. Now, if there is a conducting layer in the upper atmosphere, the situation is changed. In this case the cloud sets up a difference of electrical potential between the layer and the earth, which is handed round more or less uniformly to all parts of the earth. On submitting this matter to calculation I find, for example, that if h is the height of the cloud and H, the height of the layer, the positive charge to be found on that half of the earth which is remote from the cloud is h/H of what it would have been if the charge had been uniformly distributed, and R/H of what it would have been if the layer had been absent, R being the radius of the earth. The upshot of all this is that, if there is no conducting layer, practically all of the charge should remain on the part of the earth to which it falls, after due allowance is made for conduction back to the atmosphere on account of the conductivity of the atmosphere, whereas, if a layer is present, an appreciable part of the charge should be transferred to other regions of the earth. If then, at a number of stations in a region where it is raining, we measure the charge actually brought down by the rain, and the rate of accumulation of charge density on the ground after correction for the current back to the atmosphere, we have a means of testing whether or not the charge remains where it falls, and consequently of deducing certain information with regard to the presence or absence of a conducting layer. A practical method of making the test would be the following: Suppose we have a large insulated tray at the ground level, and exposed, of course, to the potential gradient. Allow the rain to fall into this tray after connecting it to earth and then insulating it. Keep the potential constant and zero by feeding electricity into it from a condenser in a manner of which the details will be sufficiently obvious to need no description. It will be obvious that in cases where the rain is sufficiently steady and prolonged to permit of a steady state being attained, measurements of this kind, at any rate when made simultaneously at a number of different places, will lead to information as to the rate at which charge is passing from the regions immediately below the cloud to those more remote.

Another avenue of attack on the conductivity of the upper atmosphere is to be found through the medium of wireless waves. If the transition from practical non-conductivity to high conductivity is sufficiently sharp, marked reflection of electro-magnetic waves should result; and, by suitable arrangements, the details of which I must not enter into, it should be possible to measure the time taken for wireless waves to travel to the layer and return, and so deduce the altitude of the layer.

The great importance attaching to measurements of the atmospheric elements at high altitudes, and particularly of the residual ionization in gases is so self-evident that it is hardly necessary to do more than refer to it in passing.

The problem of the conductivity of the air over the great oceans is by no means satisfactorily settled. The conductivity is far greater than can be attributed to the radio-active materials in the ocean itself, or in the air over it. It has been customary to invoke, for the explanation of the ionization over the ocean, a penetrating radiation of cosmical origin whose assistance has also been invoked to account for the ionization which is known to occur in a hermetically sealed vessel free from radio-active materials, and suitably shielded from such radiation of ordinary gamma-ray type as may come from radioactive materials in the surroundings. Unfortunately, recent experiments have thrown considerable doubt upon the reality of this penetrating radiation, at least as to its existence at low altitudes; and, it may be that, in order to account for the residual ionization in vessels and for the conductivity of the ocean's air, we shall have to resort to the assumption of a spontaneous ionization of the gas itself. A full understanding of the origin of conductivity in the lower atmosphere thus renders it imperative that we strive to solve this question of the origin of ionization in closed vessels (or more important still, the origin of the ionization which would occur in the air in the absence of the vessel) in order that we may know whether it is really necessary to invoke an external radiation, or whether the causes of the ionization are inherent in the gas itself.

I now turn to a few matters having a bearing upon the earth's magnetism. If, as seems likely, the earth's magnetism is bound up with its rotation, a knowledge of the magnetic fields of other heavenly bodies, and attempts to detect the magnetic fields of rotating bodies of laboratory size become of importance, since such information gives us a clue to the way in which the magnetic field varies with dimensions, angular velocity, density, etc. Thus to illustrate the point in question, suppose we were to seek an explanation of the earth's field on the basis that, on account of the centrifugal force of the earth's rotation, the free electrons in its interior must suffer radial displacement, with a resulting production of a magnetic field on account of the rotation of the electro-static distribution thus brought about. As a matter of fact, the field in this case can be calculated, and amounts to only 10^{-28} of the earth's field, but suppose that there had been some element in the calculation which prevented our obtaining the actual magnitude, and gave us only the form of the expression for the field. The field on the equator at the surface in this case comes out proportional to $\omega^3 a^2$ where ω is the angular velocity and a the radius of the sphere. It would thus result that on such a theory, a sphere of radius 10 centimeters rotating at a speed of 100 revolutions per second in the laboratory would have a field over a hundred thousand times that of the earth, while the sun should have a field only half that of the earth. Our knowledge of the absence of fields of the above order of magnitude in the case of bodies rotating in the laboratory is therefore sufficient to rule such a theory out of court, and the fact that the sun has a magnetic field of about 50 gauss is additional evidence in the same direction. A large class of theories give the magnetic field proportional to $\omega a^2 D$ where D is the density of the substance. On such theories, the existence of a field of the order 0.5 gauss at the earth's pole would correspond to the existence of a field of the order, 50 gauss at the corresponding place on the sun, a result well in harmony with Dr. Hale's conclusions as to the magnitude of the sun's field. On the other hand, this class of theory would give quite inappreciable fields for bodies of laboratory dimensions rotating with attainable angular velocities. Such a type of theory would, moreover, lead to the conclusion that we should have but little hope of detecting magnetic fields in the stars other than the dwarfs, for, expressed in terms of mass m rather than density, the expression makes the field proportional to $\omega\ m/a$, so that for given mass and angular velocity, the field is small in proportion as the radius is large. Even the giant stars have for the most part, masses only comparable with that of our sun, so that, on the basis of such a type of theory as that under discussion, the large radii would cut down the surface magnetic fields to very small values.

I now turn to a suggestion which is rather speculative as to its possibilities, but which is perhaps of sufficient interest to warrant mentioning. The question of the permeability of the earth's interior is one which has interested students of the subject, particularly in relation to the possible effects of combined pressure and temperature conditions to be found within the earth. It would therefore not be without interest to make a direct measurement of the average permeability of the earth. The possibilities in this connection are perhaps not as visionary as might be thought at first sight. Suppose we have two cables parallel to each other around the earth's equator, and start a current in one of them. We shall get an induced current, and a definite time integral of current in the other. The magnitude of the latter will depend upon the dimensions of the circuits, the value of the primary current, the resistance of the secondary circuit, and the average permeability of the earth, and will vary by a factor of the order of magnitude of 3 according as the average permeability is comparable with unity, or large compared therewith. Now a simple calculation will show that by starting a current of one ampere, in the primary cable, the order of magnitude of the induced flow in a secondary cable, of resistance say 10000 ohms will be 0.017 coulomb, a very large quantity when considered in relation to what is measurable with a sensitive galvanometer. It will be obvious that the ideal condition stipulated above is not necessary for the purpose of obtaining measurements from which conclusions could be drawn. It is not necessary for the circuits to be immediately adjacent, or to be drawn around the equator, or to be parallel to each other. In fact, a little consideration will show that all that is necessary is that the dimensions of the circuits shall be comparable with the depths to which we wish to extend our information as to the permeability, and their distance apart must not be greater than is comparable with their dimensions. The practical question involved is of course how far it would be possible to utilize existing cables for such a project as this.

It may perhaps be worth while to call attention to the possibilities inherent in the use of a large coil for automatically recording the instantaneous rate of change of the earth's field at any place, as distinct from what is usually measured, the actual value of the field. The matter may be illustrated by the following example: The diurnal variation of the earth's horizontal intensity is of the order of magnitude of 10^{-8} C.G.S. unit per second. A square coil with sides 100 meters long, wound with 10 turns of wire would give, on the above account, an e.m.f. of 10 electro-magnetic units, or 10^{-7} volt. If the coil were wound with wire of gauge 10, it would have a resistance of 12 ohms, and would therefore cause a current of 0.5 x 10^{-8} ampere in a galvanometer of resistance 8 ohms. A properly designed galvanometer would give a deflection of about 2 mm. for this current at a scale distance of 2 meters. Of course 2 mm. is not a large deflection, but greater deflections could be obtained by the use of a larger coil, or a coil with larger number of turns of the same kind of wire, in conjunction with a suitably chosen galvanometer, and the above calculation will serve to indicate the possibilities in this connection.

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STATUS, SCOPE, AND PROBLEMS OF THE SECTION OF OCEANOGRAPHY

G. W. Littlehales

As an emissary from the Section of Oceanography, by designation of its Executive Committee, I am to make the summary statement of the scope and problems in relation to the science whose field is the oceanic expanse, covering an area which exceeds, by more than 80 million square miles, the total land surface of the globe and occupying basins whose cubical content is estimated to be fourteen-fold greater than the bulk of all the lands in the world above sea level. The indications are that throughout nine-tenths of its extent, the depths are greater than one mile; throughout two-thirds of its extent, the depths are greater than two miles; and throughout 10,000,000 square miles of its expanse the depths exceed three miles. Many high mountains rise from the foundations of the oceanic basins, sometimes culminating in emergence above the surface of the waters; and many deep depressions sink below the general level of the bottom.

In the Philippine Deep, a sounding of 5,352 fathoms near the Island of Mindanao marks the greatest depression hitherto discovered in the earth's crust—a depression so profound that if the highest elevation in the world above sea level were set down within the trough, the summit would yet be 3,000 feet beneath the surface of the sea.

Oceanography is defined as comprehending "the investigations which deal with the form and divisions of all the marine areas on the surface of the globe, the winds that blow over the surface waters, the contour of the ocean bed from the sea-level down to the greatest depths, the temperature, the circulation, the physical and chemical properties of sea-water, the currents, tides, waves, the composition and distribution of marine deposits, the nature and distribution of marine organisms at the surface, in the intermediate waters, and on the floor of the ocean, as well as the modifications brought about in living things by the conditions of their existence, the relations of man to the ocean in the development of commerce, fisheries, navigation, hydrography, and marine meteorology. All this vast assemblage of knowledge, which embraces some aspects of astronomy, geography, geology, physics, chemistry, and the biological sciences, makes up the modern science of Oceanography."

It is worthy of note by geophysicists that the accepted gauge and shape of the earth's crust have been derived altogether from continental arcs without measurements over the 71% of the globe's extent occupied by the ocean.

In the foundation upon which the future superstructure of oceanography is to be raised there must be the configuration of the basins, which are the containers of the oceans; and in the other branches of geophysics, as well as in oceanography, the mapping of the two-thirds of earth's surface which is covered with water is doubtless of an importance second only to the importance of mapping the one-third which is comprised in the land areas.

May it be pardoned as an instance of the frailty of human nature if I seem to incline too far to a belief in the saying that "coming events cast their shadows before them," when I point to the recent survey made in 38 days by two vessels of the Navy of a tract of 34,000 square miles of the bottom of the Pacific Ocean, bordering the State of California from San Francisco to San Diego, as an evidence giving grounds for hope in the feasibility of accomplishing the mapping of the oceanic basins by proceeding along the avenues of co-operation among nations that have been opened in the International Union of Geodesy and Geophysics of the International Research Council.

The position occupied by water in the economy of Nature is most remarkable. Everywhere natural conditions are influenced and molded by its wonderful properties, and yet it is nowhere told how much water there is in the world. But we do know that the presence in the oceanic basins of 324,000,000 cubic miles of this substance having the highest known capacity for heat—which may be gauged by the statement that the specific heat of sea water is about thirty times that of mercury—must constitute the ocean a dominant factor in terrestrial physics, exercising a governing influence through its circulation upon heat transference and upon climate and the development of vegetation, and, by the enginery of its variations in physical and chemical condition from season to season and year to year, promising, when it comes to be understood, to enhance all those wealth-producing sources which operate in seasonal cycles.

I have elsewhere given a summary, in the publications of the American Geophysical Union, showing the means by which, from the unfathomed state of the middle of the nineteenth century, knowledge of the physical conditions in the depths of the oceans has advanced to an approximate conception of the conditions which prevail throughout the various ocean depths—conditions of movement, of temperature, of pressure, of the penetration of light, of gas content, of salinity, of life. Only a general idea of the contours of the ocean bed has been attained, and only a general idea of the bottom deposits covering that bed.

Although there is as yet little ground upon which to base general conclusions, the diversification found to characterize those tracts about which most has been learned appears to support the view that the ocean basins are extensively featured with orographical formations culminating at various depths below the surface of the ocean.

In every stage of the investigation beneath the surface of the ocean, invention has been the resource leading to progress. Our knowledge of the conditions in deep water, and all that takes place beneath the surface, must continue to be dependent on the correct working of instruments, the action of which is, for the time, hidden from sight. The sparse and disconnected observations of the physical elements obtained by means of indicating instruments, unstandardized among themselves, have advanced oceanography in the exploratory stage, but will not fully serve the ultimate purpose of investigating the ocean from the standpoint of thermodynamics. For this purpose continuous records are required, and, hence, the important problems of the present time include the production and perfection of instruments adapted for making continuous records, both at stations and in transit, of such physical properties of the sea as temperature and salinity, in vertical as well as in horizontal distribution. Oceanographers have already commenced to obey the modern tendency in physical research to replace indicating instruments by recording instruments. The later advances in the science of the tides are ascribable to the continuous recording tide-gauge.

It is by this course that the shadowy evidences of significant oscillatory, pulsatory, and vortical movements in the depths, which have been encountered in oceanographical research, can best be examined and extended for the fulfillment of the purpose of making known their origin and nature. The Japanese have shown the presence in their seas of standing waves, like those known to exist in lakes. Variations in physical condition have also been observed which are most naturally explained by the progression through the body of the ocean of great undulatory movements, like the waves which physicists observe on the boundary surface between media of different densities. The solution of the problem here presented is looked upon as one of the greatest importance both from the standpoint of dynamic oceanography and of biological oceanography. And of equal importance to the oceanographer of the future is to learn what takes place when wind blows over water, and hence the relation between wind and current. The currents of the ocean seem to be subject to a sort of pulsation of which little is known as yet.

Of great significance is the appeal of oceanography for international co-operation seen on all sides in the recognition by oceanographers of the fundamentally world-wide and hence continental and intercontinental nature of the phenomena, and the national and international character of the scientific and economic results to be developed, and the necessity for the application of resources that come only at the call of nations co-operating together. In every department of science the annals of knowledge supply abundant evidence of the importance of method. In recognition of this, it was with unanimous approval that the International Section of Oceanography, meeting in Rome in May, 1922, made provision for the preparation and publication of an International Manual of Oceanography.

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THE YEAR'S PROGRESS IN VOLCANOLOGY

ARTHUR L. DAY

The progress of volcanology suffered in two important respects as a result of the War. The German undertaking, under the leadership of Branca of the Berlin Academy, which in many respects was the most comprehensive plan for volcano study ever seriously contemplated, was dropped. A second plan, almost equally imposing, was started just before the War by Friedlaender at Naples. A beautiful laboratory overlooking the city and within sight of Vesuvius was constructed and a journal, the *Zeitschrift für Vulkanologie,* now in its ninth year, was started. This splendid laboratory was taken over by the Italian Government during the War, and, although its return has been promised, its resources have been so considerably impaired that it will be some years before a full return to the pre-war plan is possible. The *Zeitschrift* was continued by Friedlaender from Switzerland by the issue of occasional numbers, but here also a considerable time must elapse before its pre-war status is fully re-established. It is altogether likely that Friedlaender's recent visit to the volcanoes of Mexico and Hawaii will give new impetus to the activities both of the laboratory and its journal.

In compensation for this diminution of activity in Europe the establishment of systematic studies of volcanism in the Dutch East Indies promises to be most fruitful. Founded in 1917 by the Dutch Colonial Government, the records of this work already fill four volumes (*Vulkanologische Mededeelingen,* published in the Dutch language in 1921 and 1922) containing not only records of a wide variety of volcanic outbreaks, but also of sand flows and hot springs. Indeed the record of the eruptions which have occurred in Java and Sumatra during the last few years appears to be very complete.

The administration of this undertaking has taken the form of a commission modeled after the Imperial Earthquake Investigation Commission of Japan, and if appropriate support is provided will offer a continued service to the Meteorological Service.

In this country the most active work is at present being done by Professor Jaggar's Hawaiian laboratory, now under the direction of the U. S. Weather Bureau, and by the Geophysical Laboratory of the Carnegie Institution of Washington. Both of these institutions have been interested since 1912 in establishing upon a stable basis the study of the physical and chemical forces underlying volcanic activity, and both have issued a considerable number of publications in this field, some reasonably complete and some still tentative, and both have in hand more or less definite plans for future development. In particular Professor Jaggar has brought together the necessary funds for a number of borings in the vicinity of Kilauea crater and has already discovered an altogether unexpected temperature gradient both immediately adjacent to the lava basin and in the older deposits known locally as the Sulphur Banks. In the three borings so far made, one of which has reached a depth of nearly 100 feet, no temperature above the boiling point of water corresponding to that elevation has been found. In the case of the boring near the active lava basin this must have the effect of revising considerably our notions of the dissipation of heat through thermal conductivity.

On the chemical side the borings have served once more to emphasize the participation of water in all of the relations there prevailing, but have not thus far yielded any new data.

The Geophysical Laboratory has completed its survey of the outbreak at Lassen Peak from 1914 to 1917 and has announced what appears to be a plausible explanation of the volcanic phenomena there, which differ considerably from those studied elsewhere in that the temperatures have been found to be very low in view of the violence of the outbreak. This new hypothesis may be expected to have the effect of stimulating further observation of volcanoes of the explosive type in order to determine whether the hypothesis is to find general application or will merely serve to explain an unusual local condition.

A somewhat more elaborate study of the various types of thermal springs associated with volcanism has also resulted from these studies at Lassen Peak. The latter study of hot springs has not been published and the hypothesis in explanation of the mechanism of the activity is thus far only tentatively announced.

The Geophysical Laboratory was also privileged to join with the National Geographic Society in a study of the Katmai eruption of 1912 and the remarkable sand deposit and fumaroles of the Valley of Ten Thousand Smokes. These have been studied both from the geological and the chemical viewpoint, and a number of facts established which appear to be novel in the science of volcanology. So great an extent and intensity of fumarolic activity has perhaps never been witnessed before and the character of the hot sand flow covering several square miles is in many respects unique. Some of the results of these studies have already been published by the National Geographic Society.

Regarding the recommendation made by the Section last year that effort be exerted to secure a complete record of all volcanic outbreaks, it may be of interest to the Section to know that Professor Karl Sapper of Würzburg is continuing his chronological record, and that he now has in preparation a book upon this and other recent work.

On the whole, therefore, although it is not possible to announce a full return of pre-war interest in the study of volcanism, this field of activity has certainly not been lost sight of, and further intensive studies may soon be expected from Japan, from Italy, from the Dutch East Indies, and from the United States.

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STATUS AND PROBLEMS OF GEOPHYSICAL CHEMISTRY

ROBERT B. SOSMAN

The subject-matter of geophysical chemistry was defined three years ago, upon the organization of this Section, as "the physical properties and chemical reactions of the substances and aggregates that make up the Earth." The Section thus occupies the position of a kind of physico-chemical technical adviser to the other Sections of the Union. The members of Section (g) like to feel that their work is fundamental and necessary to the work of all the other Sections. They feel that if research on the physics and chemistry of Earth materials were abandoned entirely, the result would be that while progress in the other branches would by no means cease, it would be progress only of the kind that is easily foreseen, the kind of progress for which an Appropriations Committee can make detailed and specific appropriations covering several years ahead; this is desirable, but we must also strive for the kind of progress which stands out in the history of science when viewed from a distance of fifty or a hundred years, the kind which starts unexpectedly and goes in unforeseen directions. Such advances are dependent upon the development of new methods and instruments and the discovery of new facts in physics and chemistry.

In the organization report just referred to, the subject-matter was roughly classified into two principal groups: (a) Materials at the Earth's surface, and (b) Materials in the Earth's interior. Each was further subdivided into (1) Individual chemical substances, such as calcium carbonate; (2) Aggregates, such as granite; and (3) Larger units of matter, such as glaciers. There is not time for an exhaustive review of the progress of knowledge in these different branches, and I shall, therefore, simply mention various items that come to mind as illustrating the present status of research in fundamental geophysics and geochemistry, as seen by the laboratory investigator.

Individual Substances.—The Geophysical Laboratory of the Carnegie Institution is still the only endowed laboratory devoting the major part of its attention to research in this field. Important papers marking progress in our knowledge of the silicates have been published since 1920, and include papers on the system SiO_2:MgO:CaO, the augites, the melilites, potash feldspar, and the silicates of barium and strontium.

University laboratories in this country in which similar work is being done include Princeton (silicates), Oberlin (colloidal silica and other oxides), Illinois, Stanford, and Rice Institute (colloidal oxides).

Mention should also be made of valuable data on silica and other important oxides contributed by the research laboratories of manufacturing companies, such as the Harbison-Walker Refractories Company.

Governmental institutions have also contributed. For example, we have from the National Museum the continuation of important data on meteorites and the chemical properties of various minerals; and from the Bureau of Chemistry of the Department of Agriculture, work on the constitution of silicates.

The standard reference books for information on the individual chemical substances of the Earth's surface remain, as for some years past, Clarke's *Data of Geochemistry,* Hintze's *Mineralogie,* Dana's *Mineralogy,* and Doelter's *Mineralchemie.* There is a pressing need for a general work on lines more nearly related to the present current of advance in physical chemistry than those named.

Foreign Work.—Some work on the Earth-forming oxides, begun in Norway a few years ago, seems to have been abandoned for want of funds, while to balance this loss a new laboratory is under construction in Sweden, in which it is planned to do research on silicates. The impoverishment of central Europe has greatly diminished the contributions from that quarter, an important source of supply now being industrial laboratories, which in the nature of the case can publish only what is not highly valuable to the industry supporting the work. A geochemical journal, the *Chemie der Erde,* seems to have perished with the completion of volume 1 in 1920, and the *Silikat-Zeitschrift* has not been heard from since volume 2, 1914. On the other hand, *Tschermak's Mineralogische und Petrographische Mitteilungen* has been revived, after a lapse of five years, with the aid of Austrian, American, and Swedish funds.

Plans were made a few years ago for a laboratory in England similar to the Geophysical Laboratory in Washington; it is to be hoped that these plans will be put into execution soon. Some good work has been done in England on silica, principally for its industrial application. In Japan, work on the Earth-forming oxides is being done in several university laboratories, and several new periodicals, including the *Japanese Journal of Astronomy and Geophysics,* are making available, in English, papers which might otherwise remain inaccessible to most of us because of their original publication only in the Japanese language.

Aggregates.—That branch of geophysical-chemical work which I have classified as work "on the properties and reactions of aggregates" has made progress in the continuation of contributions from the Geophysical Laboratory and the Geological Survey on the composition of the igneous rocks and of the Earth's "crust" as a whole; also papers on the principles governing the differentiation of an igneous magma by crystallization under the active influence of gravity, and the development of pressure in a volcanic magma as a result of crystallization.

On the movements of magmas there is an important theoretical contribution showing how gravitational energy may be converted into heat energy within a magma by the process of extrusion from a region of higher to a region of lower pressure.

Compared with the work on igneous rocks and magmas, comparatively little geochemical or geophysical work has been done on metamorphic or sedimentary rocks. Work has been done in some of the directions suggested by Dr. Vaughan in his report on "Research on marine sediments" at the 1922 meeting of this Union, but it is rather to be classed as geological research—the study of natural processes not under experimental control.

The Larger Units.—In the group of researches on the larger units of the Earth and on the Earth as a whole there should be mentioned such contributions as those from the Coast and Geodetic Survey on the Earth's field of force, from the University of Chicago on the mathematics of isostasy, and similar strictly geophysical contributions which are, however, of such particular interest to the Section of Geodesy or other Sections that this passing mention is sufficient.

Among researches bearing on problems of the Earth's interior should be mentioned the recently published data from the Geophysical Laboratory on the compressibilities of minerals and rocks over a wide range of pressures. These data confirm and greatly extend the work of Adams and Coker, and make possible some important deductions regarding the Earth's rigidity, and its properties with respect to earthquake waves.

Future of the Section.—I have referred at the beginning of this report to the possible usefulness of this Section as a technical consultant to the other Sections of the Union. In turn the members of this Section also have interests of their own, closely allied to physics, chemistry, and geology, in which interests they can receive valuable aid by consultation with the geodesists, seismologists, and the other groups in the Union. The most useful activity of the Section at this annual meeting would seem to be, therefore, to meet jointly with one of the other Sections to discuss problems of common interest.

Such a meeting has been arranged this year with the Section of Volcanology, and the subject chosen is "The temperatures of hot springs and the sources of their heat and water supply." The formation of hot springs has often, but not always, accompanied visible or historically recent volcanic activity. On the other hand, hot springs persist in regions where the volcanic activity is of very ancient date, and even in regions where no connection with any igneous rocks can be proved by geologic evidence alone. What is the source of the heat under these varied circumstances, and—a matter of equal interest—

where does the water come from? Speaking broadly, the physicists and chemists have evidence which inclines them to believe that most of the water has come down fairly recently in the form of rain. The geologists and volcanologists, on the other hand, impressed by the evidence of the large part played by water in the formation of igneous rocks, are speculating as to where that water has gone in the past, and incline to the belief that many hot springs represent water which is now being released by crystallizing magmas.

By such meetings as the one just mentioned we may hope to eliminate those differences of opinion which are due mainly to the possession of different kinds of information by the different groups.

This plan raises the question, however, whether a separate organization of Section (g) is necessary or desirable. At the meeting of the International Union there seemed to be no interest in the formation of any corresponding international Section, the opinion being that the subject-matter is so much more closely allied to physics, chemistry, and geology that any diversion of interest is to be avoided. This is a question of organization and administration which may be best referred to the Executive Committee of the Union.

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RESOLUTIONS

RESOLUTIONS ON SCIENTIFIC QUESTIONS ADOPTED BY THE AMERICAN GEOPHYSICAL UNION, APRIL 19, 1923

(a) That the National Research Council be requested to do whatever may seem practicable to it towards inducing authors of scientific papers to give postal addresses.

(b) Whereas, The United States statutes at large, Vol. 37, p. 913, provide that

> "Hereafter the Executive shall not extend or accept any invitation to participate in any international congress, conference or like event, without first having specific authority of law to do so;" and

Whereas, Since other provisions of law make it impossible for officers of the government to attend and participate in an official capacity in any international foreign congress, convention, meeting or gathering, thus greatly embarassing the scientific departments of the government in conducting the important duties and responsibilities devolving on them, and since the principal object of such congresses, conventions, meetings, or gatherings is to secure between nations the most effective and profitable co-operation in scientific matters; therefore be it

Resolved, That the American Geophysical Union recommend to the National Research Council that appropriate action be taken to present to the Congress of the United States the disadvantages resulting from the effects of the laws referred to above with the object of securing the necessary legislation permitting the Executive and the heads of the various departments to designate duly accredited representatives to attend international foreign scientific congresses, conventions, meetings or gatherings. This legislation should apply especially to meetings of the International Research Council and its branches, and the International Meteorological Committee and like organizations.

(c) That the National Research Council be requested to consider, if found desirable, the question of the publication of seismological data; what data to publish, where to publish, when to publish, and the form of publication.

(d) That the National Research Council be requested to secure if possible a mathematical discussion of the transmission of seismic waves through the earth, and associated phenomena.

(e) That the National Research Council be requested to secure a study of the transmission of artificially produced seismic waves.

(f) Whereas, progress in meteorology and day-to-day weather forecasting demands a map as complete as possible embracing the entire northern hemisphere or even the entire globe, as far as this may be feasible, and whereas, Such an undertaking requires complete and effective co-operation between all the nations of the globe conducting systematic meteorological observations; therefore, be it

Resolved, That the American Geophysical Union endorses the project of securing a daily weather map of the northern hemisphere, based on telegraphic and radio reports of observations, to the forthcoming International Meteorological Conference to be held at Utrecht, Holland, in September, 1923, and further recommends that such action be taken as may lead to such international co-operation as will result in some one government undertaking the collection of the necessary observational reports and the preparation and publication of a daily map of the northern hemisphere and the distribution of the same to the participating governments.

With the inauguration of this work the circulation of the atmosphere, and other phenomena, as exhibited on a series of maps of this kind could be submitted to the students of the world for the advancement of the science of meteorology and improvement of the forecasting of the weather. It is suggested that the expense of such an undertaking, as far as this relates to the actual compilation and publication of the map itself, should be met either by the sale of the maps or by definite contributions from the co-operating nations.

(g) Whereas, Two resolutions reading as follows were adopted by the Section of Terrestrial Magnetism and Electricity of the International Geodetic and Geophysical Union at its meeting at Rome in 1922:

> "That national committees be requested to designate, if possible, one observatory in their respective countries for international intercomparisons of magnetic instruments, and to secure intercomparisons of magnetic instruments within their own countries at least once within the course of three years.
>
> "That it is desirable there should be in every country at least one observatory making systematic atmospheric electric observations (especially of potential gradient, earth-air currents, conductivity, and number of ions), which are intercomparable amongst themselves and comparable with similar observations made in other countries;" therefore be it

Resolved, That the American Geophysical Union designate the Cheltenham Magnetic Observatory of the United States Coast and Geodetic Survey and the Standardizing Magnetic Observatory of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, working in conjunction, as the observatories in the United

States to function in accordance with the first of the two Rome resolutions quoted above; and be it further

Resolved, That the American Geophysical Union endorses the second Rome resolution mentioned above and recommends the early establishment of such an observatory, without at present designating a particular observatory in order not to tie the hands of any organization which may find it possible to carry on the work specified in the resolution in question.

ABSTRACTS OF REPORTS AND PAPERS

SECTION OF GEODESY

ON THE ACCURACY OF TIME DETERMINATIONS

R. H. Morgan

Recent intercomparisons of wireless time signals show variations of $0^s.2$ to $0^s.3$ in the time from one observatory as compared with that from others, and suggestions have been made that such variations may be due to the astronomical observations, and that longitude work may be subject to similar uncertainties. This paper gives the results of several different investigations into the differences in clock corrections as determined from observations on the different instruments at the U. S. Naval Observatory, on over a thousand nights in the last twenty years.

From these comparisons it was found that the clock corrections determined from definitive reductions of simultaneous observations on two standard instruments rarely differ as much as $0^s.1$, and that the average difference is $0^s.03$. The probable error of a clock correction is $0^s.014$. The large instruments hold together within $0^s.03$ for a number of years at a time.

It was also found that the systematic or irregular variations of $0^s.1$ or $0^s.2$ which occur in time signals are more or less due to the provisional nature of the observations and reductions used, and that by properly planning the provisional work such variations in the preliminary times may be partly eliminated and partly corrected for by later definitive reductions; and that longitude determinations need not be subject to uncertainties of this size.

U. S. Naval Observatory,
Washington, D. C.

THE ACCURACY OF LONGITUDE DETERMINATIONS BY THE U. S. COAST AND GEODETIC SURVEY BY WIRE SIGNALS AND GENERAL FERRIÉ'S PROPOSED RADIO LONGITUDE NET

William Bowie

Almost from the beginning of telegraphy the U. S. Coast and Geodetic Survey determined differences of longitude by telegraph, and a network of lines between longitude stations has been spread over this country. An analysis of this work is opportune, as determinations of differences in longitude by wire signals will soon be superseded by work in which radio signals are used. It is important that the determinations by radio shall not be less accurate than the old work.

In 1897 an adjustment of the longitude net was made in which 63 differences of longitude were used. The average correction to a difference was $0^s.019$. There were 36 corrections less than $0^s.02$ and only two greater than $0^s.04$. Since 1897 new determinations have been fitted into the adjusted net.

Since 1897, there have been determined 98 differences of longitude forming all or parts of 18 closed loops. Of these, 57 received corrections of $0^s.01$ or less, while only 9 differences had corrections greater than $0^s.02$. No correction was as great as $0^s.03$.

During 1922, seven differences of longitude by radio signals were determined. Preliminary results indicate the same accuracy as that obtained in wire longitude determinations.

Gen. Ferrié of the Bureau of Longitudes, Paris, has a comprehensive plan to determine the longitude of fundamental stations around the world by means of radio signals. His plan has received much attention from astronomers and geodesists. Undoubtedly the work contemplated by Gen. Ferrié's plan will be done in the not distant future, but there is some doubt as to the advisibility of incurring the expense that would be involved in having many stations occupied simultaneously, as is proposed by Gen. Ferrié.

His plan is practicable, in so far as accuracy is concerned, provided that methods are adopted which will eliminate systematic errors in the time observations, and provided that the radio signals are received automatically, and that corrections are applied for the reaction time of the various pieces of apparatus used in sending and receiving the signals.

The use of radio greatly enlarges the areas within which accurate longitude determinations can be made.

U. S. Coast and Geodetic Survey,
Washington, D. C.

THE ERRORS WHICH MAY BE EXPECTED IN RADIO LONGITUDE DETERMINATIONS CAUSED BY THE AUTOMATIC SENDING AND RECEIVING APPARATUS

E. A. Eckhardt

It seems to be generally agreed that the time observations as now made by astronomical methods do not yield values which may be relied upon beyond the nearest $0^s.01$. This fixes the magnitude of the time errors which is admissible in radio transmitting and receiving apparatus when they are used for time-signal work in longitude determinations.

It is those parts of the apparatus which involve mechanical and electrical inertias, which are most likely to lead to lags and possible errors. Telephone diaphragms, relay armatures and relay coils are examples.

Electro-mechanical relays are used considerably, both in signal-sending and signal-receiving apparatus. The magnitude of the lags and their constancy are considerably affected by the manner in which the relays are used. Obviously that adjustment and arrangement which provides the quickest current changes in both the controlling and controlled circuit and the quickest motion of the armature will result in a minimum of lag. Relay users, however, frequently do not follow the practice that this would suggest. A method of studying relay performance, with special reference to magnitude and constancy of lags, was outlined and some typical results were presented and analyzed.

The problem of using relays in receiving radio signals was discussed. Constancy of lag can not be obtained if the relay operating current changes with signal strength. The nature of this difficulty was discussed in some detail and its avoidance by using a radio receiving circuit which is itself of the relay type was considered. The field-service apparatus now in use by the U. S. Coast and Geodetic Survey for radio time-signal recording is of this type. Its lag errors are within the limits required to make it available for use in exact longitude determinations. Where the astronomical observations are made at a field station instead of a fixed observatory, the field-service design of this apparatus makes it especially available for this purpose.

Bureau of Standards,
Washington, D. C.

THE PLACE OF GEODESY IN THE WORK OF THE COMMITTEE ON SEISMOLOGY OF THE CARNEGIE INSTITUTION

ARTHUR L. DAY

An outline was given of the work that has been undertaken by the U. S. Coast and Geodetic Survey in the form of precise triangulation to show whether the earth's crust in California is stable or is in movement and, if the latter, the rate and the extent of the movement. The triangulation which has already been done and that which is under way during the present year will, in the near future, be supplemented by precise leveling which will show the degree of stability of the earth's surface in the vertical direction.

It is only by geodetic work of the most precise character that movements of the earth's surface can be detected between seismic disturbances. With the repetition from time to time of the triangulation and leveling in selected areas it will be possible to make a very careful study of surface conditions and movements. With such information available the seismologist will be able to apply quantitative methods which will greatly advance the science.

Geophysical Laboratory,
Carnegie Institution of Washington,
Washington, D. C.

THE RESULTS OF THE TRIANGULATION DONE IN CALIFORNIA IN 1922 TO TEST THE STABILITY OF THE EARTH'S SURFACE

O. S. Adams

The redetermination of the positions of triangulation stations made in 1906-07, in the California earthquake region, was based upon the stations at Mt. Diablo, and Mocho, situated some fifty miles east of the San Andreas fault line. Since there was some question whether these points might not have moved with respect to each other, the third determination, also by precise triangulation, of the points in 1922 was made to depend upon Mt. Lola and Round Top, located some 160 or more miles east of the fault line.

The mean of the triangle closures was 0″.76 and the maximum closure was 1″.93. Of the 37 closed triangles, 28 had closures of one second or less. These results show that this triangulation is somewhat better than the usual requirement for precise work. There were thirty-five condition equations in the adjustment with a resulting probable error of an observed direction of ±0″.32.

The difference between the 1922 positions and those of 1906 show a displacement of a little less than four feet in a general southward direction. This is less than one part in 200,000 of the distance from Mt. Lola to Round Top, and in azimuth it represents about one second of twist. A southward displacement would require a combination of length and azimuth discrepancy. The lengths of triangulation, in general, can be depended upon to about one part in 100,000; in a scheme tied together as this is, one second in twist should hardly be expected. However, if the old determination of Mt. Diablo and Mocho should have been affected by an azimuth displacement northward and these 1922 determinations by one southward, the azimuth twist could be less even than a half second which might well be expected.

	Displacement 1922–1906 Based on Mt. Lola and Round Top		Displacement 1922–1906 Mt. Diablo and Mocho held fixed	
	Lat. ft.	Long. ft.	Lat. ft.	Long. ft.
Mt. Diablo	−3.4	−0.8	....	....
Mocho	−3.5	−2.0	....	....
Ross Mt.	−3.7	+2.8	−0.2	+4.2
Mt. Tamalpais	−3.6	+0.6	−0.1	+2.0
Sierra Morena	−3.5	−0.5	0.0	+0.9
Loma Prieta	−2.0	−2.9	+1.5	−1.5
Lick Observatory, small dome	−3.7	−2.2	−0.2	−0.8
Farrallon L. H.	−4.9	+0.6	−1.4	+2.0
Point Reyes L. H.	−4.6	+0.4	−1.1	+1.8

The minus sign indicates movement to the south and to the east in latitude and longitude, respectively.

U. S. Coast and Geodetic Survey,
Washington, D. C.

MEASUREMENT OF DISTANCE BETWEEN MOUNT WILSON AND SAN ANTONIO PEAK MADE IN CONNECTION WITH THE DETERMINATION OF THE VELOCITY OF LIGHT BY PROFESSOR A. A. MICHELSON

C. V. Hodgson

The work was undertaken in response to a request by Dr. A. A. Michelson that the Coast and Geodetic Survey determine the length of the line, over 20 miles in length, between Mount Wilson and San Antonio Mountain, with an accuracy represented by an actual error of not to exceed 1/250,000. To obtain the length of this line, which extended over very rugged country, it was necessary to measure a base in the valley lying to the south, and to carry the base length by triangulation to the line desired.

Before beginning the field work, the members of the Division of Geodesy of the Survey examined critically the modern methods of base measurement and triangulation. In the triangulation the choice of figure, the probable error of a direction, and the method of determining the differences of elevations of the stations were examined with special care. The figure selected gives the greatest possible strength in the connection between the base and the Michelson line. Field methods were adopted which would insure a probable error of a direction of not more than about 0″.2 of arc, or about half that obtained in ordinary precise triangulation, and would give the differences of elevation with an error of not more than one foot.

The investigations concerning base measurement were directed principally toward the problem of standards, the accurate measurement of the tension applied to the tape, the probable errors allowable in the grade and alignment corrections, and the general problem of adopting methods of measurement which would minimize the effect of unavoidable small systematic errors.

The location of the base and of the Michelson line made it very difficult to secure the requisite accuracy. The valley in which the base was located was under a high state of cultivation and the base changed in direction so frequently that 19 angle stations were necessary on the base line alone. Moreover, while observing from the base stations to the mountain stations the atmospheric conditions were very unfavorable, as they always are when observing from a valley toward mountains. The field reports, however, indicated a maximum triangle closure of only about 0″.7 and an average closure of 0″.4 of arc, and it therefore seems probable that the length of the Michelson line will be determined with the desired accuracy.

U. S. Coast and Geodetic Survey,
Washington, D. C.

THE DENSITIES OF DEEP-SEATED MATERIAL OF THE EARTH'S CRUST, AS DETERMINED BY CHEMICAL ANALYSIS, AND THE BEARING OF SUCH DENSITIES ON THE THEORY OF ISOSTASY

Henry S. Washington

A résumé was given of the work which had been done by the author, showing that the densities of igneous rocks found at different places on the earth's surface, as computed from chemical analyses, bear a very definite relation to the elevation of the ground at which the rocks are found. The higher the elevation of the earth's surface the less is the density of the igneous rocks. This fact fits in well with the theory of isostasy which postulates that under each topographic feature there is a compensating deficiency or excess in density. The investigations in rock densities and isostasy will give information in regard to the earth's crust that apparently cannot be gained by other means.

Geophysical Laboratory,
Carnegie Institution of Washington,
Washington, D. C.

FORCES TENDING TO CAUSE MOVEMENTS IN THE EARTH'S CRUST

W. D. Lambert

The title is extremely comprehensive. Of all the numerous manifestations of force in the earth's crust two have been selected for their bearing on the Taylor-Wegener hypothesis of continental migration: (1) a small force tending to move continents towards the equator; (2) a possible way in which the earth's axis of rotation might be displaced in the body of the earth.

Wegener's form of the hypothesis assumes that the continents may be likened to blocks of lighter matter (called "sial") floating on a magmatic substratum (called "sima"). To move these continental blocks there is needed a *selective* force, one which acts on the continents, not as on mere undifferentiated portions of the crust, but which has on the continents an effect different from that which it has on the surrounding magma. There is such a force tending toward the equator and roughly proportional to the height of the continent above the sustaining magma. It is, however, very minute (of the order of 1/1,000,000 of gravity), and is probably ineffective in moving the continental blocks against the resistance of the "sima."

Large displacements in the position of the earth's axis of rotation in the body of the earth play an important part in Wegener's scheme, although they are not a necessary part of a theory of continental migration. Wegener's explanation of these displacements of the axis or pole (taken from a memoir by Schiaparelli) is mechanically unsound. There is, however, a possibility, not considered by Wegener, of explaining fairly large polar displacements.

Since the earth's rotation is being slowed down by tidal friction, the earth in the past had presumably a greater ellipticity and its free period in the variation of latitude (now 14 months) was more nearly one year, which is the period of forced annual oscillation of the pole due to meteorological causes. This near coincidence of periods would produce resonance and the amplitude of the oscillations in latitude would be greatly increased, perhaps so greatly that the earth's crust would be ruptured under the resultant stresses, thereby producing something the same effect as if the earth were fluid, so that a new mean position of the pole might be established. Our equations for the variation of latitude are limited to small oscillations, so that we cannot follow the changes with mathematical rigor.

U. S. Coast and Geodetic Survey,
Washington, D. C.

SOME PHASES OF THE RESISTANCE OF THE EARTH'S MATERIAL TO CHANGES

C. E. Van Orstrand[1]

The rocks in the crust of the earth are constantly undergoing readjustment as a result of metamorphism, thermal changes, and redistribution of load. Apart from the evidence afforded by shore lines and regions of seismic activity, there is perhaps no well-authenticated evidence of surface movements now in progress. Artificial movements are oftentimes developed in the rocks beneath stone quarries as a result of diminishing the vertical cross section over which the stresses are distributed. It is found in this way that the rocks in certain areas are in a state of ease—practically no strain—while in other areas the stresses approach the elastic limit.

Contrasted with these small displacements are the enormous displacements which have occurred in geological time, some of which are undoubtedly vertical, while in others, the overthrust faults, it appears that large masses have been displaced horizontally over distances of several miles. The Serpent Mound Structure in Adams County, Ohio, and the Bearpaw Mountains, Montana, which are being studied respectively by Professor Walter H. Bucher of the University of Cincinnati, and Mr. Frank Reeves of the United States Geological Survey, are excellent examples of vertical displacement due apparently to gravitational forces. The Lewis overthrust, Montana, is a conspicuous example of horizontal displacement extending over a length of more than 100 miles. Mr. M. R. Campbell, of the United States Geological Survey, estimates the displacement at the southern end of the fault to be about 15 miles. A complete solution of the problem of overthrust faults is perhaps impossible, but it is not improbable that the displacements, following the lines of least resistance, are vertical in the lower portion of the isostatic column, but at or near the top of the column they may be inclined to the vertical, and in extreme cases, may even be horizontal.

The kind of equilibrium acquired by rock masses may be suggested by the after-effect curves of steel tapes. With small loads the curves approach an asymptote parallel to the axis of time, while with larger loads, but not necessarily exceeding the elastic limit, the asymptotes are inclined to the axis of time and there is no approach to a definite state of equilibrium.

Jeffreys' recent evaluation of the thermal contraction of the globe tends to show that cooling alone is more than sufficient to account for the contraction necessary to produce the mountain ranges of the continents. An obvious difficulty encountered in the thermal-contraction hypothesis of mountain building is the concentration of practically all of the thermal contraction of the globe in the small areas occupied by the mountain ranges. It is perhaps not unreasonable, therefore, to assume that the chief forces involved in mountain building are gravitational and therefore vertical—while the strains resulting from thermal contraction are released from time to time and play a minor part only in the development of a mountain range.

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[1] Published with the permission of the Acting Director, U. S. Geological Survey.

U. S. Geological Survey,
Washington, D. C.

SOME RECENT PROGRESS IN ISOSTATIC INVESTIGATIONS

WILLIAM BOWIE

The principal progress made in the theory of isostasy during the past year or two has been the recognition given the subject by scientific organizations dealing with the earth.

At the meeting of the Geological Society of America, held at Amherst in December, 1921, there was a symposium on isostasy at which were presented nine papers which dealt with relations between the theory of isostasy and geological phenomena.

At the meeting of the same Society at Ann Arbor in December, 1922, there were presented several papers in which the theory of isostasy was given consideration.

A valuable addition to isostatic literature is "The Strandflat and Isostasy" by Dr. Fridtjof Nansen, which appeared in December, 1922. The author shows that the isostatic balance was maintained almost, if not perfectly, in Norway during the loading and unloading of the area by ice caps and their disappearance, respectively. He uses the physiographic evidence to show that areas studied by him are in very close isostatic equilibrium.

Possibly the greatest progress made in the theory of isostasy during recent times has been the establishment of the fact that gravity anomalies are, to a great extent, due to the effect of local deviations from normal densities in the material of the earth's crust close to the gravity stations. In the United States there are a number of cases where stations very close together have decided differences in their anomalies. These anomalies could not show such a deviation if their cause were a lack of isostatic balance of the portion of the isostatic shell below them, for if the cause were deep-seated the effect on two stations close together would be very nearly the same. It has been shown that gravity stations on the Cenozoic formation have a decided tendency to be negative in sign, and the largest negative anomalies in the United States are on that formation. It has also been found that the gravity stations on the pre-Cambrian formation have a tendency to be positive in sign, and the largest positive anomalies in the United States are at stations on that formation.

As a result of showing that the large gravity anomalies are due to the local rather than to regional causes, we are safe in concluding that the earth's crust is more nearly in a state of equilibrium than had been supposed previously.

U. S. Coast and Geodetic Survey,
Washington, D. C.

SECTION OF SEISMOLOGY

INFORMAL COMMUNICATION OF THE SEISMOLOGICAL WORK OF THE U. S. COAST AND GEODETIC SURVEY

N. H. HECK

The proposed transfer of seismological investigation from the Weather Bureau to the Coast and Geodetic Survey, which was recommended to Congress by the Secretaries of Agriculture and Commerce, was discussed—the reason for the proposed action being given. The opportunity was taken to indicate to the Section of Seismology what this action might result in if approved. It was stated that the various seismologists throughout the country are interested in such a change. The definite program outlined included the following:

1. Establish a first-class seismological station at the Magnetic Observatory at Tucson, Arizona, without an increase of personnel at the present. The value of such a station near but not in the California earthquake region was pointed out.
2. Further development would include a similar program at Sitka, Alaska and Porto Rico. At Honolulu a new building and an instrument for measuring the vertical component would meet the needs.
3. Discussion of the further possibilities for the Continental United States was deferred.

It was pointed out that if this program resulted in the establishment of only one first-class seismological station, the effort would be worth while. The need for the assignment of one full-time mathematical physicist for interpretation work was indicated as essential to such a program.

U. S. Coast and Geodetic Survey,
Washington, D. C.

THE SEISMOGRAPH AND FRICTION

HARRY FIELDING REID

The friction at the marking point of seismographs recording on smoked paper has always been a difficulty. The direction of the frictional force is always changing. The elementary theory in which the friction is considered a constant force and at right angles to the lever has proved unsatisfactory. Galítzin attempted to improve it by assuming that the friction is a constant plus term proportional to the first and the second powers of the velocity, but he neglected the variation in the direction.

Direct experiments show that the friction is practically a constant, and that it is necessary to take account of its varying direction.

When the seismograph is recording a simple harmonic movement the direction of the movement varies harmonically; it can therefore be represented by a Fourier's series, in which the coefficients can be determined. Mathematical analysis shows that a very close approximation to the true effect can be obtained by neglecting all terms of the series except the first. The general equation of the seismograph can then be solved and the magnification of the earth movement determined. The formula obtained is similar to the known formula for the magnification of harmonic movements, except that it contains an additional term to take account of the friction. This term can readily be determined; and a series of tables makes it possible quickly to find the true magnification to a very close degree of approximation.

Johns Hopkins University,
Baltimore, Maryland.

SECTION OF METEOROLOGY

AN EXAMINATION OF THE DUST CONTENT OF THE ATMOSPHERE

Herbert H. Kimball

At the Rome meetings of the International Geodetic and Geophysical Union in May, 1922, Sir Napier Shaw, President of the Meteorological Section, proposed the purchase of twelve Owen's Dust Counters for distribution to observatories in different countries affiliated with the Union, to be used in an international study of the dust content of the atmosphere.

The importance of this study is apparent from the close relation known to exist between atmospheric dustiness and visibility. Since visibility is one of the most important meteorological factors in aviation, an accurate forecast of this phase of the weather is desirable. Hence, the importance of accurate data on atmospheric dustiness.

Also, at intervals, volcanic eruptions throw vast quantities of dust high into the atmosphere, where it may float for months or even years, covering perhaps an entire hemisphere, and diminishing the intensity of incoming solar radiation. Also, it is suspected that at times dust of cosmical origin finds its way into our atmosphere. The adequate study of these questions demands international co-operation.

One of the twelve dust counters distributed as proposed above was received by the U. S. Weather Bureau, and since December 7, 1922, a count of the number of dust particles contained in a cubic centimeter of air has been made at Washington on each working day, generally at the American University, D. C., in a suburb about 3 miles northwest of the city, but sometimes at the base and top of the Washington monument, which is in a smoky section of the city.

The number of dust particles has been found to vary from about 100 to 4,000 per cubic centimeter.

With the minimum number of dust particles the Blue Ridge Mountains in Virginia, 50 miles distant, are clearly visible. With the maximum number of particles the visibility is ½ mile or less, depending upon the humidity of the atmosphere.

Dust counts have been made on two occasions during aeroplane flights. The first of these was to a height of 12,000 feet. The number of dust particles per cubic centimeter increased from a minimum of 499 at the surface to a maximum of 1,716 at a height of 9,000 feet, and fell off to 1,357 at 12,000 feet. On the second flight the number of particles per cubic centimeter was 678 at the surface, 1,741 at 3,000 feet, and 158 at 6,000 feet, which was at the base of a cloud layer from which rain commenced to fall shortly afterwards.

On the first flight there appeared to be a relation between air movements at different levels and dustiness.

While attended with difficulties, the feasibility and utility of dust counting from aeroplanes appears to be established. Further development of this line of work is expected.

U. S. Weather Bureau,
Washington, D. C.

THE METEOROLOGICAL WORK ON THE *JACQUES CARTIER*

E. H. Bowie

The *S. S. Jacques Cartier,* a freighter of 19,300 tons owned by the Compagnie Generale Transatlantique, in addition to functioning as a freighter, serves as a training school for prospective officers of the merchant marine.

Through the co-operation of the National Meteorological Service of France (Office Nationale Meteorologique) the *Jacques Cartier* has also been employed to test the practicability of maintaining a floating weather service for the collection of meteorological observations by radio, the issue of weather forecasts and warnings for the guidance of vessels navigating the oceans. Supervising Forecaster Edward H. Bowie of the Weather Bureau, on the invitation of the Compagnie Generale Transatlantique, extended through Captain Coyecque, professor and meteorologist of the Training School, made the voyage, New Orleans to Le Havre, February 11 to March 2, 1923, and returning from Boulogne to Santa Cruz del Sur March 13-30, 1923.

In charting the meteorological observations received on board the vessel main dependence was had on the observations broadcast from Arlington and the Eiffel Tower there being no time on either the outward or the inward voyage when the vessel was not in communication with one or both of these stations. The remarkable fact remains that it is possible at times for a vessel at sea to receive observations from Constantinople on the east to the Pacific coast on the west.

A single example of the weather information broadcast by the *Jacques Cartier* will convey an excellent idea of the scope of the meteorological activities of the vessel. The example follows:

Broadcast—March 20th—Greenwich Noon

Meteo. Jacques Cartier to all ships: North Atlantic weather situation March 20th, Greenwich noon. Pressure is high over northwestern Europe; 30.36 also from Bermuda to Azores and northward also over American Atlantic States. Trough of low pressure extending from Iceland (29.60) southeastward to Gibraltar

(29.50). Disturbance of great intensity (29.00) over Newfoundland moving east-northeastward.

Forecasts: Track, Azores to Gibraltar—unsettled weather eastern portion—northerly fresh winds, squally weather and overcast weather western portion. Stop. Tracks, Azores to English Channel—overcast weather and moderate easterly winds eastern portion—fresh or strong northerly winds and squally weather western portion. Stop. Azores region—fresh northerly winds moderating today and shifting to southeast and south tomorrow—overcast weather becoming fair. Track—Azores to Bermuda—Moderating northerly winds and fair weather between longitudes 30° and 42°. Southerly increasing winds shifting to southwest west of 42°, weather becoming overcast and probably rainy.

Advisory for westbound ships over transatlantic tracks west of Ireland to south of Grand Banks: Disturbance of great intensity (29.00) over Newfoundland moving toward Iceland will be attended by strong shifting winds today and tonight between longitudes 35° and 55° followed by fresh or strong northerly winds. Winds will shift tomorrow morning between 40° and 50°.

U. S. Weather Bureau,
Washington, D. C.

INDIAN SUMMER AS A CHARACTERISTIC WEATHER TYPE OF THE EASTERN UNITED STATES

Robert De C. Ward

The author's studies of the weather maps which characterize Indian Summer weather have led him to recognize a type of pressure distribution which uniformly gives several days of weather commonly called Indian Summer. This pressure distribution was described as a more or less stagnant area of high pressure over the southeastern States and a weak cyclonic area passing slowly down the Saint Lawrence Valley. The resulting winds are light, and the moderate chill of the early morning and evening resulting from a fairly well marked diurnal range of temperature is swept away by the genial warmth of the light southerly winds, and by the strong isolation of the daytime.

He also emphasized the fact that the Indian Summer pressure type can and does occur in all seasons, but that the lack of precipitation, the prevailing haziness, and the temporary stagnation of the general atmospheric circulation are features most fully developed in autumn.

Harvard University,
Cambridge, Massachusetts.

TEMPERATURE INVERSIONS IN THE FREE AIR

W. J. Humphreys

It is well known that a temperature inversion, that is, an increase of temperature with increase of elevation for a short distance, often occurs at the upper surface of a stratus cloud. Also that a similar inversion is frequently found at about the same height when there are no clouds.

Some, perhaps all, of these inversions, especially those that occur on cloudless days, are caused, as Sir Napier Shaw has explained, by the turbulence of the lower air incident to winds. This stirring up of the lower air tends to bring it all to an equipotential temperature, hence to warm the under portion of the agitated layer and to cool the upper portion.

The less pronounced inversions in the higher portions of the atmosphere occur at boundaries between layers of wind that differ more or less in speed or direction, or, nearly always, both. In these cases turbulence, so far as it exists, warms, as explained above, the under portion of the agitated layer, and produces a temperature inversion, or, at least, a decrease in the temperature lapse with increase of elevation, at its upper surface.

However, since layers of air of different density glide over each other without rapid intermingling, it follows that the higher and less pronounced inversions of the free air are owing, chiefly, to the different origins of the winds immediately above and below them.

U. S. Weather Bureau,
Washington, D. C.

THE WIND FACTOR IN FLIGHT: AN ANALYSIS OF ONE YEAR'S RECORD OF THE AIR MAIL[1]

WILLIS RAY GREGG and LIEUT. J. PARKER VAN ZANDT

From an analysis of one year's records of the Air Mail Service between New York and San Francisco it has been found that at the average altitude of flight, about 1,500 feet above the ground, allowance must be made for a wind of 7 miles per hour from the west. In other words, westward flights will, on the average, be made at a speed of 14 miles per hour less than will eastward flights. A more detailed study of the New York to Chicago part of the route gives almost exactly the same wind factor as for the entire trans-continental route. This value of the wind factor has been verified by an examination of 8,667 free air observations with kites and pilot balloons, and the agreement is remarkably close. The importance of this agreement lies in the fact that, in fixing flight schedules in other regions or at other altitudes, dependence can be placed upon *either* method in case only one is available.

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[1] Paper presented at Washington, D. C., before the American Meteorological Society on April 16, 1923, and the American Geophysical Union, Section (c) Meteorology, on April 18, 1923.

From a further study of the data, schedules that can be guaranteed 90% of the time have been determined for aircraft of any cruising speed between 50 and 150 miles per hour. In making up these schedules allowance has been made for head winds of 36 miles per hour or more in westward flight and 20 miles per hour or more in eastward flight, these being the wind speeds that are shown by kite and balloon records to occur 5% of the time. When they do occur, the flights will be somewhat delayed, but nevertheless completed. During the remaining 5% of the time flights are likely to fail altogether or be seriously delayed because of exceptionally unfavorable weather, such as severe rain or snow storms, poor visibility, etc.

Finally, a general wind curve has been prepared by means of which various schedules based on other percentage performances, greater or less than 90% and for different lengths of routes, may be determined. With the normal and high cruising speeds known quantities, allowance is made for an adverse wind such that the flight can be made in any desired time. The percentage frequency of this adverse wind is then read from the curve and thus is determined the percentage of the time that the desired schedule can be guaranteed.

U. S. Weather Bureau (Gregg),
Washington, D. C.
U. S. Air Service (Van Zandt),
San Diego, California.

PROGRESSIVE CHANGES IN THE FIELD OF METEOROLOGICAL OBSERVATIONS OF THE U. S. WEATHER BUREAU

A. J. Henry

Two maps were shown. The first portrayed the distribution of stations reporting by telegraph to the Central Office of the Signal Service in Washington, D. C., at the end of the first year of operation as a Federal Weather Service. The number of stations was 55. The second map showed the location of stations reporting to the Weather Bureau at the present time both by the electric telegraph and cable lines and by radio transmission from ships at sea. The number of stations reporting at the present is 332 distributed over widely separated parts of the globe. The Asiatic coast and the Philippines in the Far East, the Islands of the Pacific, Alaska and the West Indies are some of the regions from whence reports are received.

U. S. Weather Bureau,
Washington, D. C.

SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY

REPORT OF THE COMMITTEE ON ATMOSPHERIC-ELECTRIC PHENOMENA AND MEASUREMENTS IN THE TROPOSPHERE AND STRATOSPHERE[1]

W. F. G. SWANN, *Chairman*

Last year I reported that three members of the committee had undertaken the preparation of reports, Dr. Austin having promised a report on Wireless Strays, Dr. Blair a report on Instrumental Technique pertaining to upper air investigation, and Dr. Mauchly a report on the Diurnal Variations of the Atmospheric Electric Elements. Dr. Humphreys, to whom we already owe much of our knowledge concerning thunderstorms, promised to look into the question again at this stage, and see whether the time was ripe for a further report. He has concluded that the amount of new material which is not already summarized in existing reports is not sufficient to warrant the preparation of a further account at this time.

Dr. Austin has tendered his report, and has enriched our knowledge of the field by the publication of several papers on the subject of wireless strays and allied phenomena. Dr. Mauchly has also completed a valuable report on the Diurnal Variation of the Atmospheric Potential-Gradient.

In addition to the foregoing, individual members of the committee and those working in their laboratories have been independently active in investigations pertaining to the fields of our interests. It is perhaps unnecessary to give a detailed list of publications, but will suffice to say that the activities in question have comprised investigations on the Penetrating Radiation with special reference to work at high altitudes, on the Variation of Residual Ionization in Gases with Pressure, on the Nature of Ionic Recombination in the Atmosphere, on the Theory of Ionization by Rapidly Moving Particles, on the Action of the Heavisidean Layer, on Wireless Strays and Allied Phenomena, on the Diurnal Variation of the Potential Gradient, and on certain instrumental matters, for example, two investigations pertaining to the Ebert Ion-Counter. On all of these topics papers have appeared, have been presented at learned societies, or are in process of publication.

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[1] Presented at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923. Members of the Committee: L. W. Austin, W. R. Blair, W. J. Humphreys, S. J. Mauchly, R. A. Millikan, and W. F. G. Swann, chairman.

Last year certain tentative plans of a certain group of scientists for an Algæ Survey were brought to the attention of the Committee, and the opportunity for the prosecution of atmospheric electric observations was pointed out. The committee took cognizance of this information, and the section passed resolutions in sympathy with the project, promising more active co-operation at such time as plans had been more definitely formed by the organization. A letter has been received from one of the members of the organization, the purport of which is to the effect that the plans are temporarily in abeyance pending the consummation of a plan for financing the undertaking. The outcome is expected to be known in a very short time, but until that time no further action can of course be taken by the committee.

With regard to the Committee's plan for initiating the prosecution of atmospheric electric observations at such educational institutions as would be willing to undertake such work, I may say that a beginning has been made at the University of Minnesota, where Professor Zeleny has undertaken to pursue such work, and is making a beginning by installing a potential-gradient apparatus.

University of Minnesota,
Minneapolis, Minn.

REPORT OF THE COMMITTEE ON EARTH CURRENTS AND POLAR LIGHTS[1]

S. J. Mauchly, *Chairman*

The resumption, in February 1922, of earth-current registrations on both the N-S and E-W lines of the Ebro Observatory at Tortosa, Spain, following an interruption of several months, is considered most fortunate. While the need for the recent repairs and replacements is regretted, it is hoped that the changes made will not materially affect the value of the records when these are considered as additions to what is already the longest available series of continuous earth-current registrations. The resumption of registration is especially fortunate, also, because of the opportunity which it will afford for comparative study of the Ebro results with those soon to be obtained at the Watheroo Magnetic Observatory.

The Department of Terrestrial Magnetism of the Carnegie Institution of Washington has well under way the completion of its preliminary earth-current installation at Watheroo, Western Australia. The initial N-S and E-W lines for this installation are each 2 miles (3.2 km.) long and will constitute the only special installation for earth-current registration in the Southern Hemisphere. However, much interest attaches to the new installation apart from its geographic position and its availability as a comparison station. Briefly stated these additional features are as follows: (1) use of a multiple-point recording potentiometer as recording instrument; (2) automatic comparison of results from 1-mile (1.6 km.) and 2-mile (3.2 km.) lines, respectively, on both N-S and E-W lines; (3) comparison of results obtained by the use of overhead lines with those obtained from underground cables laid in insulating conduit. In addition, it is planned to make a resistivity survey of the entire region occupied by the system for each of the two seasons which differ most regarding the moisture and temperature of the soil, and to devote considerable attention to a study of the characteristics of several types of electrodes as regards suitability for continuous earth-current registration. The progress, to date, of the work on this installation makes it appear very probable that registration will be under way by the end of 1923.

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[1] Presented at the annual meeting of the Section of Terrestrial Magnetism and Atmospheric Electricity of the American Geophysical Union, Washington, April 18, 1923. Members of Committee: J. H. Dellinger, N. H. Heck, A. J. Henry, A. G. McAdie, S. J. Mauchly, chairman, and W. E. Parker.

As a result of the work of the MacMillan Baffin Land Expedition of 1921-22 there is now available a record of the aurorae observed from November 1921 to June 1922 at Bowdoin Harbor (64°4 N, 78°0 W) which was Dr. MacMillan's winter quarters. An auroral log is also being kept at Sitka, Alaska, by Mr. F. P. Ulrich, observer in charge at the Sitka Magnetic Observatory of the United States Coast and Geodetic Survey. Since aurorae are also being observed by the "Maud Expedition," under Captain Amundsen, the observations of Mr. Ulrich may prove to be of unusual value in conjunction with those from other stations co-operating with the Expedition.[3]

Some interest has been aroused in the possibility of a connection between aurorae and the phenomena of radio communication by the fact that the failure of a recent attempt to relay a radio message across Canada was accompanied by aurorae. It has been suggested by the Committee that interested radio operators who are suitably stationed could probably obtain valuable data which might be of assistance in determining the nature and magnitude of any interrelation which may exist between the two phenomena. For example, without going into the details of procedure, two co-operating observers, far apart along an approximate east and west line, could readily determine the existence of any effect upon communication between their stations and the nature and intensity of such effects. Precisely analogous remarks would apply to two observers separated by a great north-south distance. It is of course presumed that, of each pair of co-operative radio observers, at least one is at a sending station, and at least one in a region of high auroral frequency; also that stated periods of simultaneous recording of both radio and auroral conditions have been agreed upon by the observers. From a study of the data from several series of such co-operative observations, especially if they cover simultaneous observations during aurorae, for both the E-W and N-S directions, it should be possible to draw certain preliminary conclusions of value both from the view-point of radio communication and as regards indirect evidence concerning the ionization of the upper regions of the atmosphere during aurorae. [The foregoing ideas will be carried out, so far as possible, during the ensuing year, through the use of a specially prepared form by radio stations in Canada relaying radio messages to and from the MacMillan Arctic Expedition, 1923-24.]

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[3] At the time this report was prepared for publication Dr. MacMillan had completed preparations for an expedition to the Arctic regions in 1923–24, with an observational program including auroral observations similar to those made in 1921–22.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

PROGRESS REPORT OF THE COMMITTEE ON RELATIONS BETWEEN TERRESTRIAL MAGNETISM, TERRESTRIAL ELECTRICITY, AND SOLAR ACTIVITY[1]

LOUIS A. BAUER, *Chairman*

Measures of Solar, Magnetic, and Electric Activity.—The compilation and computation of measures of solar activity, magnetic activity, and electric activity, based on all available data, have been continued by the Department of Terrestrial Magnetism. Thanks to the cooperation of Professor Wolfer of Zürich, we are promptly supplied with his relative sun-spot numbers, which are published as soon as possible in the *Journal of Terrestrial Magnetism and Atmospheric Electricity* in order to make the data available to others. We have at present the final sun-spot numbers through 1918 and the provisional ones complete from 1919 to March 1923, inclusive. From these data, it appears that for the present cycle, the minimum sun-spot activity occurred in 1913, and an extraordinarily large maximum (103.9 for the mean of the year) in 1917; it is not yet possible to say whether the minimum has again been reached. The mean number for 1922 was 14.7, and the mean monthly numbers for January to March 1923 are, respectively: 5.3, 1.6, and 4.0. There has been no such high maximum as that of 1917 since 1870, when the mean number for the year was 139.1. For the first time since 1871 has the monthly mean number exceeded 150, as it did in August 1917, viz, 154.5.

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[1] Presented at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923. Members of Committee: C. G. Abbot, Louis A. Bauer, chairman; R. L. Faris, George E. Hale, and C. F. Marvin.

An important publication during the year has been that by Hale on *"Invisible Sun-spots."*[2] At the Rome 1922 meeting of the International Astronomical Union, various committees on solar phenomena were combined into one under Hale's chairmanship; there are 21 widely-distributed members, among whom, of our Union, are Hale, Abbot, and Bauer.

Abbot has reported to the Committee the general results of the *solar-radiation observations,* October 1920 to September 1922, made at the two stations of the Smithsonian Institution, Harqua Hala, Arizona, and Montezuma, Chile. He calls attention to the high solar-constant values until 1922, usually exceeding the normal value of about 1.94 calories. Nothing so marked as the change in 1922 to low values has occurred since the observations were begun at Mount Wilson in 1905. (Dr. Abbot also personally presented his report at the meeting of the Section; it will be found published, in expanded form, in the *Monthly Weather Review* for February 1923, pp. 71-81.)

Marvin reports continuation of the attempts to find *relationships between meteorology and solar activity.* An outline of this work was presented by him at the meeting of the American Meteorological Society at Washington, on April 16, 1923.

Bauer has continued the disclosing of *relationships between terrestrial magnetism, terrestrial electricity, and solar activity.* The twelve-year series of observations at the Observatorio del Ebro, Tortosa, Spain, 1910-1921, pertaining to earth currents, atmospheric electricity, and terrestrial magnetism, have been subjected to discussion and analysis, as also the available atmospheric-electric data at other observatories. The following general conclusions[3] are drawn:

1. The relationship between sun-spot activity and atmospheric electricity turns out to be, for locally undisturbed stations, as definite as in the case of terrestrial magnetism; the sun-spot influence on the periodic variations of the atmospheric potential-gradient is, in general, as great as on the periodic variations of terrestrial magnetism; and as concerns the effects on the absolute values, the sun-spot

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[2] *Proc. Nat. Acad. Sciences,* July 1922, pp. 168–170; see also abstract in the *Year Book of the Carnegie Institution of Washington* for 1922, p. 212.

[3] See *Terr. Mag.,* vol. 27, 1922, p. 30.

influence is about 300 times greater in atmospheric electricity than in terrestrial magnetism. The potential gradient of atmospheric electricity, and, presumably, the Earth's total negative electric charge, is furthermore subject to an annual or secular change, which may vary in magnitude and sign from one sun-spot cycle to another.

2. The potential gradients of earth currents, as recorded at the Ebro Observatory, 1910–1920, varied during the sun-spot cycle, decreasing in the direction of normal flow of current with increased sun-spot activity; the diurnal range, however, increased with increased sun-spot activity.
3. There is evidence of a similar six-hour wave in atmospheric electricity, earth currents, and terrestrial magnetism.

The bearing of conclusion 1 on the question of the origin and maintenance of the Earth's electric charge needs no elaboration here.

The interesting six-hour wave referred to in the third conclusion, the amplitude of which increases with increased solar activity, just as does that of the 24-hour wave, has received further attention. The Fourier analysis of the potential gradient of atmospheric electricity has been made in the Department of Terrestrial Magnetism for each month of the mean year of the 11-year series for the Ebro Observatory. Once more the six-hour wave is found to be of peculiar interest. For every month of the mean year, excepting November, December, and January, its amplitude is larger than that of the eight-hour wave, and on the average for the year the amplitude is about one-third that of the 24-hour wave and nearly half that of the 12-hour wave. Its phase angle, relatively, shows the same constancy within less than half-hour, during the year, as does the 12-hour wave. It is not possible to enter into further details, but my colleague, Mr. Mauchly, will also have something interesting to report in his paper regarding a six-hour wave. Unfortunately previous analyses have frequently stopped with the third term, and it was assumed that the series would thereafter converge. The flaring up of the amplitude of the fourth wave, even at such an exceptionally favorably located station as is the Observatorio del Ebro, is a matter of considerable interest.

Another feature of interest is the double wave shown by the annual variation in the diurnal variation of the potential gradient, as disclosed by the combined amplitude, as well as by the amplitudes of the 12-hour and the 6-hour waves separately. The maximum amplitudes occur near the equinoctial months, the minimum near the solstitial months. Striking similarities in the annual variation of the following phenomena have now been shown to subsist: Magnetic disturbances, earth-current disturbances, polar-light frequencies, atmospheric-electric disturbances, and the earth-effect on solar activity.

Even the lag in the second maximum, occurring in October instead of September, is shown by all phenomena.[4]

The annual variation of other geophysical quantities has likewise required attention in the pursuit of the relations sought for by the Committee. For example, the annual variation of the potential gradient of atmospheric electricity, as observed at stations distributed over the Earth from the Arctic to the Antarctic regions, shows phenomena similar to those resulting from a system of vertical electric currents caused by the translatory motion of an electrically-charged sphere through the ether, for example, by the charged Earth during its orbital motion about the Sun; this hypothesis is being further investigated. The available data reveal the following types:

Type *a*. From the Arctic regions to about parallel 33° North and from about 40° South to the Antarctic regions, the maximum potential gradient occurs near the December solstice and the minimum near the June solstice.

Type *b*. In the region from about 33° North to 40° South, or over about half of the Earth's surface in the lower latitudes, some of the stations show a reversed annual variation to that of type *a*, hence, maximum potential gradient near June solstice and minimum near December solstice.

Type *c*. In region for *b*, or between *a* and *b*, there are certain stations showing a mixed type of *a* and *b*.

On the average, from the Arctic to the Antarctic, the annual range of the potential gradient is about 60 per cent of the average potential gradient for the year; the data in the North Hemisphere seemingly indicate that the range decreases as the region for type *b* is approached. There are some indications that the bounding parallels between regions of types *a* and *b* may be found to be magnetic parallels, rather than geographic ones.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

REPORT OF THE COMMITTEE ON MAGNETIC STANDARDS, AND INSTRUMENTS AND METHODS OF AERIAL MAGNETIC MEASUREMENTS[1]

J. A. Fleming, *Chairman*

Comparisons of magnetic and electric methods for determining the horizontal intensity of the Earth's magnetic field.—With respect to magnetic standards, the outstanding work during the year was the completion of an indirect comparison between three electro-magnetic instruments for determining the horizontal intensity of the Earth's magnetic field, viz, the Schuster-Smith magnetometer of England, the Watanabe electric magnetometer of Japan, and the Carnegie Institution of Washington sine galvanometer designed by Dr. Barnett and constructed by the Department of Terrestrial Magnetism. These comparisons were obtained by Messrs. Parkinson and Brown of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington using standard magnetometers of the Department for the comparisons at the National Physical Laboratory, Teddington, England, and at the magnetic observatory, Kakioka, Japan; the magnetometers were later carefully compared with the sine galvanometer at Washington. The results show that the Schuster-Smith magnetometer and the sine galvanometer give values differing by less than 1/10,000 part of the value of the horizontal intensity, H, of the Earth's magnetic field, or less. The Watanabe instrument, however, for some reason indicates values greater by 1/3,000 part of H, but field instruments were used for this work and there is somewhat greater uncertainty in the result than for the other comparisons.

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[4] See *Terr. Mag.*, vol. 26, 1921, pp. 113–115, vol. 27, 1922, p. 27.

[1] Presented at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923. Members of Committee: C. G. Abbot, J. P. Ault, S. J. Barnett, L. J. Briggs, J. A. Fleming, chairman, and N. H. Heck. The Committee carried on its duties by correspondence, and the report was compiled largely from sub-reports submitted by Messrs. Ault, Briggs, Fleming, and Heck.

The provisional international magnetic standard for H adopted in 1914 by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, as based on all available data respecting the results of inter-comparisons of magnetic instruments made the world over by various organizations and the Department, was found from extensive comparisons made at Washington, D. C., in June and August 1921 and again in February and March 1923 to be in practical agreement with the sine galvanometer. Thus it is found that the provisional H-standard gives results agreeing within 1/5,000 part of H or less with those from electro-magnetic methods (excluding Japan), and hence is well within the limit of error for all practical and theoretical requirements.

Dr. Chree reports that the comparisons of his standard magnetometer with the Schuster-Smith magnetometer also show substantial agreement between the magnetic and electric instruments.

Maintenance of standards provisionally adopted by the Department of Terrestrial Magnetism.—The results of comparisons at Washington between the standard magnetometer and earth inductor and magnetic instruments of various designs, as well as the comparisons with the electric instrument referred to above, show a satisfactory maintenance of the standards well within the limits of error of observation. The compilations of results again show that for the desired precision it is necessary to exercise close control on the magnetometer constants, and particularly on changes in the moment of inertia of the oscillating magnet and its suspension and on possible changes in the distribution factors for the magnet systems used.

Comparisons of magnetic standards at observatories.—Comparisons of magnetic standards have been made by the Department of Terrestrial Magnetism, since the last report of the Committee, at 14 observatories in France, England, Scotland, Denmark, Finland, Germany, Holland, Belgium, Spain, Portugal, Algeria, Egypt, Japan, and Australia. Mr. W. C. Parkinson of the staff of the Department of Terrestrial Magnetism made the comparisons at 12 of these observatories, while Messrs. F. C. Brown and D. G. Coleman of the Department secured the comparisons in Japan and Australia. The results indicate again the necessity of frequent control comparisons, with well-determined standards, for all observatories. It is particularly emphasized, as a result of these comparisons, that the earth inductor should replace the dip circle for the determination of magnetic inclination.

Magnetic standards of the United States Coast and Geodetic Survey.—Since the establishment of the Cheltenham magnetic observatory the regular absolute instruments of that observatory have been regarded as the standard instruments of the U. S. Coast and Geodetic Survey and all other absolute instruments of the Bureau have been referred to them either directly or indirectly. In the case of field instruments, direct comparisons are usually made at the beginning and end of each field season; in the case of the instruments at other observatories, direct comparisons have been made at every opportunity, as when an instrument is sent to the office for repairs, but main dependence is placed on indirect comparisons through field instruments.

Comparisons at Cheltenham with instruments of the Department of Terrestrial Magnetism showed that the Cheltenham instruments gave results in substantial agreement with the provisionally adopted international magnetic standards of that Department, so far as dip and declination were concerned, but that for horizontal intensity the results were too great by about one part in one thousand. The same thing was indicated by the average of the various field magnetometers of the Coast and Geodetic Survey. Accordingly it was decided, beginning with 1913, to adopt as the standard for horizontal intensity the results with magnetometer No. 26 at Cheltenham diminished by 1 part in 1,000.

The constancy of this H-standard has been controlled by repeated comparisons with other magnetometers, in particular magnetometer No. 40, an India Survey pattern instrument held in reserve as a spare observatory magnetometer.

In 1918 this spare magnetometer was compared with the standard magnetometer of the Department of Terrestrial Magnetism and was then sent for comparison to the Tucson, Honolulu, and Sitka observatories, and returned for another comparison at Cheltenham in 1921. In 1922 a magnetometer of the Carnegie Institution type was compared successively at Cheltenham, Sitka, Tucson, and Cheltenham.

As regards dip, a spare earth inductor has not heretofore been available for indirect comparison between the one at Cheltenham and those at the other observatories, but one is expected from the maker shortly. However, direct comparisons were made in 1922 with the instruments from Porto Rico and Tucson when they were sent to the office for repairs.

Additional control of the instruments at the Honolulu observatory was obtained through comparison with the instruments of the *Carnegie* in 1921.

These various comparisons indicate a satisfactory constancy of the observatory instruments in recent years, but it is not safe to assume that such a condition will continue indefinitely. The same remark applies with greater force to field instruments, which are subjected to more variable and less favorable working conditions.

Horizontal intensity is the element for which the instrumental constants are most likely to undergo change; of these the moment of inertia and the distribution factors require most attention. The apparent variation of the distribution factor from month to month and from year to year deserves careful study.

Development of instruments and methods for aerial measurements. —At the Bureau of Standards considerable work has been done in the development of an Earth-Inductor Compass, designed especially for use in airplanes. The instrument differs from previous compasses of this type in the following particulars:

(1) It employs a null method for its indications, and hence possesses the advantage of sensitivity characteristic of null methods as a class.

(2) An iron core is used in the earth inductor to reduce the size of the instrument.

(3) A method of distant electrical control is used, enabling the earth inductor itself to be located as remotely as necessary to avoid disturbances from magnetic material.

(4) Rolling and pitching errors are eliminated.

The armature is cross-shaped with long thin cores to reduce as much as possible the self-demagnetizing effect. The cores are made of 3½ per cent silicon steel with a length 20 times their diameter. The winding is of the usual closed coil pattern. The shaft of the armature is kept in a vertical position by supporting it from gimbal rings. The bearings of these rings are frictionally damped and the armature is weighted. By this means a good rolling and pitching compensation is obtained. The armature is wind driven, using cup-shaped vanes, the connection being made through a flexible shaft. The instrument may be located in the rear part of the fuselage away from the local magnetic disturbances of the engine.

Two sets of brushes of 90 degrees take off current from a 4-section commutator. A closed potentiometer circuit is provided in the form of a square and the 4 brushes are connected to the 4 corners of this square. Two wiping contacts 180 degrees apart draw current from the square frame and are connected to the galvanometer. These wiping contacts are controlled by a dial mounted coaxially with the frame and carrying compass graduations. A position of the dial can always be found in which the galvanometer current is zero, and there is a one-to-one correspondence between this neutral position of the dial and the angle of orientation of the aircraft. A square resistance frame is used to avoid a non-uniformly graduated dial which would result from a circular frame.

It has been found practicable to use this compass in the air for an accuracy of perhaps 2 or 3 degrees.[3]

Attempts have been made at the Bureau of Standards to use an especially devised dip needle in the air with the idea of determining the latitude of the aircraft approximately through reference to charts of magnetic dip. These attempts have proved unsuccessful owing to the peculiar effects of the vibration of the aircraft on the motion of the dip needle. The subject is, however, still under investigation. An earth inductor for determining dip has been constructed and is now under test. The level base of reference in this instrument is a bubble.

One of the chief efforts of Army and Navy Aviation experimenters during the past year has been to try to improve the magnetic compass. Several officers from these Departments have visited the Department of Terrestrial Magnetism and have been given such assistance as was possible to further their efforts in this direction. The attitude of practical aviators seems to be to demand a better compass, and some have felt that if the compass could be depended upon for 90 per cent of the time, all other instruments could be thrown overboard.

Dirigibles would seem to be the better adapted for aerial magnetic work than airplanes on account of their longer cruising radius, convenience for mounting observing outfits, greater stability, etc. The problem of supplying a non-inflammable gas is not yet solved. Helium is too expensive as yet for general use and is regarded as not necessary for commercial use. As an extra safeguard against fire, the all-metal airship is strongly recommended. The all-metal envelope is a protection against static electricity. If a hole is torn in this envelope and the gas catches fire, it will burn as a jet without exploding the airship if the hole does not enlarge. If all-metal airships become the rule, the mounting of magnetic measuring instruments would be complicated unless special metals are used in the construction of certain parts of the airship.

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[3] A full description of the instrument may be found in *Proc. Am. Phil. Soc.*, vol. 61, No. 1, 1922.

The present tendency in the development of aerial navigation instruments is to emphasize the need of perfecting the radio compass and direction-finder instead of magnetic instruments.

The Loth magnetic cable is being successfully developed to guide an airship from point to point, and in case of landing in a fog the cable is used to guide the aviator to the landing field.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

REPORT OF THE COMMITTEE ON RELATIONS BETWEEN LOCAL MAGNETIC DISTURBANCES, GRAVITY ANOMALIES, AND GEOLOGICAL FORMATIONS[1]

D. L. HAZARD, *Chairman*

During the past year considerable time has been devoted to the study of the relation between local magnetic disturbances and geological formations by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, and the U. S. Coast and Geodetic Survey.

The months of July, August, and September were spent by Mr. H. W. Fisk of the Department of Terrestrial Magnetism and his assistant, Mr. J. T. Howard, in Bermuda, where a special study of the magnetic disturbance there was made.

Observations at five primary stations of 1907 show that the secular change between 1907 and 1922 is sensibly the same at all stations as regards horizontal intensity and dip. In declination there is evidence of a sensible difference in different regions.

Diurnal-variation observations carried out simultaneously at two stations in regions of maximum and minimum values of each element indicate that the diurnal variation in horizontal intensity is not affected by the anomaly to a degree measurable by available field methods.

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[1] Presented at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923. Members of Committee: D. L. Hazard, chairman; N. M. Fenneman, G. W. Littlehales, and W. J. Peters.

The diurnal-variation curves for declination where this element has widely different values appear to be identical, but for points where the horizontal intensity has widely differing values there is evidence of a slight difference.

Observations were made at over 100 secondary stations and in greater detail in regions of special interest from which it is concluded that the disturbance may be separated into two parts: (*a*) The primary disturbance affecting large areas, connected with the underlying volcanic structure. (*b*) The secondary disturbance affecting very limited areas, apparently arising from magnetic properties of the soil of varying thicknesses. The intermediate coral limestone was found to be not magnetic.

At the request of Mr. W. R. Crane, Superintendent of the Southern Experiment Station of the Bureau of Mines, an investigation was undertaken by the U. S. Coast and Geodetic Survey during the summer of 1922 to determine whether deposits of non-magnetic iron ore could be detected by means of magnetic observations. Some observations which Mr. Crane had made indicated the possibility that such a method might be used with success, and it was considered worth while to test it by using a more sensitive instrument and more refined methods.

The investigation was carried out in Alabama, the observations comprising three parts: (1) Observations in the brown iron ore district, about 30 miles southwest of Birmingham. (2) Observations at Red Mountain Gap, just outside of Birmingham. (3) Detailed observations near a large specimen of brown iron ore (limonite) and a large specimen of red iron ore (hematite) on the grounds of the University of Alabama at Tuscaloosa.

The owners of the brown iron ore property kindly furnished data regarding the underlying strata as derived from the drill records.

The results indicate that iron ores of the character under investigation do not produce an effect on the earth's magnetism large enough to be detected by ordinary magnetic observations even when the greatest care is taken. In the brown iron ore district the results at the different stations do not differ more than is to be expected, taking into account the error of observation and the uncertainty of the correction for diurnal variation, and there appears to be no relation between the small observed differences and the underlying ore bodies revealed by the drilling. The results at Red Mountain Gap show greater differences, but there is no apparent relation between them and the thickness of the ore body.

The vessels of the Coast and Geodetic Survey, operating in southeastern Alaska, continued during 1922 the investigation of areas of marked local disturbances in that region with a view to the determination of more details as to their extent and magnitude both on land and over water areas; these data will be valuable for the study of the relation of the observed disturbances to geologic formation as soon as information on the latter subject is available.

During the season of 1922 a number of gravity stations were established by the Coast and Geodetic Survey in Kansas, Oklahoma, and Texas at the request of the Director of the U. S. Geological Survey in order to throw some light on subsurface densities of the earth's material. A computation and adjustment of the gravity stations and a reduction of the stations for topography and compensation were completed during the month of March 1923, and the resulting gravity anomalies were furnished to the Director of the Geological Survey, where they will be studied by Dr. David White.

No additional information has become available since last year regarding the possible relation between gravity anomalies and local magnetic disturbances, and, until new instruments or new methods are available which will make it possible to secure gravity determinations more rapidly and in less accessible places than is possible with the pendulum, much progress is not to be expected. The Geophysical Laboratory, in co-operation with the Coast and Geodetic Survey, has been at work on the problem of devising such an instrument, but it is still in the experimental stage.

U. S. Coast and Geodetic Survey,
Washington, D. C.

REPORT OF ROME MEETING OF THE SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY OF THE INTERNATIONAL GEODETIC AND GEOPHYSICAL UNION, MAY 1922[1]

LOUIS A. BAUER, *Delegate*

A well-attended meeting of the Section of Terrestrial Magnetism and Electricity of the International Geodetic and Geophysical Union was held at Rome, May 4 to 9, 1922. Representatives from Australia, Belgium, Brazil, France, Great Britain, Italy, Japan, Poland, Spain, Sweden, and the United States were present at various sessions. Besides discussion of the Agenda, which contained leading questions pertaining to terrestrial magnetism and electricity, reports were presented by national and other committees and by leading investigators.

Professor Tanakadate (Japan) tendered his resignation as president of the Section, because of his inability to attend to the duties involved. Dr. Charles Chree (Great Britain) was then chosen president and Professor Luigi Palazzo (Italy) vice-president. According to the statutes, the secretary and director of the Central Bureau, Dr. Louis A. Bauer, continues in office until the next meeting, which will be held at Madrid, Spain, in 1924. Directors J. Jaumotte (Belgium) and Ch. Maurain (France), and Professor A. Tanakadate (Japan) were elected additional members of the Executive Committee. It was agreed that administrative matters should be left to the officers of the Section, who constitute the Bureau.

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[1] Presented at the meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

Five committees were appointed: (1) Committee on Magnetic Surveys and International Comparisons of Instruments (Louis A. Bauer, chairman); (2) Committee on Observational Work in Atmospheric Electricity to report on Objects, Instruments and Methods (G. C. Simpson, chairman); (3) Committee on Measures of Magnetic Characterization of Days; (4) Committee on Best Methods, Instruments, and Compilations of Polar Light Observations; (5) Committee to Consider and Report on Best Methods and Instruments for Earth-Current Observations (Sir Arthur Schuster, chairman). The appointment of chairmen of Committees 3 and 4 was deferred pending the early entrance into the Union of additional countries.

The representatives of the American Geophysical Union on these international committees are: Louis A. Bauer and N. H. Heck (No. 1); W. F. G. Swann (No. 2); R. L. Faris (No. 3); J. A. Fleming (No. 4); and S. J. Mauchly (secretary of No. 5).

The results of the meeting were embodied in the form of 20 Resolutions, which are published in the secretary's general report.[2]

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

THE BERMUDA ANOMALY[1]

H. W. Fisk

The peculiar interest in the local magnetic disturbance in Bermuda arises from the geological conditions with which it is connected. The island is a submerged mountain of volcanic origin covered with a coral limestone cap which, at the only point where a boring has been made, extends 250 feet below sea-level.

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[2] *Terr. Mag.*, vol. 27, 1922, pp. 89–100. For the complete transactions see Bulletin No. 3 of the Section of Terrestrial Magnetism and Electricity of the International Geodetic and Geophysical Union (Johns Hopkins press, Baltimore, Maryland, October, 1923).

[1] Abstract of paper presented before Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

The Department of Terrestrial Magnetism made a magnetic survey of the island in 1907 and repeated a part of the work in the summer of 1922, carrying out at the same time some special investigations relating to local disturbances. In this later work extensive tests were made which failed to discover any magnetic properties in the limestone either in fragments or in large masses. The magnetic distribution, however, indicates that, besides the general disturbance affecting widely separated areas, there are others of local character that have their origin much nearer the surface. The compass-variometer, an instrument designed and constructed by the Department for the detection of small differences of horizontal intensity, was used to study a few limited areas in detail. In one such there were found differences of as much as 700 gammas (0.00700 C.G.S.) within less than 200 feet horizontal distance. There was no evidence of an artificial source for this disturbance. The soil at various points on the island was examined and found to be magnetic, though the distribution of these local disturbances could not be directly related to the depth or to other apparent properties of the soil. There are, therefore, a major or primary source of disturbance lying deep in the lower structure of the submerged mountain, and other minor or secondary sources lying in or near the surface, probably in the soil.

Diurnal-variation observations with field instruments were made in declination simultaneously by two observers stationed at two points between which the difference was near a maximum. The simultaneous differences remained practically constant, and no evidence of any change in the diurnal variation either in range or phase due to the existence of the disturbance was found. A similar series of observations was made for the inclination using earth inductors, obtaining a value every 15 or 20 minutes, and for horizontal intensity using the method of deflections at one distance repeated at similar intervals. For these elements good diurnal-variation curves were obtained, although, because of the magnitude of unavoidable errors of observation, they were not a satisfactory basis for any conclusions.

In the course of the work in 1907, five stations were chosen as repeat stations and carefully marked for reoccupation. Two of these were reoccupied in 1910 by the *Carnegie* party, and all were reoccupied in 1922. The small differences in the 15-year change in inclination and horizontal intensity at the different stations show a slight tendency to vary with the value of the element. When the results from the nearest magnetic observatories are available, by which an approximate reduction of observations at all stations to a common epoch may be made, it may be possible to determine whether this tendency is real. In the case of the declination, the evidence is positive. The two stations which were occupied in August 1907, in January 1910, and in August 1922 were found to have declinations which differed by 3° 53′, 3° 59′, and 4° 18′ on the three occasions respectively, showing a steady increase in the difference with time. This increase seems to be related in some way to the declination at the two stations and is suggested also by the results at the other stations; for if the 15-year change is plotted for each of the five stations, using the declination as abscissae, the points fall near a line within limits on the order of the uncertainties of determination. It is hoped that a further refinement of the intensity observations may make it possible to reveal a connection between this phenomenon and the components of the disturbance field.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

THE EARTH-INDUCTOR COMPASS[1]

P. R. HEYL

The earth-inductor compass for airplanes, as developed at the Bureau of Standards contains several novel features:

1. A null method is used for the indications. As long as the ship holds the prescribed course, the needle of the indicating galvanometer reads zero. A deviation from the course on either side is accompanied by a corresponding movement of the needle, which thus acts as a turn-indicator.
2. An iron core is used in the indicator, enabling the instrument to be designed of a small size.
3. Rolling and pitching errors are eliminated by a specially damped gimbal suspension.
4. A means is provided of distinguishing between north and south, or, in general, between any two opposite bearings.
5. A distant electrical control is used, by means of which the inductor can be located at a distance from the engine, and the course-setting device and indicator be located at the instrument board.

Bureau of Standards,
Washington, D. C.

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[1] Abstract of paper presented before Section (d) of the American Geophysical Union, April 18, 1923.

MAGNETIC METHODS FOR LOCATING IRON ORE[1]

N. H. Heck

At the request of Dr. W. E. Crane, Superintendent, Southern Experiment Station, Bureau of Mines, Birmingham, Alabama, observations were made by the Coast and Geodetic Survey to determine whether or not so-called non-magnetic iron ore affected the magnetic needle sufficiently to use this method in the investigation of iron ore deposits. The possibility was rather doubtful, but the commercial importance of such a method warranted the investigation.

The total intensity was obtained from plain dip, and dip with loaded needle, the dip needle not being reversed in polarity during the season, and the loaded needle lying nearly horizontal. The reduction factor and temperature coefficient of loaded needle were determined at Cheltenham and verified at Birmingham. The total intensity was obtained from the expression $F=P \cos I' \csc (I-I')$, where F=total intensity, I' is dip with loaded needle, I is dip, and P is constant.

P reduced to 30° C. shows very slight change. It was found that an increase of 1° C. lowers loaded end of needle 3.09 minutes of arc. The regions of red ore (hematite) and brown ore (limonite) were each examined; observations were also made over large specimens of each ore in an undisturbed region.

Probable error of dip was 1 minute; loaded dip, 2 minutes; total intensity, 30γ. Diurnal variation was taken from Cheltenham, one magnetic storm occurring during observations.

Results: In brown ore region, variations in observations 10 feet apart were as great as for 100 feet; mean difference between adjoining poles was $3'.9$ with maximum difference of $9'.7$ in the loaded dip. The total range over the entire area was 18′. Less range was found in the other cases.

Conclusions: The observed differences are too near those of the probable error of observation to be significant. The needle is slightly affected by the presence of ore, but not sufficiently for interpretation.

AREAS OF MARKED LOCAL DISTURBANCE

The recently discovered large areas of local disturbances in southeastern Alaska are described. The Port Snettisham area is described in the Journal of Terrestrial Magnetism for December 1922. An area of 70 square miles in Lynn Canal has been found to have local disturbances with a range of declination of 25°. A new area was found in Union Bay, Ernest Sound, with variation of 38° from the normal value for the region. Seven square miles of this area were investigated.

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[1] Abstract of paper presented before Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

In other cases the investigation covered both land and water areas, the land observations being along the shores. At Port Snettisham and Union Bay two points of actual disturbances were indicated by observations. For the water areas the north end of the needle is drawn towards the shore where the maximum disturbance is noted, except in one case in Chilkat Inlet, where the needle apparently pointed away from the disturbed region and is deflected towards the area where normal values were observed—this is probably a local effect.

Attention was called to the large deflections in depths of water of from 400 to 800 feet. Need for geological investigation in this region was indicated.

U. S. Coast and Geodetic Survey,
Washington, D. C.

OBSERVATORY EQUIPMENT FOR RECORDING PHOTOGRAPHICALLY THE CONDUCTIVITY OF THE AIR[1]

S. J. Mauchly

The atmospheric-electric program at the observatories of the Department of Terrestrial Magnetism at Watheroo, Western Australia, and Huancayo, Peru, provides for separate continuous records of each of the unipolar conductivities, λ_+ and λ_-. The conductivity recording equipment for a given observatory consists of two subsidiary units, each of which is a modification of the well-known Gerdien conductivity apparatus similar, in general, to that described by Swann,[2] in 1917, but including certain features and improvements which have resulted from several years' experience, at Washington, with the original apparatus.

The records are obtained photographically by means of a quadrant electrometer and suitable accessories. One pair of quadrants of the electrometer is connected to the insulated central cylinder of the main condenser and the other to the case of the electrometer, which is maintained at a potential of the order of 100 volts. The two pairs of quadrants are permanently connected through a high-resistance radioactive cell, of the type developed in the Department of Terrestrial Magnetism by Swann and the author,[8] and the apparatus is thus readily adjusted for a linear relation between electrometer deflection and conductivity. The arrangements for calibration and for the elimination of spurious effects due to leak and to radioactive materials in the atmosphere are similar in principle to those described by Swann (l.c.).

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[1] Abstract of paper read at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

[2] See "Apparatus for Automatically Recording the Electric Conductivity of the Air," by W. F. G. Swann, in "Annual Report of the Director of the Department of Terrestrial Magnetism" for the year 1917, *Year Book of the Carnegie Institution of Washington,* 1917, p. 279.

In each of the two subsidiary units the "outer cylinder" of the portable Gerdien apparatus is represented by the middle section of a vertical air-flow tube extending through both the roof and the floor of a specially designed instrument house. The air flow in each tube is maintained by a fan driven by a small electric motor, both the motors and the electric recording lamps being operated from a 32-volt storage battery. Air is taken in through a suitable hood above the roof and discharged into an open space about 40 cm. high between the raised floor and the level of the ground. There is thus little chance of air being drawn a second time through the apparatus, and the maintenance of satisfactory insulation is greatly facilitated by the fact that dust and moisture from the air stream are prevented from depositing on the insulators of the electrometers and accessory apparatus.

As a result of changes in the fan and the lower part of the tube, the rate of air flow has been increased sufficiently to insure against saturation current between the inner and outer cylinders even under the most severe recording conditions.

Characteristic performance data were given, and normal conductivity records obtained at the Watheroo Magnetic Observatory were shown.

Fuller description will appear in the *Journal of Terrestrial Magnetism and Atmospheric Electricity.*

Department of Terrestrial Magnetism,
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Washington, D. C.

PROGRESS OF THE MAGNETIC SURVEY OF THE UNITED STATES[1]

D. L. HAZARD

The first general magnetic survey of the United States, begun by the Coast and Geodetic Survey in 1900, was practically completed in 1915. It involved the determination of the three magnetic elements, declination, dip, and horizontal intensity, at nearly every county seat in the country, and the reoccupation from time to time of a sufficient number of repeat stations in all parts of the country, to determine the change of the magnetic elements with lapse of time and to make it possible to reduce the observed values to any desired common epoch. The collected results of this survey reduced to January 1, 1915, were published in United States Magnetic Tables and Magnetic Charts for 1915, together with magnetic charts showing graphically the distribution of the declination, dip, horizontal intensity, vertical intensity, and total intensity for that date.

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[8] *Terr. Mag.*, vol. 22, 1917, pp. 1-21.

[1] Abstract of paper presented at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

This general survey showed that there are many regions in the United States where more detailed observations are required to develop the irregular distribution of the earth's magnetism, the so-called "areas of local disturbance." Since 1915 the field work has been largely devoted to the continued occupation of repeat stations and the establishment of more stations in disturbed areas. During and immediately after the war the force of observers was so small that little field work could be done other than to keep the secular-change data up to date, but in 1922 it was possible to resume the filling-in process.

While there has been an increasing number of requests for information regarding the earth's magnetism, particularly as to the change of declination with lapse of time in connection with the reestablishment of lost boundaries, it is clear that the use of the magnetic stations of the Coast and Geodetic Survey and the data available in the office of the Bureau is still far short of what it should be. Accordingly in 1922 the names of all county surveyors, so far as possible, were obtained from the State authorities and a circular letter was sent to each one having two objects in view: First, to call attention to the data in this office at his disposal, and second, to find out the present condition of the magnetic stations in his vicinity. This has resulted in a large increase in the number of requests for information and has brought out the fact that a large percentage of the magnetic stations have disappeared or are no longer suitable for use. Up to the end of March, 1923, reports had been received from 441 stations (about one-tenth of the whole number) of which 254 are no longer available. The establishment of new stations to meet local demands will be one of the most important practical features of the work in the immediate future.

Work of this character had already been undertaken early in 1922 in response to the request from the Florida Engineering Society for the establishment of a meridian line and magnetic station at every county seat.

The work of 1922 also included an investigation in Alabama of the possibility of using magnetic instruments for the location of deposits of non-magnetic iron ore.

The investigation of areas of local disturbance in southeastern Alaska was continued over both land and water areas, the observations being limited to declination.

The magnetic observatories at Vieques, P. R., Cheltenham, Md., Tucson, Ariz., Sitka, Alaska, and Honolulu, Hawaii, have continued in operation. Receiving sets have been in use at Cheltenham and Honolulu for some time for obtaining wireless time-signals, and one is about to be installed at the Porto Rico Observatory on Vieques Island. This Observatory soon may have to be discontinued or removed, due to poor living conditions at Vieques.

At the Cheltenham Observatory preparations are under way for an investigation of the effect produced on magnetic variometers by earthquake waves, a test of Professor Reid's theory, and an investigation of the control of the temperature coefficient of variometers, based on Mr. Hartnell's discussion of theory of such variometers.

At the Rome meeting of the International Section of Terrestrial Magnetism and Electricity a resolution was adopted "that it is desirable there should be in every country at least one observatory making systematic atmospheric-electric observations * * *." The Coast and Geodetic Survey is in sympathy with this resolution and is prepared to undertake such observations as soon as the necessary instruments can be secured, at either the Cheltenham or the Tucson Observatory. It also desires to add the study of earth currents to the program of one of its observatories when funds are available for the purpose.

In the reduction and publication of the observatory results several new methods have been adopted to facilitate the work and reduce the cost. These methods were discussed in detail.

Among the important publications may be mentioned "Compass Surveys" and "Magnetic Declination in the United States for January 1, 1920," issued primarily for the use of local surveyors.

The paper on "Horizontal-Intensity Variometers" by George Hartnell, Magnetic Observer, published in 1922, gives in detail the mathematical theory of the instrument and will be of great assistance in its operation and adjustment.

The routine computations and publication of field and observatory results have been carried on systematically, the latest publications of observatory results being for 1919 and 1920 and of field results for 1921.

U. S. Coast and Geodetic Survey,
Washington, D. C.

SUMMARY OF RESULTS OF A RECENT ANALYSIS OF THE EARTH'S MAGNETIC FIELD FOR 1922[1]

LOUIS A. BAUER

A new analysis of the Earth's magnetic field was made for 1922 by the writer, with the assistance of various members of the Department of Terrestrial Magnetism, as free as possible from assumptions as to the systems composing the entire magnetic field, and restricted, for the present, to the region of the Earth (86 per cent), from 60° North to 60° South. A spherical harmonic series to the sixth degree, and in some cases to the seventh, was established separately for each of the rectangular components, X (positive towards North), Y (positive towards East), and Z (positive towards nadir); the Earth was treated as a spheroid of revolution. Only some of the chief conclusions can be given here; for details, reference must be made to the full publication:[2]

It is found that for a satisfactory representation of the observed data, it is necessary to recognize the existence of an internal magnetic system, I, an external system, E, and a non-potential system, N, or of three equivalent systems, X, Y, Z. The I-system constitutes about 94 per cent of the total magnetic field, and E and N, each about 3 per cent.

The secular variation of the Earth's magnetism is caused both by changes, with lapse of time, in the direction of magnetization and by changes in the intensity of magnetization; the latter quantity has been steadily diminishing during the past 80 years at the annual average rate of about 1/1,500 part.

The north end of the magnetic axis of the I-system during the past 80 years has been moving slowly towards the west, and apparently at the same time slowly towards the equator. Were we to suppose that the magnetic axis made a complete revolution around the Earth's axis of rotation, the indications are that the period would not be a few hundred years, but many thousand years.

For the Earth's internal uniform magnetic field, the following data apply for 1922: The magnetic moment, M, is 8.04×10^{25} C.G.S.; the components of M, respectively parallel and perpendicular to the Earth's axis of rotation, are $M_p=7.88\times10^{25}$ C.G.S., and $M_e=1.60\times10^{25}$ C.G.S.; $M_p=4.93\ M_e$. Were the Earth's magnetism uniformly distributed through the Earth, as it probably is not, the average intensity of magnetism would be 0.074 C.G.S. The magnetic axis intersects the North Hemisphere in latitude 78° 32′ North and longitude 69° 08′ West of Greenwich.

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[1] Abstract of paper presented at the meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

[2] See *Terr. Mag.* for March-June 1923, pp. 1-28, and for September 1923, pp. 61-81.

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NOTES ON THE ORIGIN OF THE EARTH'S MAGNETIC AND ELECTRIC FIELDS

W. F. G. SWANN

No abstract is given for this paper which was presented at the meeting of the Section of Terrestrial Magnetism and Electricity, April 18, 1923, as the gist of it is contained in the address by the author before the second general session of the Union, April 17, 1923 (see pp. 60–68).

University of Minnesota,
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OUR PRESENT KNOWLEDGE CONCERNING THE ATMOSPHERIC DISTURBANCES OF RADIO-TELEGRAPHY[1]

L. W. AUSTIN

Our present knowledge concerning the atmospheric disturbances of radio-telegraphy is very limited, but in the following I have attempted to make a brief résumé of the known facts and generally accepted hypotheses.

Atmospheric disturbances, popularly called static, form the chief obstacle to the progress of radio-telegraphy and telephony. They give rise to troublesome noises in the telephone receivers, which frequently make the reception of signals at great distances impossible, even in the case of the most powerful transmitting stations. They are generally most violent during the warmer months, and are especially severe in the tropics. They differ in strength from year to year, and their intensity usually increases rapidly with the wave-length to which the receiving apparatus is tuned.

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[1] Sub-report of Committee on Atmospheric-Electric Phenomena and Measurements in the Troposphere and Stratosphere presented at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

The disturbances have been classified by Eccles according to their sound in the telephones as clicks, hissing, rumbling, and crashing.

The clicks and hissing are of comparatively little importance, the first being thought to be due to distant flashes of lightning, and the second to the discharge of dust, snow, or rain, striking the antenna.

The rumbling type (grinders), which is the most common type of disturbance, is generally believed to consist of strongly damped electrical wave trains or of untuned single pulses. That this type of disturbance has sometimes a certain amount of tuning is shown by the fact that the individual discharges are not always heard simultaneously at different wave-lengths, but it is believed that the discharges are so numerous that since they occur at different frequencies, they form a kind of disturbance spectrum and thus appear in the receiving apparatus at every wave-length to which the antenna may be tuned. While nothing is definitely known as to the exact nature of the source of the rumbling disturbances, it is believed that they are probably produced in the upper atmosphere by electrical readjustments. The resulting electrical waves expand in a more or less spherical manner until the lower portions of the wave front strike the earth when they spread out guided by the earth and move off with a wave front which soon becomes practically vertical, exactly like the electrical waves started from the transmitter of an airplane. The more powerful rumbling disturbances seem to come frequently from definite centers which often appear to lie above mountainous regions; for example, a large proportion of these disturbances which have been noted along the coast of Oregon and Washington appear to come from the direction of Mt. Rainier. Other well-marked centers have been observed in the mountains back of San Francisco and San Diego. The centers mentioned appear to be nearly constant in position and have been continuously active since their first discovery in 1920. It is believed that there is also a center of great energy somewhere in southern Mexico which frequently produces receiving difficulties throughout the eastern United States where disturbances from the Allegheny Mountains are also noticed. Among other reported sources of rumbling disturbances are large cities, perhaps on account of their ascending currents of heated air, thunder clouds, though here they are apparently unconnected with the visible lightning flashes, which produce clicks, and the advancing edges of rain areas. It may be said that, in general, the rumbling disturbances more commonly originate above the land as ships far out at sea are comparatively little troubled by them.

The intensities of disturbances of the rumbling type seem to be closely connected with the seasonal variations in the height of the sun's path. Some observations in Africa, reported by Dr. R. B. Goldschmidt and R. Braillard, tend to show that radio stations close to the equator have two well-marked maxima during the year at the time when the path of the sun passes through the zenith.

When the disturbances are directive and come from a direction different from that of the signal which is being received, it is often possible to eliminate a large part of the disturbances and thus improve the reception by making use of unidirectional receiving apparatus. Unfortunately it generally happens that there are secondary centers, such as local clouds or near-by mountains, which act upon the receiving station from several directions at the same time.

Some situations have been found, notably in California, where the disturbances come almost entirely from one point. In these places reception from the direction of the Pacific Ocean can be made nearly free from atmospheric troubles.

In addition to the rumbling disturbances, crashes are often heard in the telephones which seem to differ in origin and character from the rumbling type. The difference in sound of the two types is not easily distinguishable for untrained observers, since the rumbling is frequently preceded by a crashing noise. The true crashing disturbance, however, is not followed by rumbling. There is little evidence that the crashes are directive in character, while there are a number of observations which tend to show that they occur simultaneously at widely distant stations; for example, at San Francisco and Honolulu. It has been suggested that they may be connected with solar outbursts, although there is little direct proof to this effect. Sometimes they appear to occur in large number on days when earth-current disturbances are noted on telegraph and cable lines. It is also thought by radio operators that the crashes are entirely untuned in character, since the individual crashes are observed frequently simultaneously on widely different wave-lengths.

THE CHIEF PROBLEMS OF ATMOSPHERIC DISTURBANCES

1. There is need of systematic daily observations on the intensity of the atmospheric disturbances at a large number of points in all parts of the earth. It is desirable, where possible, to have this intensity measured in terms of electric voltage on the antenna expressed in volts per meter height. Eventually recording apparatus should be devised for automatically taking these observations.

2. Measurements are also needed on the prevailing direction of the directive types of disturbances at widely scattered stations. As soon as possible, recording instruments should be used for this purpose. A direction recorder has, I believe, already been constructed in Eng-

land. These observations will probably show daily and seasonal shifts of prevailing direction, and will eventually indicate certain points as disturbance centers.

3. When these disturbance centers are identified, a study should be made of the meteorological and electrical conditions prevailing, which will perhaps throw light on the conditions under which the disturbances are produced.

4. Observations should be made on the simultaneity of individual disturbances at widely distant points, identifying as far as possible the type of disturbance.

5. The question of the directivity of the crashing type should be settled.

When sufficient information regarding these points has been collected, comparisons can be made between the disturbances and other natural phenomena, solar activity, the earth's electrical and magnetic phenomena, and those of seismology and volcanology.

U. S. Naval Radio Research Laboratory,
Bureau of Standards,
Washington, D. C.

THE WORK OF THE INTERNATIONAL UNION OF SCIENTIFIC RADIO TELEGRAPHY[1]

J. H. Dellinger

The International Union of Scientific Radio Telegraphy, begun in July 1919, is organized: (1) To promote the scientific study of radio communication. (2) To aid and organize researches requiring co-operation on an international scale and to encourage the discussion and publication of the results of such researches. (3) To facilitate agreement upon common methods of measurement and the standardization of measuring instruments. The work is actually carried on through National Sections, which have been formed in the United States, Belgium, England, France, Italy, and Norway. Steps are being taken to create such committees also in Australia, Holland, Japan, and Spain.

The meeting at Brussels in 1922 organized four international "commissions" as follows: (1) Methods of measurement and standards. (2) Radio wave transmission phenomena. (3) Atmospheric disturbances. (4) "Liaisons."

The American Section, under the auspices of the National Research Council, in Washington, is made up of an Executive Committee and six technical committees. These technical committees are: (1) Radio Wave Transmission Phenomena. (1a) Variations of Radio Wave Direction. (2) Atmospheric Disturbances. (3) Methods of Measurement and Standards. (4) Measurement of Radiation Causing Interference. (5) Electron Tubes. (6) "Liaison."

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[1] Abstract of paper presented before the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

There is fruitful work for many years in the programs as outlined for the committees and it should have value not only for the better understanding and use of radio but also in the problems of geophysics, composition of the atmosphere, etc. Observations at sea are likely to be of particular value, as they have been fewest in the past, and because the radio phenomena are in general less complicated than on land.

It is believed that radio is unique among the few fields having special adaptability to a large-scale international research program. The phenomena that must be studied are world-wide in extent, and yet are in large measure subject to control of the experimenters. It is a field of work that has large possibilities for the future, and the surface only has been touched in the work done so far. Great progress can be made in the understanding of radio phenomena when widespread observations are taken, similar to the organized observational work on weather, earthquake, and terrestrial magnetic phenomena.

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Bureau of Standards,
Washington, D. C.

ON THE DIURNAL VARIATION OF THE POTENTIAL GRADIENT OF ATMOSPHERIC ELECTRICITY[1]

S. J. MAUCHLY

Two years ago the author read before this section a paper giving the results of an analysis based upon all the diurnal-variation observations for potential gradient, conforming to an adopted standard, which had then been made aboard the *Carnegie*.[2] It may be recalled that when the observations for the Atlantic, Pacific, and Indian Oceans, respectively, were tabulated in the usual manner, according to the local time of observation, the mean curves showed the diurnal variation over each of the oceans to consist *almost entirely* of a well-defined 24-hour wave although the times of occurrence of, say, the maximum phase of this wave differed greatly for the several oceans.

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[1] Sub-report from the Committee on "Atmospheric-Electric Phenomena and Measurements in the Troposphere and Stratosphere," read at the annual meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union, April 18, 1923.

[2] *Bulletin of the National Research Council,* No. 17, 1922, pp. 73–77.

It was also shown that mean curves, for the several oceans, obtained after first referring each series of observations to G.M.T. indicated a roughly simultaneous occurrence of maximum and of minimum, respectively, over all the accessible ocean areas. It was further shown that the mean diurnal-variation curves from many leading observatories, especially those corresponding to high latitudes and to winter months in the Northern Hemisphere, were in better agreement when plotted according to G.M.T. than according to local time.

Since the publication of the foregoing conclusions additional observations aboard the *Carnegie* have increased by 50 per cent the number of ocean observations of the diurnal variation of the potential gradient. The results obtained from a separate reduction of the new material have confirmed those obtained in 1921, both as regards the preponderance of a 24-hour wave progressing approximately according to universal time and also as to the times of occurrence of its characteristic phases.

Enough data are now available to justify grouping according to time of year in order to determine whether the diurnal variation undergoes any material changes during the course of the year. Four mean curves, each representing all the observations in a given three-month period, show only minor differences in the variation. The general constancy as regards the chief features of the diurnal variation is perhaps most readily indicated by the results of a Fourier analysis which was made of the data corresponding to the several curves. It should be noted that the observations were all referred to G.M.T. and, therefore, that the phase angles given in the table refer to 0^h, midnight, G.M.T. (The absolute values of the potential gradient given in the table are subject to an increase of the order of 20 per cent on account of a corresponding increase in the adopted reduction factor over the one originally used.)

TABLE 1.—*Fourier analysis of the diurnal variation of the potential gradient* (*P*) *from observations aboard the Carnegie 1915–1921.*

$P = P_m + c_1 \sin(\theta + \varphi_1) + c_2 \sin(2\theta + \varphi_2) + \cdots$, θ is counted from 0^h, midnight G.M.T. In the table c_1, c_2, etc., are expressed in percentages of P_m.

Months	P_m	φ_1	φ_2	φ_3	φ_4	c_1	c_2	c_3	c_4	c_2/c_1
	V/m	°	°	°	°	%	%	%	%	
Feb.–Apr.	114	197	273	208	352	19	4	1	1	0.24
May–July	94	163	209	119	223	10	7	2	1	0.72
Aug.–Oct.	97	168	218	250	337	16	4	1	2	0.24
Nov.–Jan.	105	204	225	243	14	14	2	4	1	0.15
Year	102	187	221	228	348	15	4	1	1	0.25

While there is an apparent variation in ϕ_1 with time of year it should be borne in mind that the number of observations now available for each quarter is not sufficient to insure the reality of such differences as are shown in Table 1. Similarly, the large value of c_2 for the months *May, June, and July,* may be due, at least in part, to the fact that this quarter is represented by somewhat fewer observations than the others.

A special method was employed to determine, if possible, whether any part of the diurnal variation of the potential gradient over the oceans enters in as a local-time effect. For this purpose a separate study was made of the observations from 24 selected days (12 pairs), the places of observations represented by each pair differing in longitude by 180°. A mean curve, *according to local time,* of these "compensated" observations shows a distinct 6-hour wave, but no evidence of any other wave of appreciable amplitude. According to Fourier analysis the amplitude of the 6-hour wave is about 3 per cent. of the mean-of-day value, P_m, and the corresponding phase angle, at local midnight, is zero. These results, so far as they go, are in good accord with the 6-hour wave found by L. A. Bauer in his analysis of the potential-gradient data from the Ebro Observatory[8] although they must, of course, be accepted with reserve on account of the relatively small amount of observational material from which they are derived.

With the confirmation of the occurrence, in the diurnal-variation of the potential gradient over the oceans, of a 24-hour wave progressing approximately according to universal time, increasing interest attaches to the investigation of the possible occurrence of a similar phenomenon at land stations. For this purpose a comparison was made of the results of all available Fourier analyses of the diurnal variation of the potential gradient as published by various authors from time to time, together with the results of many new analyses, made in the Department of Terrestrial Magnetism and based on published data which had not, heretofore, been thus utilized. For the purpose in hand the main comparison of stations was based on the phase angles (φ_1) of the first harmonic (24-hour wave) at 0^h, midnight, G.M.T., rather than on the corresponding phase angle (φ_1) at local midnight. The comparisons were carried out for each of the solstitial and equinoctial quarters, respectively. Space prevents giving here more than a summary of the results for months corresponding approximately to the northern winter, shown in Table 2. The symbols used in this table are briefly as follows: φ_1, the phase angle of the first harmonic at local midnight; $\overline{\varphi}_1$, the corresponding phase angle at 0^h, midnight, G.M.T.; G_1, the G.M.T.

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[8] *Terr. Mag.*, vol. 27, 1922, p. 28.

TABLE 2.—*Diurnal variation of potential gradient for months of northern winter. Results of Fourier analysis*

Place	Months	Years	Lat.	Long.	φ_1	$\bar{\varphi}_1$	G_1 (G.M.T.)	φ_2	c_1	c_2	c_2/c_1
			°	°	°	°	h.	°	%	%	
Karasjok	Nov.–Jan.	1903–04	69.3N	25.6E	189	215	15.7	155	35	14	0.41
Edinburgh	Oct.–Mch.	1912	55.9N	3.2W	222	219	15.4	161	21	6	0.30
Potsdam	Dec.	1904–08	52.4N	13.1E	206	219	15.4	179	22	5	0.25
Kew	Nov.–Feb.	1898–12	51.5N	0.3W	209	209	16.1	181	12	11	0.90
Paris	Nov.–Feb.	1895–99	48.8N	2.3E	223	225	15.0	206	22	10	0.47
Munich	Nov.–Dec.	1905–10	48.1N	11.6E	241	253	13.1	187	26	15	0.56
Kremsmünster	Nov.–Mch.	1902–11	48.0N	14.1E	212	226	14.9	172	30	13	0.43
Davos	Nov.–Jan.	1909–10	46.8N	9.8E	217	227	14.9	189	35	16	0.44
Lyons	Dec.–Feb.	1885–90	45.8N	4.8E	220	225	15.0	183	13	10	0.76
Trieste	Dec.–Feb.	1904–05	45.6N	13.8 E	226	240	14.0	212	55	5	0.09
Ebro	Dec.	1910–20	40.8N	0.5E	224	224	15.1	193	22	9	0.40
Baltimore	Oct.–Feb.	1885–86	39.3N	76.6W	255	178	18.1	239	14	9	0.66
Washington	Jan.	1918–21	39.0N	77.1W	257	180	18.0	161	19	9	0.48
Algiers	Nov.–Feb.	1912–17	36.8N	3.1E	234	237	14.2	114	32	8	0.26
Batavia	Oct.–Mch.	1887–90	6.2S	106.8E	70	177	18.2	220	64	23	0.36
Rio de Janeiro	Sept.–Feb.	1910–11	22.9S	43.2W	286	243	13.8	171	32	23	0.72
Melbourne	Dec.	1858–62	37.8S	145.0E	63	208	16.1	210	25	24	0.96
Cape Evans	Nov.–Jan.	1911–12	77.6S	166.4E	66	232	14.5	243	21	4	0.22
Carnegie						204	16.4	—	14	2	0.15

corresponding to the maximum phase of the 24-hour wave; φ_2, the phase angle of the second harmonic at local midnight; and P_m, c_1, and c_2, respectively, the mean-of-day value, and amplitudes of the 24-hour and 12-hour waves, expressed in percentages of P_m.

From an inspection of Table 2 it appears that the G.M.T. corresponding to maximum phase of the 24-hour wave (G_1) shows no greater variations than might quite properly be attributed, in the main, to local causes. For both equinoctial quarters the agreement is nearly as good as during the months of northern winter. Only during the months which correspond to the northern summer is there a wide scattering as regards the G.M.T. values of G_1. However, even during this quarter, the majority of the stations show only small changes in G_1, from the values for the remainder of the year. Throughout the year there are relatively few exceptions to the general rule that only at stations for which c_2 is greater than c_1 is there marked departure from the value of G_1 as derived from the corresponding observations aboard the *Carnegie*.

There seems, thus, no reason to doubt that a 24-hour wave, progressing approximately according to universal rather than local time is in evidence at most land stations as well as over the oceans, and that its apparent absence at any given station is, in general, due to the fact that it is masked by local effects of greater magnitude.

Fuller publication and discussion of the results of this investigation will appear in the *Journal of Terrestrial Magnetism and Atmospheric Electricity* for September 1923.

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SECTION OF OCEANOGRAPHY

REPORT OF THE COMMITTEE ON SEDIMENTATION AND MARINE DEPOSITS

T. WAYLAND VAUGHAN, *Chairman*

The members of this Committee are H. F. Moore, C. E. Van Orstrand, H. S. Washington, and T. Wayland Vaughan, Chairman. During the year two meetings were held, one at the residence of the Chairman on November 17, 1922, in conjunction with the members of the Committee on Sedimentation of the Division of Geology and Geography, N. R. C., resident in Washington, and one at Ann Arbor, Mich., on December 27, 1922, in conjunction with a general meeting of the Committee on Sedimentation above mentioned.

During the past year the following investigations on marine sediments have been conducted or begun in the United States.

Delta deposits at the mouth of the Mississippi River, by A. C. Trowbridge, for the U. S. Army Engineers. A report setting forth the results of his investigations has been prepared by Professor Trowbridge and it is hoped that it will be published. A considerable body of valuable information on the physical features of the sediments, on the effect of salt water on the flocculation of river-borne material, and on the influence of currents in distributing the material on the sea-bottom was acquired.

Deposits in Pago Pago Harbor, Samoa.—M. N. Bramlette has described a set of bottom samples from Pago Pago Harbor for the Department of Marine Biology of the Carnegie Institution of Washington. Mechanical analyses of the samples were made by the U. S. Bureau of Soils and chemical analyses were made by L. G. Fairchild of the Chemical Laboratory of the U. S. Geological Survey. A report on these samples, accompanied by maps and diagrams, is almost ready for the printer.

Coral reef sands from the Bahamas.—An elaborate study of sands from behind the reef off Cocoanut Point, Andros Island, Bahamas, has been completed by M. I. Goldman of the U. S. Geological Survey, and a report has been prepared for publication. This study was made according to the method of Doctor Goldman's report on samples from Murray Island, Australia, in Carnegie Institution of Washington Publication 213.

Deep-sea samples from off the Bahamas, the Caribbean Sea, and off the south coast of Central America.—The mechanical analyses of about 50 samples from the regions above indicated have been made by the U. S. Bureau of Soils and the amount of material in a colloidal state in each estimated from the amount of water-vapor absorbed. Chemical analyses of fractions of the same samples have been completed by L. G. Fairchild of the U. S. Geological Survey. T. W. Vaughan has examined and classified a number of the samples but has not been able to complete a report on them.

Muds from the shoal waters of the Bahamas and Florida.—Much additional work has been done on the calcium-carbonate muds of the shoal waters of the Bahamas and Florida, as follows: Mechanical composition by the U. S. Bureau of Soils; the bacterial flora and the bacterial precipitation of calcium carbonate by K. F. Kellerman and N. R. Smith of the U. S. Bureau of Plant Industry; and the physical features and contained remains of organisms by T. W. Vaughan. C. B. Lipman concludes from studies at Tortugas, Florida, that denitrifying bacteria do not precipitate calcium carbonate from pure ocean water; but the experiments of Messrs. Kellerman and Smith show that these organisms do precipitate calcium carbonate when a small amount of organic matter is added to the bottom mud, and chemical analyses by E. T. Erickson of the U. S. Geological Survey show that the bottom mud contains organic matter. The actual rôle of bacteria in precipitating calcium carbonate has not been determined, but the fact seems established that as bottom living organisms they are competent agents.

Beach pebbles along the New England coast.—C. K. Wentworth has published a paper on this subject as one of the Shorter Contributions to Geology of the U. S. Geological Survey.

Bottom deposits of the Gulf of Maine.—An investigation of the bottom deposits of the Gulf of Maine has been initiated through co-operation between the U. S. Bureau of Fisheries, the U. S. Geological Survey, and the U. S. Bureau of Soils. The mechanical analyses are being made at the Bureau of Soils, and the chemical analyses are being made in the Chemical Laboratory of the Geological Survey. A report will be prepared by W. S. Burbank, who will describe the samples and discuss the significance of the results.

Miscellaneous investigations.—Besides the studies above enumerated, a number of other investigations need to be noticed. R. C. Wells has continued his experiments on the cementation of calcium-carbonate sands; E. M. Kindle has published in the Journal of Geology a paper on mud cracks on the surface of calcium-carbonate muds in southern Florida, and R. W. Sayles has made qualitative experiments to determine the possibility of the occurrence of seasonal banding in marine deposits. The last subject has also been considered by A. C. Trowbridge and Max Littlefield in connection with their studies of the sediments at the mouth of the Mississippi River. The discussions at the joint meeting of the Committee on Marine Sediments and of the Committee on Sedimentation of the Division of Geology and Geography, N. R. C., will be mimeographed and distributed as a part of the report of the Committee on Sedimentation.

This Committee is gratified to report so much progress on the investigation of marine deposits during the past year.

U. S. Geological Survey,
Washington, D. C.

REPORT OF COMMITTEE ON CONFORMATION AND TOPOGRAPHY OF OCEANIC BASINS

G. W. LITTLEHALES

In connection with a contribution published among the communications to the First Pan-Pacific Scientific Conference, entitled "The Problem of Ascertaining the Configuration of the Basin of the Pacific Ocean," occasion has been taken to refer to the manuscript sheets of the bathymetrical chart of the world, prepared by the U. S. Hydrographic Office, setting forth the authentic deep-sea soundings, as evidence of the small extent to which the oceanic basins have been sounded and as showing the inadequacy of existing measurements to define the contours of configuration beyond the continental shoulder. While it must be remembered that not all the recorded soundings could be entered in some parts of this chart, yet, where none are laid down, none have ever been measured: and in regions where soundings are represented, if all the recorded soundings could be included instead of the selection which is given where the room does not allow more to be shown, there would yet be too few in general to serve for the delineation of contours of configuration of the bottom of the ocean. Nevertheless, there are some parts of the ocean where the number and distribution of the measurements have proved sufficient to afford an approach toward a fulfilment of this purpose, notably in the lane extending from California to Hawaii, of which a contoured map and profiles are published in United States Senate Executive Document No. 153, 52nd Congress, 1st Session, being the Report of the Results of the Survey for the Purpose of Determining the Practicability of Laying a Telegraphic Cable between the United States and the Hawaiian Islands. And, recently, the method of measuring the depth of the ocean by means of the Sonic-Depth Finder has been employed in the survey of a tract of the bottom of the Pacific Ocean 34,000 square miles in extent, bordering the coast from San Francisco to San Diego and extending seaward to the depth of 2,000 fathoms. A map of the topography of the ocean floor resulting from this survey by two vessels of the Navy in the course of 34 days was presented with the report.

This map is of great significance to the Committee on Conformation and Topography of the Oceanic Basins because it seems to indicate that the mapping of the oceanic basins has been brought within the range of feasibility. Other important accessions, from American sources, to the accumulated data in relation to the configuration of the oceanic basins have resulted from the measurements by sonic depth finder at close intervals across the Atlantic from Narragansett Bay to Gibraltar and thence through the Mediterranean Sea to the Suez Canal, of which profiles were presented.

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A MATHEMATICAL THEORY OF THE TEMPERATURE DISTRIBUTION IN WATER DUE TO SOLAR RADIATION, EVAPORATION AND CONVECTION

George F. McEwen

The distribution of heat, chemical properties, and substances dissolved in the water of reservoirs, lakes and oceans depends both upon external agencies and the internal processes of conduction, and diffusion. But the latter phenomena taking place in large bodies of water, are entirely different in kind from what controlled laboratory experiments reveal. The well-known laws of conduction and diffusion deduced from *laboratory* experiments can not be carried over into the *field* where the corresponding phenomena are of a large scale type peculiar to nature.

This paper presents an attempt to formulate a set of assumptions, underlying phenomena of heat conduction, based upon field observations, and to deduce from them corresponding mathematical formulae involving the external agencies, solar radiation and evaporation as well as the internal convective or alternating circulation of the water. The convective process resulting in a mingling of small portions of the water is always present, and at the same time a general drift or current either vertical or horizontal may exist and can be taken into account in the theory, but is not considered here. A brief statement of assumption follows. The solar radiation penetrating the surface decreases in intensity according to a simple exponential law. The rate of evaporation is the same throughout the whole surface area, and the resultant increase in specific gravity of any small volume at the surface destroys its equilibrium, thus causing it to descend. It is assumed that the amount of increase in specific gravity required to cause the descent of any particle varies throughout the surface. Thus, the number of particles of a given specific gravity descending at any time decreases as the specific gravity increases, while the downward velocity of each is assumed to be proportional to the difference between its specific gravity and the general average value at that level. There is a continual disturbance of the equilibrium resulting in a succession of overturning motions each following a sufficient increase of stress in the system. Only those particles whose specific gravity exceeds the average value at any given level will descend below that level, and displace upward an equal amount of water per unit time.

Mathematical formulae corresponding to these assumptions have been derived. By means of these formulae, from observations of temperature and salinity alone, the rate of evaporation at the surface, the rate at which solar radiation penetrates the surface, and the absorbtion coefficient of radiation in water can be computed. Preliminary computations based upon both lake and ocean data have given values in good agreement with what would be expected from available direct observations of their values. However, it is necessary for a satisfactory test of the theory that all these related quantities be simultaneously *observed* in the same body of water. But, the approximate verification already obtained indicates that the basal idea already developed is applicable, although certain refinements of the theory will likely be found to increase the accuracy of computed results.

If the mechanism of the convective circulation thus deduced from principles of mechanics and heat conforms closely enough to actual conditions, the same theory also affords a means of dealing with phenomena of diffusion and even fluid friction. Such an extension of the application of the theory may be expected from the generally accepted idea that both diffusion and viscosity "so called" in large bodies of water are really due to convection or the interchange of water particles or small portions of water having different properties.

A paper presenting, in detail, the development of the theory and its application to a variety of field data is in preparation.

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NOTE ON THE RELATIVE ACCURACY OF THE DETERMINATION OF MEAN SEA-LEVEL, AND EXTENSIVE LINES OF LAND LEVELING

W. Bell Dawson, M.A., D.Sc., M.Inst.C.E.

To obtain mean sea-level from tidal observations, a summation of the heights at each hour throughout the year is made; the heights being measured from any low-level plane of reference which is maintained at a constant elevation. The average value of this summation is the elevation of mean sea-level above the plane of reference. The variation between the determination in any one complete year and in a long series of years, will rarely exceed 0.10 foot, plus or minus.

The average value of mean sea-level, based on 3 or 4 consecutive years of observation, has thus a high degree of accuracy; provided that the observations are taken with the precautions explained in the paper. The variation from year to year, is undoubtedly actual, as can be shown; but its cause we need not discuss, as we are here dealing merely with the question of accuracy.

An error of 0.10 foot as above, corresponds with an error in land leveling, as determined by the usual formula, on an extent of some 300 or 400 miles. Hence there is an advantage, in equating errors in leveling, in checking when practicable to do so, with determinations of mean sea-level on the seacoast, within the limits of this distance.

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ESTUARY TIDES: A COMPARISON OF METHODS OF COMPUTATION

W. Bell Dawson, D.Sc., F.R.S.C., *Superintendent of the Tidal and Current Survey of Canada*

Estuary tides are of great importance; because many of the largest harbours of the world are in estuaries. Ocean transportation is thus carried as far inland as possible. The situation of estuary harbours seems to be determined by the point on the estuary where the tide attains its maximum range, before the height of the tide begins to be cut off by the river slope.

The most generally used method for the calculation of tide tables, is the harmonic analysis; by which tidal constants are determined, to serve as a basis for the prediction of tides in a future year. This method answers well so long as the tide curve is symmetrical, as it is on the open coast of the ocean. The tidal constants are then almost wholly of an astronomical character, in accordance with the positions of the sun and moon. But when the tide passes over some hundreds of miles of relatively shallow water, or up an estuary of any considerable extent, other elements become too large to be ignored. These elements are hydraulic in character rather than astronomical; since they are due to the balance between the momentum of the tidal streams and the retardation of bottom and lateral friction; as well as to the contrary flow of the river against the incoming tidal undulation. The tide curve itself is modified; becoming steeper during the rise of the tide and longer and flatter during the fall.

To meet these changed conditions, additional factors are introduced into the harmonic series, known as "Compound tides" and "Overtides." But with those usually employed, the resulting predictions are often unsatisfactory. From investigations by the Tidal Institute of Liverpool, it would appear necessary to introduce an indefinite number of additional terms for these Compound tides and Overtides; and in analysing for these, it could not be known in advance which of them would prove important and which of them negligible. The analysis itself thus becomes almost hopelessly tedious; and further, when these constants are found, there is no Predicting Machine which could handle so many of them as would result. (Discussion invited on this question.)

In supplying tidal data for large regions, especially where there are land-locked areas and estuaries, it is only for a few carefully selected reference stations that primary predictions are made from tidal constants determined by harmonic analysis. Computations for secondary tide tables and for the prediction of the turn of tidal streams, must in turn be based on differences of time and on ratios; which are not necessarily constant, but may vary in some lunar period. The question therefore arises whether estuary ports cannot better be dealt with on this principle. The methods that have been tried for Quebec, in the St. Lawrence estuary, will furnish a good example of the two ways of arriving at the result desired.

There were 19 years of tidal observation at Quebec which had been submitted to harmonic analysis; but the predictions obtained were not satisfactory. The whole record was therefore re-examined, as the ice conditions in some winters caused serious disturbance; and in the spring months of some years the high freshet levels were not likely to be well represented by the harmonic terms (Sa and Ssa) which are intended to give this variation. Accordingly, seven years were selected from this long series which appeared to be quite satisfactory. The average constants derived from these years were used to fill gaps in seven other years in which some winter months had to be cut out because of unreliable curves due to ice obstruction in the river; and these seven years were then submitted to re-analysis. Even with this revised basis comprising the fourteen best years, the predictions are not what they should be; as there is at times a discrepancy of as much as 30 minutes as compared with observation, and largely in one direction. The trouble may be due in part to the freshet, which rises suddenly in April with the melting snows, and falls off till the end of June or July. Its rise also differs greatly in amount in different years; and it is questionable whether the harmonic terms that are supposed to represent this rise, do so adequately.

The alternative method is to begin with a port in the open estuary below Quebec, where the tide curve is still symmetrical, and may be assumed to be represented accurately by the harmonic constants. From the predictions for this port, the time of the tide at Quebec is then computed by some system of variable tidal differences. This method has also the advantage of dealing in a simple and adequate way with the freshet, as the variation in the general average difference for successive months will cover the freshet effect in retarding the tidal undulation.

Father Point, the Pilot station in the St. Lawrence estuary, was taken as the primary tidal station. The predictions there, are now based on sixteen years of harmonic analysis. It is 180 miles below Quebec, where the estuary is 28 miles wide; and it is within seven miles of the 100-fathom line, and is thus oceanic in its tidal characteristics. The tide curve is still quite symmetrical at Father Point, as shown by the following figures for the duration of the rise and fall of the tide at localities along the estuary:

Quebec	Duration	of rise,	$5^h\ 02^m$	Duration of	fall, $7^h\ 23^m$
Grosse Isle	"	"	$5^h\ 22^m$	"	" $7^h\ 03^m$
Orignaux Point	"	"	$6^h\ 00^m$	"	" $6^h\ 25^m$
Rivière du Loup	"	"	$6^h\ 09^m$	"	" $6^h\ 16^m$
Father Point	"	"	$6^h\ 13^m$	"	" $6^h\ 12^m$

The time which the tidal undulation takes to travel from Father Point to Quebec is approximately 4½ hours for High Water and 5½ hours for Low Water. The average time is subject to variation however, in the three leading lunar periods; which are (1) The Synodic month, from New moon to New moon; the variation from Spring tides to Neap tides recurring in the half month. Its length is 57 half lunar days or tide-intervals. (2) The Anomalistic month of the moon's distance, from Perigee to Perigee; length 53¼ tide intervals. (3) The Declinational month; its length between the moon's successive crossings of the equator in the same direction, being 52¾ tide-intervals.

In the simultaneous observations of the tide at Father Point and

Quebec during five years, the tidal differences were numbered in accordance with these three periods to determine the corresponding variations in the difference of time. In each year, the months of January and February were omitted because of disturbance of normal conditions by ice; and the months of March, April and May were averaged separately to determine the effect of the freshet in the spring. As a rule, the greater depth of water during the freshet increases the rate of travel of the positive undulation at High Water, whereas at Low Water the restricted channel-ways and greater speed of the current due to the additional freshet volume, retard the negative undulation. These effects are manifest in the river above Quebec, as shown by simultaneous observations in the tidal section of the river.

For the computation of High Water at Quebec, the result proved simple. The general average value of the time-difference, after the freshet months, was 4 h. 23 m. and the variation in the value from Springs to Neaps during the Synodic month was only one minute, plus or minus, which is negligible. In the freshet months, the value was 13 m. less than the general average in April and 6 m. less in May. (As early as March, the conditions seemed still to be disturbed; as the averages differed in different years.) As the calculation value for High Water is thus a constant, the lower values in the spring months can readily be allowed for.

For Low Water the general average of the time-difference was 5 h. 37 m. and distinct variation in the value was evident in each lunar month as follows:

Synodic, from Spring to Neaps	Variation 34^m
Anomalistic, with the Moon's distance	Variation 18^m
Declinational	Extreme Variation 19^m

The moon's declination causes a heavy alteration in the successive differences in accordance with its upper and lower transits; and to treat this as a correction would involve a series of plus and minus values alternately, which would be inconvenient. The variation with declination was therefore used as the primary series of differences, which thus become all of one sign; the extreme values in the series being 5 h. 41 m. and 5 h. 23 m. The variations in the Synodic and Anomalistic months are then applied to this series as plus and minus corrections; the correction having the plus sign for quarter or half a month consecutively and the minus sign likewise; which is more convenient than an alternation, and reduces the chances of accidental error in the computation.

In determining these variations, the numbering and tabulation of the observed time-differences in their respective periods, enabled the three variations to be reduced to the form of curves. From these, the primary values and the two series of corrections were read off in accordance with their respective numberings. In making the computation, the predicted times for Father Point during the coming year are numbered in three series in correspondence with the moon's position, which enables the values and corrections to be applied appropriately; the result affording the desired time of the tide at Quebec.

In investigating the freshet effect upon Low Water, little deviation from the general average was found in the spring months. This appears to result from the considerable depth which extends most of the way from Father Point to Quebec; so that the increase of depth during the freshet months compensated for the increased speed of the river flow against the tidal undulation. Above Quebec, where the depth is less, the time of Low Water is appreciably retarded during the freshet, as compared with other months.

It was therefore assumed in dealing with the Low Water values, that the form and amplitude of the various curves of variation would not be modified in the freshet months, even if the axis of the curve, which represents the general average, were altered in position. The freshet months were accordingly included, to extend the number of months on which the variations were based; making ten in each of the five years with one exception, or 49 months in all.

Correct tide tables for Quebec are unusually important, because the time of the tide at up-river points is derived in turn from them, as far as Lake St. Peter at the head of tide water, 105 miles above Quebec. The crucial point in St. Lawrence navigation is in this stretch, at a point named Cap à la Roche, where rock dredging is still in progress to bring the channel to standard depth. The tide at this point is computed from Quebec by means of two series of differences of time, for High Water and Low Water respectively, in successive months, as the level of the river falls from spring to autumn. In the case of Low Water, the variation during the course of the Synodic month is also allowed for. The computation for time is thus based entirely upon Quebec; the rise of the tide being determined on an independent system.

A test for the accuracy of the method herein described, was based upon the "average error" during the month, between the tide as observed and the time of the tide as predicted or calculated. This average error is the mean amount, irrespective of the plus or minus sign; which affords a consistent comparison.

The Low Water values were used for the test, as calculated for Quebec by the three series of variables, during three months in

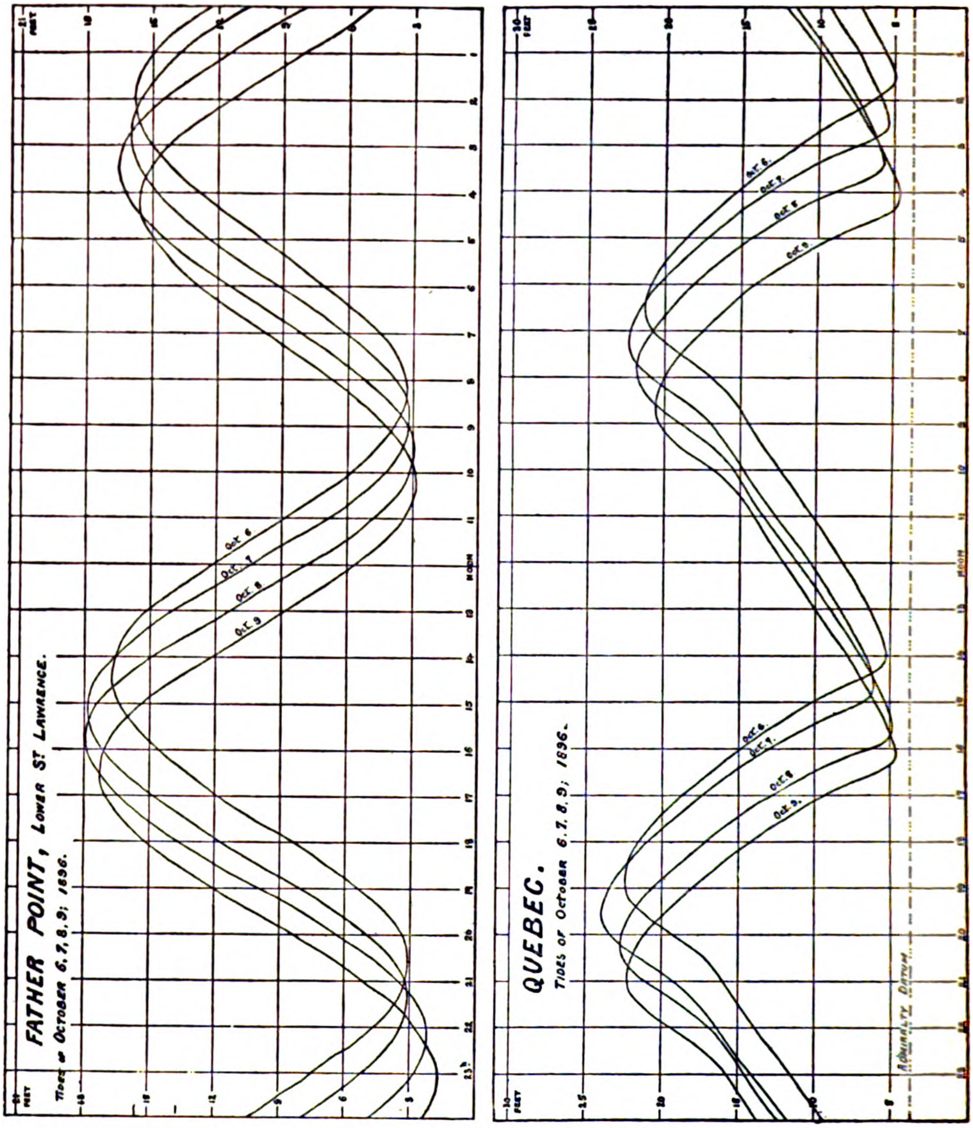

FIGURE 1.—Simultaneous tidal observations at Father Point and Quebec for the period Oct. 6–9, 1896, illustrating the smooth oceanic tidal curve at Father Point and the irregular river tide at Quebec.

1922; the variables being applied to the predictions for Father Point as obtained from the harmonic constants. The improvement obtained, as compared with the harmonic predictions for Quebec, is as follows:

Predictions from Harmonic analysis. Average error in the month, 26 minutes.
Calculation by method explained. Average error in the month, 6 to 8 minutes.

As a further test, the time of Low Water at Quebec was calculated by this method from the time as actually observed at Father Point. The average error in the month was substantially the same as above, namely 6 to 7½ minutes.

The predictions for Father Point, based on fifteen years of harmonic analysis, when compared with the observed time of Low Water there, showed an average error during the month of 6 minutes. The values for Quebec, as given by this method, have thus practically as high an accuracy as the predictions for Father Point itself, on which the calculation for Quebec is based.

The High Water values at Quebec, when calculated by the constant time-difference from Father Point, also show an improvement as compared with the predictions obtained from the harmonic constants for Quebec. The average error during the month, between observed high water and predictions by the harmonic analysis, is 8 to 12 minutes; and by this new method based on Father Point, it is only 4 to 6 minutes. This is remarkably good, when the average error in high water as predicted by the harmonic constants for Father Point itself, is 6 minutes.

Tidal and Current Survey,
Ottawa, Canada.

SOME RECENT OCEAN HYDROGRAPHY BY THE U. S. COAST AND GEODETIC SURVEY

W. E. PARKER

The purpose of this paper is to show that a considerable amount of oceanographic data can be obtained by public vessels at comparatively small cost without seriously interfering with their regular duties and that it is not necessary to defer further exploration of the oceans until such time as well-equipped expeditions can be organized.

During the spring and summer of 1919, and again during the summer and fall of 1922, the U. S. Coast and Geodetic Survey had occasion to transfer several of its surveying vessels from the Atlantic to the Pacific and vice versa.

Advantage was taken of these necessary passages from one coast of the United States to the other to explore the waters through which the vessels were required to pass. Each vessel was required to carry a line of soundings from the port of departure to the port of arrival on the other coast, and each was given a slightly different route so that there would be no duplication of work.

In this way an area of the Atlantic Ocean from Cape Hatteras to the Bahama Islands, 150 to 240 miles wide, of the Caribbean Sea along the steamer route from the Windward Passage to the Panama Canal 120 miles wide, and of the Pacific Ocean from Panama to San Diego, California, 100 to 150 miles wide, was surveyed in considerable detail.

A total of 587 deep sea soundings were taken by the most accurate method known. More than 1,000 temperature observations were taken at the surface, at a depth of 200 fathoms and at the bottom, and several hundred samples of sea water and of bottom material were preserved and forwarded to Washington for analysis.

Some of the most important geographical information obtained by those vessels was, the exact depths over the bank which lies south of Jamaica, proof of the non-existence of the bank supposed to lie about seventeen miles north of Panama, definite location of the Alijos Rocks, and the discovery of a bank midway between Guadaloupe Island and the mainland.

The cost of obtaining all of this data was truly insignificant. Since the vessels were required to make the passages in going to and from their stations, the only expenses which can properly be charged against this work are for special equipment purchased for the work, and for the overhead cost of the vessels, including pay of officials and men for the time required to make the passages in excess of the time which would have been necessary had no survey work been done.

The total time devoted to this work by all of the vessels, six in number, one vessel making the passage twice, was twenty-one days. The total cost of the work, including all expenses, was a little less than $9,000. About 1,000,000 square miles of ocean was explored.

U. S. Coast and Geodetic Survey,
Washington, D. C.

SECTIONS OF VOLCANOLOGY AND GEOPHYSICAL CHEMISTRY

SYMPOSIUM ON HOT SPRINGS[1]

ROBERT B. SOSMAN

A joint meeting of the Sections of Volcanology and Geophysical Chemistry of the American Geophysical Union was held at the Carnegie Institution of Washington on April 18, 1923, and was devoted to a symposium and discussion on the temperatures of hot springs and the sources of their heat and water supply. Ten papers were presented.

The temperatures of hot springs range all the way from those found in warm springs in the mountains of Virginia, some of which are only slightly above the annual mean temperature, up through boiling springs and mud pots such as are found in our western volcanic regions, to fumarole temperatures of about 650° C., the highest temperature found in the Valley of Ten Thousand Smokes, Alaska, reported on by E. G. Zies in his paper on "Hot Springs and Fumaroles of the Katmai Region."

E. T. Allen, in his paper on "The Hot Springs of Mt. Lassen," showed that chemical reactions, such as could be inferred from the composition of the spring waters, are not adequate to account for more than a very small proportion of the heat. Similarly, radioactivity, so far as it can be inferred from the radioactivity of "The Hot Springs of Iceland," described by F. E. Wright, is also insufficient to supply much heat. The radioactivity shows no parallelism with the temperatures of the springs, as has been shown by Thorkkelsson in Iceland, and by Schlundt and Moore in the Yellowstone.

L. H. Adams discussed "Physical Sources of Heat in Springs," showing that the forced flow of a fluid from a high to a low pressure through a porous plug may develop considerable heat—for liquids, as much as 40° C. for 1000 atmospheres fall. In fact, the passage of any material from a region of high pressure to one of low pressure will usually produce a rise in temperature. Such a process may be a source of heat in rock magmas. G. W. Morey, in his paper on "Relation of Crystallization to the Water Content and Vapor Pressure of Water in a Cooling Magma," showed that similar heat effects could be obtained by the forced flow of steam. These physical sources of heat have not been considered heretofore, and geological observations are lacking to indicate to what extent forced flow is a factor in natural processes.

There seemed to be general agreement, however, in the papers and in the discussion, that the source of heat is in subcrustal rock magmas whether it escaped by the familiar processes of conduction and convection, or whether the process of flow played an important part. C. E. Van Orstrand, in his paper on "Temperatures in Some Springs and Geysers in Yellowstone National Park," estimated from temperatures in deep borings that most of the heat might be coming from depths between 3400 and 8000 ft.

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[1] Reprinted, with minor changes, from Journal of Industrial and Engineering Chemistry, vol. 15, No. 6, p. 610, June 1923.

Agreement was not so general as to the sources of the water supply. In the Yellowstone, as described by Van Orstrand, a considerable proportion of the water must be meteoric—that is, from rain and snow. The same is probably true in Iceland, where there is a large supply from snow fields and glaciers. Allen believes that the probability is strong that over half of the water in the springs of Mt. Lassen is from such sources, and A. L. Day showed that the water of "The Hot Springs of 'The Geysers' Region of California" is also mainly meteoric.

On the other hand, it was brought out in the discussion that the amount of water which may be dissolved in a batholithic magma of the chemical character described by Morey may be sufficient to supply small hot springs for hundreds of thousands of years without any addition of surface water, and Morey also showed that the conditions of crystallization of such a magma may yield water vapor under considerable pressure. It seems very reasonable to suppose that a considerable proportion of the water in such places as "The Hot Springs of Vulcano and Kalamaki," described by H. S. Washington, might be original magmatic water. This seems almost a necessary conclusion for some of "The Hot Springs of Southern Idaho," described by O. E. Meinzer. It was shown very clearly in the paper by Meinzer that there is no artesian source of water for many of the springs. It was also shown that on the whole the hotter springs are the smaller, indicating that dilution with ground water from moderate depths is responsible at the same time for low temperature and large volume. These springs come through Paleozoic rocks, Cretaceous granite, etc., and have no direct relation to the Snake River basalts, which are now quite cold.

A paper by Kirk Bryan on "The Hot Springs of Arkansas" was presented only by title as Mr. Bryan had to be absent on field work.

The publication in full of the papers and discussion is being considered. If they are so published it will be possible to add several other papers on other hot-spring areas of the United States, as well as communicated discussion, which will make this a valuable summary of the present status of our information on the subject.

Geophysical Laboratory,
Carnegie Institution of Washington,
Washington, D. C.

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BULLETIN
OF THE
NATIONAL RESEARCH COUNCIL

COOPERATIVE EXPERIMENTS UPON THE PROTEIN REQUIREMENTS FOR THE GROWTH OF CATTLE–II

REPORT OF THE SUBCOMMITTEE ON ANIMAL NUTRITION
Presented for the Subcommittee by E. B. Forbes, Chairman, with the collaboration of S. I. Bechdel, C. D. Jeffries and Max Kriss

PUBLISHED BY THE NATIONAL RESEARCH COUNCIL
OF
THE NATIONAL ACADEMY OF SCIENCES
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1924

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INTRODUCTION

This project of cooperative investigation of the protein requirements for the growth of cattle may be regarded as having had its inception as long ago as the year 1908 in Dr. Armsby's report as Chairman of the Committee on Organization of the American Society of Animal Nutrition, this committee having been instructed "to make a survey of the field of animal husbandry work and present suggestions for plans of cooperation among various experiment stations."

This report emphasized the need for cooperative investigation, as a means of improving the quality of research work, and, for the furtherance of this idea, there was included in the constitution of the new society a provision for a standing Committee on Experiments.

During the next four years Dr. Armsby actively fostered the idea of cooperative research, for three years as President of the Society, and during the whole of this period as Chairman, or as a member, of the Committee on Experiments.

In 1910 Dr. Armsby presented, for the committee, a "Scheme for Cooperative Investigations upon the Optimum Protein Supply of Fattening Cattle," and during the next two years repeated efforts were made to enlist the active interest of the experiment stations in this plan of research, but without success.

In Dr. Armsby's report as Chairman of the Committee on Experiments in 1911 he said, "Two members of the Society replied to the Committee's circular letter indicating an intention of taking part in the work. One of these has since found it impracticable to do so this year; the second still adheres to his purpose, although it does not yet appear with whom he is to cooperate."

This project was once more commended to the Society, and to the experiment stations of the country, in the report of this committee for 1912, after which the idea lay dormant for a period of five years, the expiration of which, in 1917, found a large part of the civilized world in the throes of warfare, and at least threatened by starvation.

Under the stimulus of this situation, the idea of cooperative experiments on the protein requirements of cattle was revived, once more under the direction of Dr. Armsby, but this time under the auspices of the National Research Council.

The history of the project from this date, through the year 1918 and a part of 1919, is given by Dr. Armsby in the introduction of the First Report of the Subcommittee on Protein Metabolism in Animal Feeding.[1]

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[1] Bulletin of the National Research Council, Vol. 2, Part 4, No. 12, June, 1921.

The present paper, covering investigations during the years 1919 to 1923 inclusive, reports the continuance of the project, under the same plan of experimentation, the work of organization and discussion of the results having devolved upon the writers through the lamented death of Dr. Armsby, which occurred on October 19, 1921.

For the full statement of the plan of this investigation see below.

The several cooperating institutions varied much in their success in carrying out the plans as submitted. In fact, it is our feeling this report should be considered as much a contribution on the subject of method and practicability of cooperative experimentation as a contribution to our knowledge of the protein requirements of cattle. In the nature of the case some of the more important deductions must be drawn by the reader in the light of his own discrimination.

In full justice to the conception of a program of cooperation in research it should be borne in mind that the work herein discussed was conducted during the years immediately following the World War, and that various unfavorable conditions as to scientific personnel, labor, and material supplies prevailed, these being freely reported by the individual cooperators as undoubtedly affecting the quality of the results. Under more favorable conditions, especially with closer and more detailed oversight by the coordinating agency, a more successful outcome of such a program might fairly be anticipated.

In the following report the several articles are ascribed to the individuals who conducted the investigations. The presentation of a condensed and unified discussion, however, required that the materials upon which it is based be thoroughly worked over by the authors of the report as a whole. The text and conclusions, therefore, and much of the computation and tabulation of results are our own. With this explanation we hope that no false impressions will be created by our method of presentation and signature.

PLAN OF EXPERIMENTATION

For convenience of reference the plan as originally proposed, and as used for the guidance of the cooperators in the experiments discussed in this paper, as well as in the earlier report, is reprinted in full as follows:

Statement of Problem.—A productive ration must contain digestible protein at least equal to the maintenance requirement plus the amount of protein contained in the product.

Some recent investigations indicate that at least moderate production may be secured with rations containing but little more digestible protein than the minimum just indicated.

Other investigations, on the contrary, seem to show that a much more liberal protein supply is at least advantageous, if not necessary, and the current feeding standards for growing animals accord with this view.

In most of these investigations little or no attention has been paid to the quality of the proteins consumed, to the influence of accessory substances (vitamines), or to variations in the ash of the rations.

Method of Attack.—The general idea is to feed either beef or dairy calves old enough to consume exclusively dry feed upon two different planes of protein intake but with equal net energy supply. The low-protein ration is to supply little more than the minimum amount theoretically required while the high-protein ration is to supply about the amount demanded by current feeding standards. The energy supply is intended to be such as will support normal growth but not cause material fattening.

The proteins of the two rations are to be derived in fixed proportions from identical feeding stuffs and an ample supply of ash ingredients and vitamines is to be assured. Any necessary adjustments in the bulk of the ration are to be made by variations in the amount of straw, and necessary adjustments of the energy supply by the use of greater or smaller amounts of commercial starch.

Animals.—Obviously the animals to be compared must be as nearly alike as possible. The plan contemplates the comparison of the animals by pairs, one animal of each pair receiving the high-protein and one the low-protein ration. The two animals of each pair should be of substantially the same breed (not necessarily pure-breds) and as nearly as possible of the same age and weight at the beginning of the experiment.

It is desired that as many pairs be used as is consistent with the keeping of individual records. Lot records are of far less value. The more nearly alike the several pairs are the better, but uniformity in this respect is not so essential as between the two animals of each pair.

It is desirable to select as young animals as can be expected to eat the proposed rations.

Feed Requirements.—Rations are to be computed for different ages to furnish the amounts of digestible true protein and net energy per 1000 pounds live weight shown in the following table, age and not live weight being the controlling factor. The first column shows the theoretical minimum of digestible protein computed in the manner already indicated. It is proposed to increase this by about 35%[1] in the low-protein ration, in part to compensate for the probable depression of the digestibility, and to make the high-protein ration correspond substantially to Haecker's standards as reported in Henry and Morrison's "Feeds and Feeding," 15th Ed., p. 670. The estimates for net energy are those made by the writer. Figures for intermediate ages can be estimated with sufficient accuracy by interpolation.

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[1] 20% in the first year.

ESTIMATED RATIONS PER 1000 POUNDS LIVE WEIGHT
(TO BE COMPUTED IN DIRECT PROPORTION TO THE LIVE WEIGHT)

Age	Estimated minimum of digestible protein	Digestible true protein of ration			Net energy
		Low		High	
		1st year	2nd year		
Months	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Therms*
4	1.50	1.80	2.03	3.00	18.0
5	1.30	1.56	1.76	2.70	16.1
6	1.20	1.44	1.62	2.40	14.5
9	1.00	1.20	1.35	2.10	12.7
12	0.85	1.00	1.15	2.00	11.8

Feeding Stuffs.—It is proposed to supply proteins from a mixture of alfalfa hay, corn meal, linseed meal (O. P.) and high-grade peanut meal, as free from hulls as possible, in such proportions that the digestible true protein of the mixture will be supplied in the following proportions by the several feeding stuffs:

	Per Cent
From alfalfa hay	30
From corn meal	10
From linseed meal (O. P.)	25
From peanut meal	35
	100

It is believed that this will be a well-balanced protein mixture. These particular proportions have been selected because they seem to lend themselves well to the formulation of rations of the desired make-up.

All computations are to be based on the so-called true protein, i.e., crude protein minus non-protein, the non-protein being assumed to be entirely digestible. The factor 6.25 for protein is to be used throughout.

Quality of Alfalfa Hay.—One important function of the alfalfa hay in the ration is to supply basic ash ingredients and the accessory food substances.

In view of the very considerable variations in the quality of alfalfa hay care should be taken to secure a supply of good quality and especially one containing a normal proportion of leaves since these appear to be the principal carriers of ash and vitamines.

Rations.—To the foregoing mixture of feeding stuffs there is to be added sufficient oat straw and commercial starch to make up rations having the required protein content and energy value per 1000 pounds. Should it seem desirable, sugar may be substituted for part of the starch in amount sufficient to yield the same energy value. Simply as an illustration the following rations have been computed per 1000 pounds live weight for the age of six months on the assumption that the feeding stuffs have the composition and energy values shown in Bulletin No. 142 of the Pennsylvania Station, but using for corn meal the corrected net energy value of 85.20 Therms per 100 pounds reported in the Journal of Agricultural Research, v. 10, p. 612 (Sept. 17, 1918). Starch is estimated at 81.79 Therms, and sugar at 81.20 Therms per 100 pounds dry matter.

High protein ration	Dry matter	Digestible true protein	Net energy value
	Pounds	*Pounds*	*Therms*
10 lbs. oat straw	8.85	0.08	3.48
10 lbs. alfalfa hay (all analyses)	9.14	0.70	3.42
3.6 lbs. corn meal	3.19	0.23	3.07
2.0 lbs. linseed meal (O. P.)	1.82	0.57	1.78
2.0 lbs. peanut meal	1.79	0.82	1.87
	24.79	2.40	13.62
1.27 lbs. starch	1.08	....	0.88
	25.87	2.40	14.50

Low protein ration*	Dry matter	Digestible true protein	Net energy value
	Pounds	*Pounds*	*Therms*
14 lbs. oat straw	12.39	0.11	4.87
6.34 lbs. alfalfa hay (all analyses)	5.79	0.45	2.17
2.34 lbs. corn meal	2.08	0.15	1.99
1.35 lbs. linseed meal (O. P.)	1.15	0.38	1.18
1.28 lbs. peanut meal	1.14	0.53	1.20
	22.55	1.62	11.41
4.45 lbs. starch	3.78	....	3.09
	26.33	1.62	14.50

* For the second year.

The actual rations to be used should, however, be computed on the basis of the composition and estimated digestibility of the feeding stuffs actually used and should take account as far as possible of variations in the moisture content of the feeding stuffs.

Feeding.—It is contemplated that the proportions of the several feeding stuffs in the high-protein and low-protein rations respectively, computed in the manner just illustrated, shall be continued unchanged for the respective animals throughout the experiment. Amounts of these rations sufficient to supply the energy requirements per 1000 pounds live weight given in the table on page 6 for the different ages will furnish almost exactly the computed amounts of digestible protein required.

It is desirable to adjust the per head ration to the increasing live weight of the animal as frequently as seems practicable. In doing so, allowance should be made for the increase to be expected during the next interval. Thus if the ration is recalculated bi-weekly the new ration should be based on the live weight at the time of changing plus the increase to be expected during the following week.

It is quite possible, however, that some modification of this plan may prove necessary. If the proportion of roughage in the rations proves to be too great, the quantity of oat straw may be reduced for both animals of a pair by the same amount per 1000 pounds live weight, disregarding the slight change in the protein thus caused or slightly increasing the amount of grain fed in order to keep the protein constant, while the starch is correspondingly decreased so as to keep the energy content unchanged.

It may be necessary also to adjust the energy supply of the animals in order, on the one hand, to prevent fattening or, on the other, to maintain good condition. This may be effected by varying the amount of starch used, but the energy supply of both animals of a pair must be kept the same per 1000 pounds live weight.

Measurement of Results.—By far the most satisfactory measure of the growth made is the amount of protein stored in the body as shown by the nitrogen balance of the animal. It is therefore urged that, if possible, the income and outgo of nitrogen be determined at intervals. This, of course, requires a quantitative collection of feces and urine and a determination of their nitrogen content. With male animals a separate collection of the two is not particularly difficult. With female animals it is believed that if necessary the two may be collected together and satisfactorily sampled, thus giving data for the nitrogen balance although not for the digestibility.

When a separate collection of the feces is made the data of the ordinary digestion experiment should be determined if practicable. This not only gives a control of the amount of digestible protein actually consumed but aids in estimating the actual energy values of the rations. If the heats of combustion of the feeds and excreta can be determined a still more accurate estimate is possible.

If determinations of the nitrogen balance are not possible, it is believed that useful, even if less accurate, conclusions may be derived from live weight results coupled with careful control of the general condition of the animals. In this case, however, it is especially important to avoid any material fattening, while at the same time maintaining what may be regarded as a good thrifty condition, by an adjustment of the energy supply in the way already indicated. The employment of as many animals as practicable tends to reduce the uncertainties of live weight experiments.

Systematic measurements of a few dimensions suitable for comparing the skeletal growth might aid greatly in the interpretation of the results.

Length of Experiment.—It does not seem desirable to stipulate the duration of the experiment nor the frequency with which the digestibility and the nitrogen balance shall be determined. Obviously, the longer the experiment is continued and the more frequent the nitrogen balances or digestion trials, the more information may be hoped for. It is suggested that in the interest of securing results with reasonable promptness a total duration of five or six months may be the most desirable. It may be anticipated that the younger the animals the more decisive will be the results.

Summary.—The most essential requirements for successful cooperation in these experiments may be briefly summarized as follows:

1. Careful selection of animals as regards comparability.
2. All experimenters to use the same protein mixture in as nearly as possible the same relative amounts.
3. The animals to be compared must receive the same relative energy supply at the same age.
4. Growth is to be measured in terms of the nitrogen balance if possible; if not, in terms of live weight and dimensions.

A Supplementary Control.—If practicable, a useful check upon the results with the low-protein animals would be afforded by adding a third set of animals receiving the same low-protein ration but with skim milk substituted for linseed and peanut meals as a source of protein, the energy supply being adjusted by varying the amount of starch.

It is not desired, however, that this be undertaken to the detriment of the principal comparison.

As in the first report on this subject, normal growth for animals of a particular age and breed was computed in accord with the results of Eckles's[1] study on the normal growth of dairy cattle.

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[1] **Missouri Agricultural Experiment Station, Research Bulletin No. 36.**

INVESTIGATIONS AT THE PENNSYLVANIA AGRICULTURAL EXPERIMENT STATION

S. I. BECHDEL AND P. S. WILLIAMS

EXPERIMENTS OF 1921

Two pairs of Holstein calves were the subjects of this experiment, each pair consisting of a male and a female. The average initial age was 8 months, and average initial live weight, 473 pounds.

The experiment consisted of four digestion periods of 5 to 9 days' duration, with preliminary or intermediate periods of varying length, the time during which the experimental rations were fed previous to the collection periods being from 2 to 15 days' duration, and the length of the total periods of growth covered by experimental weighings being 132 or 133 days.

Of the 16 individual periods (four animals, four experimental periods) 6 were complete balance periods, involving the accounting for feed, urine and feces, and making possible, therefore, the computation of protein retained, while in the remaining 10 the urine was not collected, and the data regarding metabolism consist of digestion coefficients only.

The details as to the experimental animals and the schedule of the experiment are set forth in Tables I and II.

TABLE I.—*Description of the Animals*

No.	Sex	Breed	Dropped, 1920	Age in months at start of experiment	Level of protein intake	Initial weight
Pair I:						*Pounds*
974	Heifer	Holstein	June 10	8¾	Low	526
978	Heifer	Holstein	July 3	8	High	428
Pair II:						
982	Bull	Holstein	Aug. 3	7	Low	470.3
980	Bull	Holstein	July 9	7⅚	High	468.0

It is our opinion that the preliminary periods of 2, 3, 4 and 5 days were of insufficient duration, for the intended purpose, as also all of the collection periods (5 to 9 days' duration).

The average daily rations for the experiment as a whole, computed to 1000 pounds live weight, are set forth in Table III, the rations being composed, as contemplated by the plan, of alfalfa hay, oat straw, corn meal, linseed meal, peanut meal and starch.

It will be noted that the digestible crude protein intake was 1.69 pounds for the low-protein and 2.41 pounds for the high-protein calves, the corresponding dry matter of the rations being 19.91 and 18.95 pounds, the net energy, computed from the digestible nutrients as found in this experiment, being 11.87 and 9.95 Therms, respectively.

TABLE II.—*Schedule of Experiment*

Period	Animal No.	Level of protein intake	Length of preliminary feeding	Length of trial proper	Character of trial	Date of trial
			Days	*Days*		
I........	974	Low	5	8	Balance..	Mar. 8-15
I........	978	High	5	8	Balance..	Mar. 8-15
I........	980	High	8	8	Dig. trial.	Mar. 26–Apr. 5
I........	982	Low	4	8	Dig. trial.	Mar. 26–Apr. 5
II........	974	Low	10	9	Balance..	Apr. 12-20
II........	978	High	10	9	Balance..	Apr. 12-20
II........	980	High	11	9	Dig. trial.	Apr. 27–May 5
II........	982	Low	11	8	Dig. trial.	Apr. 27–May 4
III........	974	Low	11	8	Balance..	May 10-17
III........	978	High	12	7	Balance..	May 10-16
III........	980	High	3	7	Dig. trial.	May 20-26
III........	982	Low	2	7	Dig. trial.	May 20-26
IV........	974	Low	15	5	Dig. trial.	July 6-10
IV........	978	High	15	5	Dig. trial.	July 6-10
IV........	980	High	15	5	Dig. trial.	July 6-10
IV........	982	Low	15	5	Dig. trial.	July 6-10

TABLE III.—*Average Daily Rations per 1000 Pounds Live Weight*

Feeds	Low protein				High protein			
	Fresh weight	Dry matter	Digestible crude protein	Net energy	Fresh weight	Dry matter	Digestible crude protein	Net energy
	Pounds	*Pounds*	*Pounds*	*Therms*	*Pounds*	*Pounds*	*Pounds*	*Therms*
Alfalfa hay....	6.42	5.73	....	....	10.15	9.06	....	....
Oat straw.....	3.69	3.24	....	....	2.24	1.97	....	....
Corn meal.....	2.63	2.23	....	....	3.84	3.25	....	....
Linseed meal..	1.25	1.12	....	....	1.99	1.78	....	....
Peanut meal...	2.06	1.90	....	....	1.65	1.52	....	....
Starch........	6.58	5.69	....	....	1.59	1.37	....	....
	22.63	19.91	1.69	11.87	21.46	18.95	2.41	9.95

The comparatively slight differences in dry matter were incidental, the direct objects in the compounding of the rations being to vary the protein (as it was varied) while maintaining the net energy value of the intake at the same level. The intake of energy-producing food was computed from the data of Pennsylvania Agricultural Experiment Station Bulletin 142, and the net energy value of the rations was computed from the digestibility of the rations as actually determined in this experiment. The marked difference between the two net energy values, which were intended to have been the same, signifies that the high-protein rations were less well digested in this particular experiment than as previously computed.

From the detailed data on the digestibility of the several classes of nutrients and the amounts of nutriment digested, as set forth in

TABLE IV.—*Digestibility of Rations*

Period	Calf No.	Level of protein intake	Digestion coefficients				
			Dry matter	Crude protein	Crude fiber	Ether extract	Nitrogen-free extract
I	982	Low	70.85	61.52	41.59	60.08	85.19
	980	High	64.46	69.43	48.93	79.03	71.47
II	982	Low	67.20	55.13	62.39	53.79	74.36
	980	High	67.66	72.00	51.47	75.00	76.00
III	982	Low	75.23	64.84	45.54	50.39	87.09
	980	High	66.69	70.59	44.36	76.11	77.38
IV	974	Low	60.82	45.40	34.59	66.15	75.18
	978	High	63.93	66.00	45.51	72.30	70.28
	982	Low	69.01	57.61	29.56	65.69	82.09
	980	High	66.01	69.44	51.30	68.16	76.60

TABLE V.—*Nutrients Digested per Day and Head*

Period	Calf No.	Level of protein intake	Nutrients digested (pounds)				
			Dry matter	Crude protein	Crude fiber	Ether extract	Nitrogen-free extract
I	982	Low	6.921	0.814	0.685	0.158	5.233
	980	High	5.679	1.333	1.032	0.294	2.743
II	982	Low	7.255	0.758	1.274	0.156	4.913
	980	High	7.463	1.494	1.224	0.306	3.853
III	982	Low	9.290	0.994	0.879	0.128	6.997
	980	High	7.132	1.500	1.019	0.325	4.009
IV	974	Low	7.407	0.656	0.814	0.215	5.648
	978	High	7.322	1.343	1.167	0.308	3.503
	982	Low	8.622	0.878	0.579	0.224	6.244
	980	High	8.138	1.602	1.364	0.319	4.578

Tables IV and V, it is apparent that the higher net energy value of the rations fed to the low-protein calves was due more largely to the greater digestibility of the nitrogen-free extract than to any other factor, since protein, crude fiber and ether extract were more efficiently digested by the high-protein than by the low-protein calves. In addition to this factor of higher digestibility of the nitrogen-free extract, other factors contributing to the above-mentioned preponderance of the net energy value of the low-protein rations are the greater proportion of nitrogen-free extract and the greater total dry matter intake.

Table VI.—*Gains in Live Weight (pounds)*

Period	Ration			
	Low protein (Animal No. 974)	High protein (Animal No. 978)	Low protein (Animal No. 982)	High protein (Animal No. 980)
At end of 14 days	3.6	6.0	20.7	17.3
At end of 17 days	42.7	36.3	28.6	17.0
At end of 16 days	5.0	4.3	19.0	31.7
At end of 19 days	....	....	28.0	19.6
At end of 33 days	44.0	51.4	....	....
At end of 24 days	23.7	13.0	....	....
At end of 37 days	....	....	72.4	42.4
At end of 14 days	51.0	39.6	32.3	50.3
At end of 15 days	−2.6	.1	12.9	−1.3
Totals	143.7	151.6	213.9	177.0

It appears then, that from the rations provided, the low-protein calves digested 1.19 times as much energy-producing food as did the high-protein calves, a situation which had the effect to give to the low-protein calves an appreciable advantage in comparison with the high-protein calves which was not contemplated by the plan of the experiment.

The gains in live weight as exhibited in Table VI show that the data were insufficient in extent to indicate the respective value of the rations. Among the four subjects the two low-protein calves occupied the intermediate positions. These data on live weight are without great value.

The supply of digestible crude protein and net energy per 1000 pounds live weight for each individual, in the several periods, is as exhibited in Table VII.

The protein was varied according to plan in each period, but in only one period was there a pair of calves, one on high-protein and the other on low-protein intake, in which the net energy of the rations appeared to have been approximately the same. This successful exception was in the case of Calves 974 and 980 in Period IV.

The protein supply is shown in relation to the retention of protein in Table VIII. The intake of protein varied between 0.95 and 1.69 pounds per day per 1000 pounds live weight with the calves on the low-protein rations, and between 2.32 and 2.73 pounds with the calves on the high-protein rations.

Assuming a maintenance requirement of 0.6 pound of protein, there remained available for growth 0.35 to 1.09 pounds of protein with the low-protein calves, and 1.72 to 2.13 pounds with the high-protein calves.

TABLE VII.—*Supply of Digestible Crude Protein and Net Energy per 1000 Pounds Live Weight*

Period	Calf No.	Level of protein intake	Digestible crude protein	Net energy
			Pounds	*Therms*
I	982	Low	1.64	12.93
	980	High	2.58	8.68
II	982	Low	1.35	11.57
	980	High	2.73	9.98
III	982	Low	1.69	14.88
	980	High	2.65	16.54
IV	974	Low	0.95	7.54
	982	Low	1.30	11.33
	978	High	2.32	10.16
	980	High	2.48	7.89

The observed retention of protein was 0.31 and 1.06 pounds with the low-protein calves, and 0.42 to 1.17 pounds with the high-protein calves.

As compared with the computed normal gain of protein, the gain with these calves exceeded this standard in all cases except with Calf 974 in Period IV, on the low-protein ration.

The protein supply on the low-protein ration, then, appeared to be generally more than sufficient for normal gain of protein.

The gains of protein from the high-protein rations were of the same general magnitude as from the low-protein rations, but averaging somewhat higher. That the protein supply in the high-protein rations was decidedly in excess of the requirement for normal growth, however, is shown by the markedly inferior utilization of the protein available for growth with the high-protein calves. The utilization with the low-protein calves was 86.54 to 97.25 per cent of that which was available, while the utilization with the higher-protein calves was 24.42 to 59.09 per cent of the available.

The retention from the low-protein rations shows that the proteins of the component feeds supplied an efficient combination of amino acids; and the gain in relation to the normal gain shows that with such an efficient protein combination there may be at the same time normal growth and very high utilization of the protein supplied in excess of the assumed standard maintenance requirement.

TABLE VIII.—*Supply of Digestible Crude Protein and Daily Gain of Protein per 1000 Pounds Live Weight during Digestion Periods*

Period	Calf No.	Age in days	Digestible crude protein intake	Digestible crude protein for maintenance	Digestible crude protein available for growth	Observed retention of protein	Normal retention of protein	Percent of available protein utilized
			Low Protein					
			Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*	
I	982	230	1.64	0.60	1.04	0.90	0.54	86.54
II	982	270	1.35	0.60	0.75	0.67	0.47	89.33
III	982	290	1.69	0.60	1.09	1.06	0.44	97.25
IV	982	335	1.30	0.60	0.70	0.55	0.38	78.57
IV	974	390	0.95	0.60	0.35	0.31	0.33	88.57
			High Protein					
			Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*	
I	980	260	2.58	0.60	1.98	1.17	0.48	59.09
II	980	290	2.73	0.60	2.13	1.03	0.44	48.36
III	980	315	2.65	0.60	2.05	0.79	0.40	38.54
IV	980	360	2.48	0.60	1.78	0.72	0.36	40.45
IV	978	365	2.32	0.60	1.72	0.42	0.35	24.42

TABLE IX.—*Gains in Live Weight Compared with the Normal as Determined by Eckles*

Calf No.	Protein in ration	Observed			Computed normal		
		Total gain	Gain per day	Per cent gain	Total gain	Gain per day	Per cent gain
		Pounds	*Pounds*		*Pounds*	*Pounds*	
974	Low	143.7	1.06	20.15	126.0	0.93	27.69
978	High	151.6	1.12	35.42	141.0	1.04	33.18
980	High	177.0	1.31	37.82	144.3	1.07	34.44
982	Low	213.9	1.58	45.48	154.5	1.14	39.72

TABLE X.—*Body Measurements (centimeters)*

	Measurements								Percentage increase			
	Initial				Final							
Ration	Low protein		High protein		Low protein		High protein		Low protein		High protein	
Animal number	974	982	978	980	974	982	978	980	974	982	978	980
Period	Mar. 3–July 15	Mar. 5–July 15	Mar. 3–July 15	Mar. 5–July 15	Mar. 3–July 15	Mar. 5–July 15	Mar. 3–July 15	Mar. 5–July 15	Mar. 3–July 15	Mar. 5–July 15	Mar. 3–July 15	Mar. 5–July 15
Height at withers	107.5	108.0	104.0	105.5	117.25	118.3	116.75	116.2	9.07	9.53	12.26	10.14
Height at hips	115.5	112.25	110.5	113.25	123.75	124.5	122.0	112.25	7.14	10.91	10.41	−0.88
Depth of chest	53.75	49.75	50.5	52.25	58.75	58.0	57.25	58.5	9.30	16.58	13.37	11.96
Depth of paunch	54.75	55.25	53.0	56.25	63.75	62.25	58.75	62.0	16.44	12.67	10.85	10.22
Width of chest	30.75	30.75	30.5	27.50	33.25	33.12	29.25	33.12	8.13	7.71	−4.10	20.44
Width of paunch	46.25	48.5	42.0	43.75	54.5	50.12	48.9	47.0	17.84	3.34	16.43	7.43
Width of shoulder points	36.75	35.0	32.5	35.25	38.5	41.0	35.5	39.75	4.76	17.14	9.23	12.77
Width of hips	34.5	32.0	32.0	31.0	39.5	38.0	37.12	36.5	14.49	18.75	16.00	17.74
Circum. of chest	137.16	132.08	130.81	133.35	153.67	152.40	125.73	151.77	12.04	15.38	−3.88	13.81
Circum. of paunch	163.83	154.31	153.67	161.93	185.42	180.98	170.18	175.26	13.87	17.28	10.74	8.23
Circum. of shank	16.51	15.58	15.24	15.88	17.78	17.78	16.18	17.78	7.69	14.11	6.17	11.94

A consideration of the gains and the efficiency of utilization of protein from the high-protein rations shows clearly that the supply was excessive, and uneconomical, and without positive advantage as compared with that of the low-protein rations.

The low-protein rations, supplying 0.95 to 1.69 pounds of digestible crude protein per 1000 pounds live weight, seem to have been much closer to the practical optimum than were the high-protein rations. It must be borne in mind, however, as previously noted, that the basis for comparison was appreciably imperfect.

The observed gains in weight as compared with Eckles's normals (Table IX) show that in all cases the gain exceeded this standard. The calves, therefore, may be considered as of normal character; the feed was abundant and nutritious, and the response of the animals was within the usual range.

The body measurements and the percentage increase in the same during the period of March 5 (or March 3) to July 15 (132 or 134 days) are set forth in Table X. These data do not add in important ways to the evidence; that is, they do not show any unmistakable difference in the character of the gain in weight by the low-protein and the high-protein calves. The factor of error in measurements of cattle is so great that a considerable volume of evidence is required in order to bring out comparatively small differences in an accurate manner.

CONCLUSIONS

General conclusions from this experiment are not warranted. The most significant individual data are those relating to the growth of Calf No. 982, aged 7 to 12 months, which in a feeding period of 133 days in the course of which were four digestion periods of 5–8 days length, consumed during these periods feed containing digestible crude protein in amounts of 1.3 to 1.69 pounds per 1000 pounds live weight, the total rations having net energy values of 11.33 to 14.88 Therms per 1000 pounds live weight, made an average daily gain in live weight of 1.58 pounds, which may be compared with a computed normal gain of 1.14 pounds, and stored protein at rates of 0.55 to 1.06 pounds per day, which may be compared with computed normals of 0.38 to 0.54 pounds.

This calf, on a so-called low-protein ration, therefore, seems to have received protein and total nutriment more than sufficient for normal growth.

INVESTIGATIONS AT THE PENNSYLVANIA STATION

EXPERIMENTS OF 1921–1922

The second year's work consisted of two digestion experiments with each of four calves, and a feeding experiment with the same subjects lasting 243 days.

The subjects were two Holstein heifers and two Jersey bull calves, in their seventh and eighth months, weighing between 203 and 338 pounds at the beginning.

TABLE I.—*Description of the Animals*

No.	Sex	Breed	Dropped, 1921	Age at start	Level of protein intake	Initial weight	No. of digestion trials	Length of trials (days)		Length of preliminary feeding (days)		Length of experiment
								I	II	I	II	
						Pounds						*Days*
1006	Heifer	Holstein	Jan. 29	7 mo. 16 da.	High	338	2	9	10	9	10	243
1013	Heifer	Holstein	Feb. 25	6 mo. 21 da.	Low	257	2	9	10	9	10	243
1016	Bull	Jersey	Mar. 11	6 mo. 4 da.	High	208	2	9	10	9	10	243
1017	Bull	Jersey	Mar. 11	6 mo. 4 da.	Low	203	2	9	10	9	10	243

The rations were composed of the standard components—alfalfa hay, oat straw, linseed oil meal, peanut meal, corn meal, and starch, the average amounts fed, per 1000 pounds live weight, during the experimental program as a whole, being set forth in Table II.

TABLE II.—*Average Daily Rations per 1000 Pounds Live Weight*

Feeds	Low protein				High protein			
	Fresh	Dry matter	Digestible crude protein	Net energy	Fresh	Dry matter	Digestible crude protein	Net energy
	Pounds	*Pounds*	*Pounds*	*Therms*	*Pounds*	*Pounds*	*Pounds*	*Therms*
Alfalfa hay....	4.24	3.75	....	....	6.89	6.09	....	....
Oat straw.....	4.51	4.01	....	....	4.82	4.28	....	....
Linseed meal..	1.09	0.98	....	....	1.77	1.59	....	....
Peanut meal ..	0.95	0.85	....	....	1.61	1.44	....	....
Corn meal....	1.35	1.20	....	....	2.00	1.78	....	....
Starch........	3.93	3.53	....	....	3.89	3.49	....	....
Totals....	....	14.32	0.659	6.76	....	18.67	1.571	8.34

In comparing the feed of the high-protein and low-protein calves it will be noted that the computed digestible crude protein differed between 0.659 and 1.571 pounds per 1000 pounds live weight per day, which was, in general, as contemplated by the plan. The dry matter and the net energy, however, also differed materially, which was not as contemplated. These differences may be considered as due to the adjustment of the amounts of feeds to the individual appetite of the subjects. Since they did not choose to eat the low-protein ration in amounts supplying as much net energy as did the high-protein ration, the conditions for the comparison of the results of the feeding on the basis of the plane of protein intake were appreciably imperfect.

After exhaustive consideration of the analytical data for the excreta, and of results based upon these analyses, it seemed to us inadvisable to use them. Either the condition of the experimental subjects was so highly abnormal that the results were without significance in relation to animals in a state of health, or the recorded analytical data for the excreta are in important respects incorrect, or, possibly both of these unfavorable conditions prevailed.

TABLE III.—*Gains in Live Weight (pounds)*

Period	Ration			
	High protein (Animal No. 1006)	Low protein (Animal No. 1013)	High protein (Animal No. 1016)	Low protein (Animal No. 1017)
At the end of 0 days.......	0	0	0	0
At the end of 13 days.......	9.0	10.0	3.0	−2.0
At the end of 19 days.......	25.0	15.0	14.0	11.0
At the end of 19 days.......	19.0	−3.0	1.0	1.0
At the end of 12 days.......	11.0	−12.0	−2.0	8.0
At the end of 25 days.......	30.0	25.0	22.0	3.0
At the end of 22 days.......	28.0	2.0	1.0	16.0
At the end of 35 days.......	62.0	17.0	40.0	1.0
At the end of 47 days.......	44.0	6.0	45.0	10.0
At the end of 18 days.......	11.0	3.0	14.0	5.0
At the end of 33 days.......	37.0	27.0	28.0	5.0
At the end of 243 days.......	276.0	90.0	166.0	58.0

The gains in live weight during 243 days are as in Table III. The gain of the high-protein calves was much superior to that of the calves receiving the low-protein ration, which is consistent with the facts that the high-protein calves consumed considerably more energy-producing food than did the low-protein calves, and that the latter received protein and energy-producing food only about sufficient for maintenance.

The rate of gain of the four subjects (Table IV) shows that one only was growing at a rate equal to the normal. The gain in weight by the other three was below normal, and very irregular.

TABLE IV.—*Comparison of Gains in Live Weight with Normal*

Animal No.	Total gain	Gain per day	Per cent gain	Normal gain	Normal gain per day	Per cent normal gain
1006	276.0	1.14	33.72	217.6	0.90	22.11
1013	90.0	0.37	14.40	231.3	0.95	25.20
1016	166.0	0.68	33.50	240.5	0.99	37.47
1017	58.0	0.24	16.75	240.5	0.99	37.47

CONCLUSION

The results of this experiment do not warrant general conclusions. The more noteworthy data are those relating to Calf No. 1006, on the so-called high-protein ration. During a feeding trial of 243 days this calf was 7½ to 15½ months of age. With an average daily intake of 1.571 pounds digestible crude protein and 8.34 Therms net energy per 1000 pounds live weight, it made an average daily gain in live weight of 1.14 pounds, which may be compared with a computed normal gain of 0.9 pound.

INVESTIGATIONS AT THE PENNSYLVANIA STATION

EXPERIMENTS OF 1922–23

The third year's work at the Pennsylvania Station involved two Holstein heifer calves, aged 9 and 10 months, respectively, at the beginning of the experiment. There was a feeding period of 150 days' duration, and one digestion and balance period for each subject, the urine and feces being collected separately.

In the digestion trial the preliminary feeding covered 17 days, and the collection period, 9 days. This preliminary period was undoubtedly adequate, and the collection period was fairly satisfactory as a digestion test, but apparently not satisfactory as a nitrogen balance period.

The rations were composed of the standard components. The high-protein ration, as fed during the 150 days, supplied 2.176 pounds of digestible crude protein per 1000 pounds live weight, and the low-protein ration, 1.305 pounds, the computed net energy values of these rations being 11.137 and 10.366 Therms, respectively. The high-protein ration, therefore, had an energy value 7.4 per cent greater than the low-protein ration.

In the digestion trial the digestible crude protein supplied by the high-protein and low-protein rations was 1.44 and 0.825 pounds, respectively, per 1000 pounds live weight, and the computed net energy, 8.775 and 8.564 Therms, respectively. The basis for comparison of the rations in the digestion trial in terms of protein supplied, therefore, was satisfactory.

The digestion coefficients of the rations, as determined, are exhibited in Table IV. The evidence is insufficient in extent to furnish a basis

TABLE I.—*Description of Animals and Experimental Periods*

Animal No.	Breed	Sex	Dropped, 1921	Age at beginning	Level of protein intake	Initial weight in pounds
1032	Holstein	Female	Oct. 20	10 mo.	Low	461
1033	Holstein	Female	Nov. 16	9 mo.	High	395

TABLE I.—*Concluded*

Animal No.	Experiment began, 1922	Experiment ended, 1923	Number of digestion trials	Length of digestion trials	Length of preliminary period
				Days	*Days*
1032	Aug. 17	Jan. 14	1	9	17
1033	Aug. 17	Jan. 14	1	9	17

TABLE II.—*Average Daily Ration per 1000 Pounds Live Weight*

	High protein		Low protein	
	Pounds	*Therms*	*Pounds*	*Therms*
Alfalfa hay	6.97	....	3.99	....
Corn meal	2.22	....	1.26	....
Linseed meal	1.82	....	1.03	....
Peanut meal	1.57	....	0.92	....
Oat straw	2.97	....	5.56	....
Starch	4.18	....	6.13	....
Digestible crude protein	2.173	....	1.305	....
Net energy	....	11.120	....	10.366

TABLE III.—*Protein and Energy Supply per 1000 Pounds Live Weight per Day in Digestion Trials*

	Digestible crude protein	Net energy
	Pounds	*Therms*
Calf No. 1032, Low protein	0.825	8.564
Calf No. 1033, High protein	1.440	8.775

for judgment as to whether the differences observed, in the digestibility of the several classes of nutrients in the two rations, are reflections of the individuality of the animals, or are related to the very considerable difference in protein content of the rations.

The utilization of the protein of the rations is as indicated in Table V. By subtraction of the computed maintenance requirements of protein from the intake we ascertain that the high-protein and

TABLE IV.—*Digestion Coefficients of Rations*

	High protein (Animal No. 1033)	Low protein (Animal No. 1032)
	Per cent	*Per cent*
Dry matter	62.936	68.362
Crude protein	52.851	50.000
Ether extract	55.909	44.944
Crude fiber	34.225	53.545
Nitrogen-free extract	77.258	79.246

TABLE V.—*Protein Supply and Daily Retention of Protein per 1000 Pounds Live Weight*

Animal No.	Age in days	Digestible crude protein consumed	Estimated maintenance requirement	Available for growth	Observed gain of protein	Estimated normal gain of protein
		Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*
No. 1033—High protein	372	1.440	0.600	0.840	0.009	0.344
No. 1032—Low protein	399	0.825	0.600	0.225	−0.151	0.322

TABLE VI.—*Gains in Live Weight*

Period	High protein (Animal No. 1033)	Low protein (Animal No. 1032)
	Pounds	*Pounds*
At end of 11 days	−6	6
At end of 17 days	21	15
At end of 22 days	14	24
At end of 22 days	17	10
At end of 21 days	27	26
At end of 9 days	−22	−24
At end of 35 days	49	65
At end of 13 days	13.3	3.6
At end of 150 days	113.3	125.6

low-protein rations supplied 0.840 and 0.225 pound, respectively, of protein available for growth; but the retention from the high-protein ration was only 0.009 pound of protein, and with the low-protein ration there was a loss of 0.151 pound protein—which signifies that either the experimental conditions were unsatisfactory or the technic faulty. The recorded response of the subjects to the experimental treatment did not truly represent the value and the character of the ration.

TABLE VII.—*Comparison of gain in live weight with the normal gain*

Animal No.	Average daily gain	Total gain	Percentage gain	Average normal daily gain	Normal total gain	Normal percentage gain
	Pounds	*Pounds*		*Pounds*	*Pounds*	*Per cent*
No. 1033—High protein..	0.755	113.3	28.7	0.853	128	27.4
No. 1032—Low protein..	0.837	125.6	27.2	0.733	110	21.4

TABLE VIII.—*Body Measurements* (*centimeters*)

Measurements	Initial		Final		Percentage increase	
	Aug. 28, Low protein (Animal No. 1032)	Aug. 28, High protein (Animal No. 1033)	Jan. 14, Low protein (Animal No. 1032)	Jan. 14, High protein (Animal No. 1033)	Low protein (Animal No. 1032)	High protein (Animal No. 1033)
Height at withers	110.50	112.50	117.00	110.5	5.88	−1.78
Height at hips	116.0	116.75	121.25	113.25	4.53	−3.00
Width of hips	33.0	32.25	37.25	30.5	12.88	−5.43
Width of shoulder	32.25	29.25	32.25	31.75	0.0	8.55
Width of chest	29.25	27.25	30.75	30.5	5.13	11.93
Width of paunch	43.25	42.75	30.0	45.0	−30.64	5.26
Heart girth	137.16	128.27	152.40	146.05	11.11	13.86
Paunch girth	156.85	152.40	171.45	161.29	9.31	5.83
Right shank (circumference)	16.51	14.61	20.96	17.78	23.95	21.70
Depth of paunch	59.50	47.75	52.25	52.25	−12.18	9.42
Depth of chest	51.25	47.50	54.5	52.25	6.34	11.00

The gain of the two calves in live weight during the 150 days is recorded in Table VI. The low-protein calf made the greater gain. In view of the individual variability in the gains of cattle, and the magnitude of the factor of error in live weights of cattle, however, the difference in gain of these two calves can not be regarded as possessing definite significance.

Table VII shows that the percentage of gain in weight was about the same with the two calves, and that the rate was fully up to the normal.

The body measurements of the subjects as exhibited in Table VIII show clearly the difficulty of taking significant measurements of cattle. Safe conclusions from measurements can be drawn only from extensive evidence of the most critical character.

CONCLUSION

The most significant data from this experiment are those relating to Calf No. 1032, which during a feeding experiment of 150 days' duration was 10 to 15 months of age, and received the low-protein ration.

With an average daily intake of 1.305 pounds digestible crude protein, and 10.366 Therms net energy, per 1000 pounds live weight, it made an average daily gain in live weight of 0.837 pound which may be compared with a computed normal gain of 0.733 pound.

INVESTIGATIONS AT THE NORTH DAKOTA AGRICULTURAL EXPERIMENT STATION

F. W. CHRISTENSEN and T. H. HOPPER, with the collaboration of J. W. INCE and L. T. ANDEREGG

EXPERIMENTS OF 1919 AND 1920

On account of difficulties in obtaining labor and certain supplies the plan of the experiments, as submitted, was modified to suit the possibilities of the situation. Thus calves not being available, yearling steers (coming twos) were used as subjects in the first experiment. In the second test calves 6 to 7 months of age were used; and peanut meal was not to be had; also the use of starch in the compounding of the rations was eliminated, as a measure of economy. The protein of the rations as compounded differed not only quantitatively, as contemplated, but also qualitatively.

TABLE I.—*Schedule of Experiments*

Breed of subject	Subject No.	Age at beginning[1]	Level of protein intake	Initial live weight	Length of feeding period	Digestion periods	Preliminary feeding periods
		Days		*Pounds*	*Days*	*Days*	*Days*
Grade Shorthorn..	3A	600	Low	648.3	64	10	17
Grade Shorthorn..	7B	600	High	670.0	64	10	17
Grade Shorthorn..	5C	600	Low	703.3	64	10	17
Grade Shorthorn..	9D	600	High	641.6	64	10	17
Grade Hereford...	A	180–210	Low	416.6	155	10	10
Grade Hereford...	B	180–210	High	391.4	155	10	10
Grade Hereford...	C	180–210	Low	314.6	155	10	10
Grade Hereford...	D	180–210	High	315.0	155	10	10

[1] Estimated.

A general outline of the experiments composes Table I. A digestion experiment of 10 days' duration was conducted with four grade Shorthorn steers during a feeding period of 64 days in the spring of 1919; and two digestion experiments, likewise of 10 days' duration, were conducted in February and May, 1920, with four grade Hereford steers, during a feeding period of 155 days. The preliminary feeding periods in these two series of tests were 17 and 10 days, respectively. The ages of the subjects, as stated, was estimated, the exact ages being unknown.

The rations were compounded from oat straw, alfalfa hay, corn meal and linseed meal. It will be noted that the proportions of the protein-rich foods, one to another, were not constant, a condition resulting from the method chosen of balancing the rations without the use of starch.

During the feeding periods the rations were adjusted as to quantity at intervals of about 10 days so as to provide for an expected daily gain of 1.25 pounds per 1000 pounds live weight.

During the early part of the experiment and the first digestion trial the animals received decidedly less feed than as contemplated, because the oat straw and alfalfa hay proved to be much higher in moisture content than as assumed in the computation of the rations.

Table II.—*Average Daily Rations during Digestion Trials—Pounds Dry Matter*

Date of trial	Level of protein intake	Number of animal	Oat straw	Alfalfa hay	Corn meal	Linseed meal
1919	Low	3A	5.97	4.62	1.59	0.23
	Low	5C	4.78	4.94	2.54	.13
April 16–25	High	7B	1.66	9.43	.14	1.59
	High	9D	1.55	8.82	.14	1.48
1920	Low	A	2.89	4.50	3.45	.20
	Low	C	2.18	3.41	2.61	.16
Feb. 10–19	High	B	2.75	4.30	1.90	1.55
	High	D	2.26	3.53	1.53	1.28
1920	Low	A	5.26	3.72	4.08	.11
	Low	C	4.02	2.84	3.12	.08
May 12–21	High	B	3.83	2.72	2.10	2.13
	High	D	4.28	3.02	1.72	1 74

The digestion coefficients of the nutrients in the several rations are set forth in Table III. The variability of results was greater than prevails under favorable conditions of work. The one difference in digestibility between the high-protein and the low-protein rations which was unquestionably significant, in the sense of not being due to either individuality or to error of work, was that in regard to protein. The digestibility of the crude protein of all of the high-protein rations was distinctly higher than that of any one of the low-protein rations. The significance of this consistent difference, however, is not clear, since it may be due in part, at least, to difference in kind of protein, or to the metabolic nitrogen, which was not estimated, and which was probably more nearly a constant of the animal than of the feed, having the effect to lower the apparent digestibility of the protein of the low-protein as compared with that of the high-protein ration.

The nitrogen balances (Table IV) do not vary closely in accord with the nitrogen intake. The urinary elimination of nitrogen much more closely follows the differences in the intake than does either the balance or the fecal elimination.

TABLE III.—*Digestion Coefficients*

Date of trial	Level of protein intake	Animal No.	Dry matter	Crude protein	Crude fiber	Nitrogen-free extract	Ether extract
1919 April 16–25	Low	3A	50.25	50.83	45.14	61.01	22.67
		5C	58.95	57.14	50.11	71.01	46.36
	High	7B	56.27	75.23	45.06	65.67	44.01
		9D	56.54	78.00	49.64	60.04	44.21
1920 Feb. 10–19	Low	A	62.90	58.87	50.87	71.38	53.52
		C	59.31	51.72	42.31	70.21	53.59
	High	B	61.09	66.88	44.26	70.62	62.69
		D	58.76	65.89	40.01	68.46	63.55
1920 May 12–21	Low	A	60.64	54.89	48.25	70.51	56.22
		C	57.28	51.49	43.02	70.00	51.25
	High	B	60.84	67.07	44.18	71.91	63.70
		D	57.52	66.23	43.87	67.50	65.26

TABLE IV.—*Nitrogen Balances—Pounds*

Date of trial	Level of protein intake	Animal No.	Feed	Urine	Feces	Balance
1919 Apr. 16–25	Low	3A	0.1910	0.1191	0.0965	−0.0246
		5C	.2029	.1107	.0862	+.0060
	High	7B	.3224	.2136	.0878	+.0211
		9D	.3223	.2400	.0887	−.0064
1920 Feb. 10–19	Low	A	.2000	.0791	.0813	+.0396
		C	.1525	.0508	.0678	+.0339
	High	B	.2506	.1416	.0836	+.0254
		D	.2070	.0998	.0698	+.0374
1920 May 12–21	Low	A	.2113	.0826	.0952	+.0335
		C	.1613	.0526	.0783	+.0304
	High	B	.2649	.1567	.0856	+.0226
		D	.2462	.1414	.0826	+.0222

The economy of utilization of protein on the thousand-pound live weight basis is exhibited by the data composing Table V.

The amounts of available crude protein varied between the high-protein and low-protein animals in a manner generally consistent and in harmony with the protein intake.

The gain of protein per 1000 pounds live weight in the first experiment varied inconsistently, and was less than the normal except with Calf No. 7B, a high-protein calf, with which the gain of protein just coincided with the normal.

Table V.—*Protein Available for Growth during Digestion Trials as Compared with Observed and Normal Gain in Protein*

Date of trial	Level of protein intake	Animal No.	Age of animal	Available crude protein	Gain of protein per 1000 pounds live weight	Normal gain of protein per 1000 pounds live weight	Available protein utilized	Total protein utilized
			Days	*Pounds*	*Pounds*	*Pounds*	*Per cent*	*Per cent*
1919 April 16–25	Low	3A	690	0.27	−0.22	0.19	....	....
		5C	690	0.38	.05	.19	13.16	2.96
	High	7B	690	1.89	.19	.19	10.05	5.80
		9D	690	1.63	−.06	.19	....	....
1920 Feb. 10–19	Low	A	270	0.91	.52	.47	57.14	19.70
		C	240	0.60	.56	.52	93.34	22.16
	High	B	270	1.64	.34	.47	20.73	10.09
		D	240	1.58	.59	.52	37.34	18.07
1920 May 12–21	Low	A	360	0.54	.33	.36	61.11	15.88
		C	360	0.45	.38	.39	84.44	18.88
	High	B	360	1.29	.24	.36	18.60	14.12
		D	330	1.36	.25	.39	18.38	9.02

In the two later experiments more favorable conditions appear to have prevailed. The correspondence between the observed and the normal gain of protein was close; and the comparative gain by the high-protein and the low-protein calves suggests strongly that the rate of intake by the calves on the low-protein rations was close to the normal, and that no certain advantage accrued from the larger protein intake of the high-protein calves.

These observations are borne out by the data representing utilization of available protein and total protein. Assuming that the low-protein rations supplied as much protein as required for normal growth, as indicated by the data for gain of protein per 1000 pounds live weight, it would naturally follow, as was actually observed, that the utilization of the total and the available protein should have been more nearly perfect from the low-protein than from the high-protein rations.

In the feeding-for-growth periods the average daily rations were as indicated in Table VI. In the two seasons' experiments we have comparisons between an intake of 0.72 and 2.03 pounds of digestible crude protein, and between 1.08 and 1.90 pounds of the same. In the first experiment the net energy of the high-protein rations was 22.1 per cent greater than of the low-protein rations, and in the second the high-protein rations exceeded the low-protein in net energy by 6.2 per cent. The basis for comparison, therefore, was much more favorable in the second than in the first experiment.

TABLE VI.—*Average Daily Rations per 1000 Pounds Live Weight during Feeding Periods—Dry Matter Basis*

Year	Level of protein intake	Oat straw	Alfalfa hay	Corn meal	Linseed meal	Crude protein (digestible)	Net energy
		Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Therms*
1919	Low	8.64	7.35	2.36	0.26	0.72	6.13
1919	High	2.38	15.28	0.21	1.93	2.03	6.87
1919–1920	Low	6.18	9.64	7.39	0.44	1.08	9.82
1919–1920	High	7.45	7.90	3.88	3.53	1.90	10.13

The digestible nutrients as computed per 1000 pounds live weight for the individual animals in these growth periods are as indicated in Table VII. It will be noted that the fourth division of the table, representing the period March 13 to May 15, 1913, covers the three sub-periods, and that the last section, representing the period from December 23, 1919 to May 25, 1920, covers two sub-periods.

On the thousand-pound live weight basis, in the 64-day period, the comparison of protein was between 0.903 and 1.02 pounds in the low-protein rations, and 2.39 and 2.31 pounds in the high-protein rations, with corresponding net energy values of 5.43 and 6.82 Therms in the low-protein, and 7.11 and 6.64 Therms in the high-protein rations.

It will be noted that in the March to May, 1919, experiments the net energy per 1000 pounds live weight in a number of instances was less than six Therms, the presumption being, therefore, that these animals were fed less than a maintenance ration, though we do not have discriminating evidence as to the maintenance requirement of cattle throughout the range of live weights. With the energy intake at such low figures the conditions were not favorable for the expression of the effects of the different planes of protein intake.

TABLE VII.—*Average Daily Digestible Nutrients per 1000 Pounds Live Weight, during Feeding Periods*

Dates of period	Level of protein	Animal No.	Digestible organic matter	Digestible crude protein	Net energy in feed
			Pounds	*Pounds*	*Therms*
1919 Mar. 13–29	Low	3A	8.42	0.888	5.08
		5C	8.07	.919	4.86
	High	7B	10.91	2.47	7.18
		9D	9.75	2.23	6.29
1919 Mar. 30–Apr. 25	Low	3A	8.72	.881	5.45
		5C	9.69	1.04	7.34
	High	7B	9.84	2.29	6.93
		9D	9.60	2.26	6.56
1919 Apr. 26–May 15	Low	3A	9.30	.944	5.69
		5C	10.28	1.09	7.78
	High	7B	10.43	2.46	7.29
		9D	10.29	2.45	7.06
1919 Mar. 13–May 15 (64 days)	Low	3A	8.82	.903	5.43
		5C	9.44	1.02	6.82
	High	7B	10.31	2.39	7.11
		9D	9.86	2.31	6.64
1919–1920 Dec. 23–Mar. 21	Low	A	13.621	1.57	10.905
		C	12.750	1.46	9.515
	High	B	12.854	2.28	10.636
		D	12.657	2.30	9.980
1920 Mar. 22–May 25	Low	A	12.379	1.20	9.688
		C	11.876	1.12	8.883
	High	B	11.574	2.04	10.127
		D	11.835	2.04	9.638
1919–1920 Dec. 23–May 25 (155 days)	Low	A	13.100	1.41	10.395
		C	12.383	1.31	9.250
	High	B	12.317	2.18	10.423
		D	12.312	2.19	9.837

In the experiment of 1920 the net energy of the rations was, on the average, 53.5 per cent greater than in the earlier experiment, and the high-protein and low-protein rations agreed closely as to energy value. The conditions, therefore, were much more favorable for the comparison of the effects of the different planes of protein intake.

The live weights and gains in live weight are set forth in Table VIII. These live weights are averages of three weights taken on successive days.

TABLE VIII.—*Live Weights and Gains in Live Weight during Feeding Periods—Pounds*

Date of	Level of protein intake	Animal No.	Initial weight	Final weight	Total gain	Average daily gain
1919	Low	3A	648.3	696.7	48.4	2.85
		5C	703.3	725.0	21.7	1.28
Mar. 13–29	High	7B	670.0	681.7	11.7	.69
		9D	641.6	648.3	6.7	.39
1919	Low	3A	696.7	698.3	1.6	.06
		5C	725.0	736.7	11.7	.43
Mar. 30–Apr. 25	High	7B	681.7	666.7	−15.0	−.56
		9D	648.3	655.0	6.7	.25
1919	Low	3A	698.3	730.0	31.7	1.59
		5C	736.7	765.0	28.3	1.42
Apr. 26–May 15	High	7B	666.7	696.7	30.0	1.50
		9D	655.0	671.3	16.3	.82
1919	Low	3A	648.3	730.0	81.7	1.28
		5C	703.3	765.0	61.7	.96
Mar. 13–May 15 (64 days)	High	7B	670.0	696.7	26.7	0.42
		9D	641.6	671.3	29.7	0.46
1919–1920	Low	A	416.6	550.8	134.2	1.491
		C	314.6	422.4	107.8	1.198
Dec. 23–Mar. 21	High	B	391.4	503.0	111.6	1.240
		D	315.0	440.0	125.2	1.391
1920	Low	A	550.8	654.4	103.6	1.594
		C	422.4	503.0	80.6	1.240
Mar. 22–May 25	High	B	503.0	587.0	84.0	1.292
		D	440.2	525.4	85.2	1.311
1919–1920	Low	A	416.6	654.4	237.8	1.534
		C	314.6	503.0	188.4	1.215
Dec. 23–May 25 (155 days)	High	B	391.4	587.0	195.6	1.262
		D	315.0	525.4	210.4	1.357

The gains in weight in the experiment of 1919 were not consistent with the nutritive values of the rations. This part of the work must be considered unsuccessful.

The gains in weight in the second feeding experiment were much more satisfactory. The average daily gain of the low-protein steers was 1.37 pounds, and of the high-protein steers 1.31 pounds. In consideration of the small numbers of individuals involved it is our judgment that the difference observed in favor of the low-protein ration was not shown to be significant. In so far as a conclusion is justifiable it is that the protein intake with the low-protein steers was sufficient, and that the higher intake of the high-protein steers was not shown to be more efficient.

TABLE IX.—*Gains in Live Weight Compared with Normal Gain*

Level of protein	Animal No.	Age of animal	Length of feeding period	Total gain in weight	Percent gain in weight	Normal gain in weight	Percent normal gain in weight
		Months	*Days*	*Pounds*	*Percent*	*Pounds*	*Percent*
Low	3A	20–22	64	81.7	12.60	104	14.04
1919	5C	20–22	64	61.7	8.77	104	14.04
High	7B	20–22	64	26.7	3.99	104	14.04
1919	9D	20–22	64	29.7	4.52	104	14.04
Low	A	7–12	155	237.8	57.08	199	57.18
1919–20	C	7–12	155	188.4	59.89	260	82.28
High	B	7–12	155	185.6	47.42	199	57.18
1919–20	D	7–12	155	210.4	66.79	260	82.28

From the gains in live weight, in comparison with the normal gain (Table IX), it appears that the response of the animals to the feed was in general irregular and below normal. In order to have a perfect basis for comparison of the high-protein and low-protein rations the more efficient one should have produced gain in weight at least equal to the normal. One steer only in each year made a gain equal to or approximating the normal.

CONCLUSION

It is not warrantable to present a comprehensive summary. In our opinion the most useful data are those relating to Calf A in the second test (Dec. 23, 1919 to May 25, 1920), on the low-protein ration.

This calf, initial age 6 to 7 months, during a feeding period of 155 days made a gain in live weight just equal to the computed normal gain of 1.53 pounds on an intake per 1000 pounds live weight of 1.41 pounds digestible crude protein and 10.395 Therms net energy.

INVESTIGATIONS AT THE SOUTH DAKOTA AGRICULTURAL EXPERIMENTAL STATION

B. L. JOHNSON

EXPERIMENTS OF 1919–20

In the work at the South Dakota Station there were conducted two digestion trials, and a growth experiment of 408 days. The data here submitted have to do with the growth period only.

The subjects were four male calves reported to have been "about 15/16 Holstein." Their initial ages were estimated to have ranged from 6½ to 10 months, with corresponding weights of 339 to 468 pounds.

TABLE. I—*Description of Animals*

Pair	No.	Breed	Sex	Age at beginning of experiment	High or low protein ration	Initial weight
				Months		*Pounds*
1	306	Holstein	Male	6½	High	344
	307	Holstein	Male	6½	Low	339
2	308	Holstein	Male	8	High	396
	309	Holstein	Male	10	Low	468

TABLE II.—*Average Daily Rations per 1000 Pounds Live Weight during Feeding Period of 408 Days*

Feeding stuff	High protein ration				Low protein ration			
	Feeding stuff	Digestible crude protein	Digestible true protein	Net energy	Feeding stuff	Digestible crude protein	Digestible true protein	Net energy
	Pounds	*Pounds*	*Pounds*	*Therms*	*Pounds*	*Pounds*	*Pounds*	*Therms*
Oat straw....	8.91	....	....	....	11.49	....	....	....
Alfalfa hay...	8.59	....	....	....	4.54	....	....	....
Corn meal....	3.20	....	....	....	1.66	....	....	....
Linseed meal.	1.84	....	....	....	0.97	....	....	....
Peanut meal..	1.69	....	....	....	0.89	....	....	....
Starch.......	0.54	(1)	(2)	(3)	3.97	(1)	(2)	(3)
Total......	24.77	2.502	2.112	12.350	23.52	1.399	1.178	11.426

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[1] Computed from Pa. Agr. Exp. Sta., Bul. 142.

[2] Computed from Pa. Agr. Exp. Sta., Bul. 142.

[3] For computing the net energy value of starch the figure 0.8179 Therms per pound of dry substance was used, the figure for dry matter used being the average of the determinations made for the two digestion periods.

The rations were compounded from the feeds suggested in the plan submitted, as usual on two levels of protein intake, the energy value being maintained as nearly the same as practicable.

Since the feeds of the growth period were not analyzed we have computed the nutritive value of the rations by the use of published average feed analyses.

It will be noted that the crude protein intake per 1000 pounds live weight was 2.502 pounds, and 1.399 pounds, respectively, with the high-protein and low-protein groups, and that the corresponding net energy values, as computed from the average analyses used, were 12.350 and 11.426 Therms.

TABLE III.—*Gains in Live Weight*

Weight after intervals	Pair No. 1		Pair No. 2	
	High protein	Low protein	High protein	Low protein
	Pounds	*Pounds*	*Pounds*	*Pounds*
28 days	15	22	33	20
56 days	44	17	7	12
84 days	28	50	47	48
112 days	7	6	11	24
140 days	43	51	63	51
168 days	24	−1	16	30
196 days	31	42	35	18
224 days	23	37	42	59
252 days	20	22	14	24
287 days	54	44	30	36
310 days	14	46	37	36
343 days	51	26	38	43
408 days	47	95	95	124
Total	401	457	468	525

The gains in live weight appear to have been the greater with the low-protein than with the high-protein calves. In view of the small number of individuals it is not justifiable to place emphasis upon this difference. It seems unlikely that the excess of protein in the high-protein as compared with the low-protein ration was an actual disadvantage. A safer suggestion appears to be that the protein of the low-protein ration was sufficient, and that a more liberal supply was not shown to be advantageous.

The gains in live weight, in comparison with the normal, show that all of the calves were well nourished, the gain exceeding the normal in all cases, both on an absolute and on a percentage basis. Even though this should mean, in part, that the calves were passing through a period of compensation for previous subnormal gains, it signifies, none the less, that the rations were adequate to the support of such abundant increase.

The gains in body measurements are reported as a part of the record but do not permit of the drawing of conclusions. The difficulties involved in the taking of measurements of cattle are so great that safe deductions can be drawn only from extensive data.

TABLE IV.—*Comparison of Gains in Live Weight with the Normal*

High protein ration						Low protein ration					
Animal No.	Daily gain	Total gain	Percentage gain	Normal total gain	Normal percentage gain	Animal No.	Daily gain	Total gain	Percentage gain	Normal total gain	Normal percentage gain
	Pounds	*Pounds*		*Pounds*			*Pounds*	*Pounds*		*Pounds*	
306	0.983	401	117	377	102	307	1.120	457	135	377	102
308	1.147	468	93	356	82	309	1.287	525	112	334	67
Means	1.065	434	105	366	92	Means	1.203	491	123	355	84

TABLE V.—*Increase in Body Measurements during the Feeding Period of 408 Days*

Group	Animal No.	Heart girth	Height at withers	Height at hips	Width of hips	Length of body
		Inches	*Inches*	*Inches*	*Inches*	*Inches*
High protein	306	11.00	8.75	8.50	5.00	12.00
	308	12.75	8.50	8.25	4.25	11.00
Low protein	307	14.25	10.50	9.25	5.50	12.25
	309	12.25	5.75	7.50	6.75	6.75

SUMMARY

The data of greatest significance in relation to protein requirement seem to be that two calves, initial ages 6½ and 10 months, respectively, during 408 days made a gain in live weight exceeding the computed normal on an intake per 1000 pounds live weight of 1.399 pounds digestible crude protein and 11.426 Therms net energy. These calves received the low-protein ration.

INVESTIGATIONS AT THE MARYLAND AGRICULTURAL EXPERIMENT STATION

R. C. TOWLES

Experiment of 1918–19

The work at the Maryland Station consisted of a feeding experiment with two grade Guernsey-Jersey calves, males, aged 4½ months, the experiment extending over a period of 138 days.

Of these two calves one received a high-protein and the other a low-protein ration. The average daily rations fed, per 1000 pounds live weight, are stated below.

Table I.—*Average Daily Rations per 1000 Pounds Live Weight*

Level of protein intake	Corn meal	Linseed meal	Peanut meal	Starch	Mixed hay	Digestible true protein	Net energy
	Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Therms*
Low..........	5.39	0.43	1.41	0.33	19.78	1.91	14.17
High..........	0.95	2.40	4.57	0.35	18.39	3.44	14.59

Table II.—*Gains in Live Weight during 138 Days—Pounds*

Period No.	Length of sub-period	Low protein ration		High protein ration	
		Gain per period	Average daily gain	Gain per period	Average daily gain
	Days				
1..........	14	26.5		18.5	
2..........	14	−5.5		14.5	
3..........	19	7.0		9.0	
4..........	14	17.0		22.0	
5..........	21	18.0		13.0	
6..........	14	7.0		22.0	
7..........	14	10.0		2.0	
8..........	14	11.0		32.0	
9..........	14	5.0		−7.0	
			0.69		0.91

The hay used was a mixture of timothy, alfalfa and alsike clover. In the case of this experiment the feed protein was reported as "true" protein, and we do not have the corresponding figures on the "crude" basis.

Per 1000 pounds live weight the low-protein calf received 1.91

pounds digestible true protein (2.37 crude, computed) and 14.17 Therms net energy, while the high-protein calf received 3.44 pounds true-protein (3.93 pounds crude, computed) and 14.59 Therms net energy.

The gains in live weight during the 138 days are as in Table II. While the advantage is with the high-protein calf, such a difference, between two individuals, in consideration of the magnitude of the probable error of live weights of cattle, is of uncertain significance.

The low-protein calf required, per pound of gain in live weight, 0.564 pound true protein and 4.18 Therms net energy, while the high-protein calf required 0.831 pound true protein and 3.53 Therms net energy. We compute that the above amounts of true protein are equivalent to 0.697 and 0.947 pound, respectively, of crude protein.

TABLE III.—*Gain in Live Weight Compared with the Normal*

Level of protein intake	Age at beginning	Age at end	Gain in weight	Percent gain in live weight	Normal gain in live weight	Percent normal gain
	Days	*Days*	*Pounds*		*Pounds*	
Low.........	135	273	96	62.75	181	91.41
High........	135	273	126	83.44	181	91.41

Table III shows that the growth made by the two subjects was at a rate distinctly less than the normal, especially in the case of the low-protein calf.

SUMMARY

In view of the heavy feeding which prevailed, and the subnormal gain in live weight, we are of the opinion that the results of this experiment do not contribute significant evidence on the subject of protein requirements.

INVESTIGATIONS AT THE NEBRASKA AGRICULTURAL EXPERIMENT STATION

W. B. NEVINS and J. W. HENDRICKSON[1]

EXPERIMENT OF 1919–20

The work at the Nebraska Station consisted of growth experiments only. Six Jersey and two Holstein heifers were fed as four pairs, one individual of each being on a high-protein and the other on a low-protein ration.

The subjects ranged in age from 8⅔ to 17⅓ months at the beginning, and the initial weights varied from 429 to 589 pounds.

TABLE I.—*Description of Animals and Gains in Live Weight*

Pairs of calves	Individual calves	Breed of calves	Initial age of calves	Level of protein intake	Length of feeding period	Initial live weight	Final live weight	Average daily gain in weight
			Months		*Days*	*Pounds*	*Pounds*	*Pounds*
I	1	Jersey	15⅓	High	266	562	818	0.962
	2	Jersey	16	Low	266	550	751	0.756
II	3	Jersey	17⅓	High	266	583	770	0.703
	4	Jersey	14	Low	266	536	804	1.007
III	5	Jersey	11	High	252	475	858	0.941
	6	Jersey	10½	Low	252	429	855	1.047
IV	255	Holstein	9½	High	407	589	959	1.460
	256	Holstein	8⅔	Low	407	532	848	1.254

The feeding period for two pairs was 266 days, for one pair 252 days, and for one pair 407 days. The data consist of live weights, body measurements, and feed weights and analyses (See Table I).

The rations were compounded from the feeds contemplated in the plan as submitted except for the facts that a part of the alfalfa hay was fed in the form of meal, and wheat straw was substituted, in part, for oat straw because of the fact that the animals were willing to eat more of the wheat straw than of the less desirable quality of the oat straw which was available.

It will be noted in Table II that the intake of digestible crude protein in the high-protein and low-protein rations was 2.098 and

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[1] For a more extended report of this experiment see Nebraska Agr. Exp. Sta., Bul. 184.

1.361 pounds, respectively, the corresponding net energy values being 11.058 and 11.962 Therms. The low-protein rations, therefore, appear to have contained, on the average, 8.17 per cent more net energy per 1000 pounds live weight of animal than did the high-protein rations.

The monthly gains in live weight compose Table III. They exhibit the usual perplexingly high range of variation.

TABLE II.—*Average Daily Rations per 1000 Pounds Live Weight*

	High protein ration					Low protein ration				
	Calf No. 1	Calf No. 3	Calf No. 5	Calf No. 255	Mean	Calf No. 2	Calf No. 4	Calf No. 6	Calf No. 256	Mean
	Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Pounds*
Alfalfa hay	0.31	0.29	0.15	......	0.25	......	......	......	......	......
Alfalfa meal	7.88	7.46	8.03	8.15	7.88	4.83	4.95	4.69	4.88	4.84
Corn meal	3.11	2.83	2.99	2.98	2.98	1.79	1.82	1.73	1.78	1.78
Linseed meal	1.69	1.59	1.68	1.68	1.66	1.01	1.02	0.98	1.01	1.01
Starch	2.11	1.99	1.89	1.94	1.98	6.21	6.43	6.64	6.92	6.55
Peanut meal	1.65	1.55	1.64	1.63	1.62	0.97	0.99	0.94	0.98	0.97
Wheat straw	0.76	0.77	0.48	......	0.67	0.47	0.58	0.37	......	0.47
Oat straw	2.24	2.18	3.30	4.37	3.02	1.92	2.24	3.05	4.85	3.01
Sugar	......	......	......	......	......	1.01	1.01	0.59	......	0.87
Total	......	......	......	......	20.06	......	......	......	......	19.49
Digestible crude protein[1]	2.194	2.059	1.944	2.196	2.098	1.551	1.346	1.170	1.377	1.361
Digestible true protein[2]	1.858	1.746	1.617	1.853	1.769	1.117	1.139	0.973	1.157	1.097
Net energy (Therms)[3]	11.086	10.380	11.177	11.587	11.058	11.571	11.960	11.860	12.455	11.962

[1] Computed from the crude protein as determined, and coefficients of digestibility as compiled by Henry and Morrison.

[2] Computed from data in Pa. Agr. Exp. Sta., Bul. 142.

[3] Computed from data in Pa. Agr. Exp. Sta., Bul. 142, corrected for moisture content of feeds as determined.

Table IV presents the total and the average daily gains in live weight of the two groups in comparison with each other and with average normal values.

In two of the four pairs the high-protein calf made the most rapid growth, the average daily gain with the two treatments being identical.

The percentage of gain in weight in comparison with the normal shows that all of the individuals made average or more than average gains, there being insufficient basis for distinction between the two groups.

The increase in body measurements might be considered to show that the low-protein calves made a somewhat greater growth. If so, as already noted, they received 8.17 per cent more net energy in the

TABLE III.—*Monthly Gains in Live Weight*

Pairs of calves	Calf No.	Level of protein intake	Months, and gains in live weight (pounds)														Total
			1	2	3	4	5	6	7	8	9	10	11	12	13	+	
I	1	High	−9	42	50	32	13	57	17	15	39						266 days—256 lbs.
	2	Low	−2	42	42	14	4	33	15	21	32						266 days—201 lbs.
II	3	High	−1	40	44	0	1	49	43	−10	21						266 days—187 lbs.
	4	Low	0	44	55	18	24	37	36	32	22						266 days—268 lbs.
III	5	High	−10	35	56	9	19	43	21	25	19	37	40	39	12	38	407 days—383 lbs.
	6	Low	9	40	35	22	0	43	27	38	8	57	49	50	40	8	407 days—426 lbs.
IV	255	High	35	54	47	50	52	57	3	73	5						252 days—370 lbs.
	256	Low	40	53	12	58	28	40	33	54	−2						252 days—316 lbs.

ration. As much as can be said with certainty is that the protein intake of the low-protein group was not shown to be insufficient, the presumption being that it was adequate.

TABLE IV.—*Comparison of Gains in Live Weight with the Normal*

Group	Animal number	Number of days fed	Total gain	Average daily gain	Percentage gain	Normal total gain	Normal daily gain	Normal percentage gain
			Pounds	*Pounds*		*Pounds*	*Pounds*	
High protein ration	1	266	256	0.962	45.5	192	0.722	36.6
	3	266	187	0.703	32.1	199	0.748	35.6
	5	407	383	0.941	80.6	290	0.712	66.5
	255	252	370	1.460	62.8	197	0.782	40.8
	Means	....	299.0	1.02	55.2	219.5	0.741	44.9
Low protein ration	2	266	201	0.756	36.5	197	0.741	37.0
	4	266	268	1.007	50.0	179	0.673	35.6
	6	407	426	1.047	99.3	289	0.710	68.8
	256	252	316	1.254	59.4	207	0.821	45.7
	Means	....	302.7	1.02	61.3	218	0.736	46.8

TABLE V.—*Increase in Body Measurements during the Feeding Period—Centimeters*

Group	Animal No.	Height at withers	Height at hips	Width at hips	Length of body	Heart girth
High protein ration	1	4.0	7.4	5.5	11.8	18.7
	3	3.5	4.4	4.0	5.8	16.0
	5	11.6	9.8	9.6	23.0	32.0
	255	10.6	11.4	8.0	23.7	23.5
	Means	7.4	8.2	6.8	16.1	22.5
Low protein ration	2	3.6	2.5	2.9	14.1	15.3
	4	6.2	7.7	5.7	16.1	21.3
	6	13.2	12.6	9.7	21.7	33.4
	256	13.7	11.3	9.0	20.4	25.0
	Means	9.2	8.5	6.8	18.1	23.7

SUMMARY

The results of greatest significance in connection with protein requirements are those relating to the low-protein group.

Four heifers, aged 16, 14, 10½ and 8⅔ months, respectively, in feeding periods of 266, 266, 252 and 407 days, the average initial weights being 550, 536, 429 and 532 pounds, with intake of digestible crude protein of 1.551, 1.346, 1.170, 1.377 and 1.361 pounds, and net energy of 11.571, 11.960, 11.860, 12.455 and 11.962 Therms, per 1000 pounds live weight, made average daily gains in weight of 0.756, 1.007, 1.047 and 1.254 pounds, which in each case exceeded the computed normal gain.

DISCUSSION OF RESULTS FROM ALL EXPERIMENTS AND EXPERIMENT STATIONS

In consideration of the nature of the results it seems impossible to group them for summary treatment, and that in relation to protein requirements for growth the most significant data are those representing the performance of those individuals which made a growth at least equal to the computed normal growth on the smallest intake of protein.

On this basis, then, we have selected the following data. As to their significance, however, it should be borne in mind that the evidence is in all cases slight, and that these individual records represent isolated observations in the sense of not being derived from experiments in which there was such a gradation of protein intake, by small steps, as would be necessary accurately to determine the minimum intake capable of supporting normal growth.

For comparison with these data we have numerous feeding standards, but certainly no other one is so widely used in the United States as the Morrison standard, which in its earlier form has been known as the Modified Wolff-Lehman standard. The protein requirements of the Morrison standard, as applying to growing dairy cattle, are set forth below.

Growing Cattle, Dairy Breeds

Age in months	Average live weight per head	Digestible crude protein per 1000 pounds live weight
	Pounds	*Pounds*
2–3	150	4.0
3–6	300	3.0
6–12	500	2.0
12–18	700	1.8
18–24	900	1.5

A comparison of the foregoing experimental data with this standard indicates that virtually all of the growth represented was made from protein intake distinctly less than the standard.

On various accounts we do not regard these results as conclusive, but, so far as they go, they indicate that greater economy of protein intake than as suggested by this standard would be practicable. At least these results suggest that an exhaustive investigation of the subject is desirable, and that current practice, as represented by the most widely used feeding standard, wastes protein.

This deduction is in harmony with the conclusions of Armsby from the earlier experiments of this research as reported in the first paper on the subject.

Selected Data Applying to Those Individual Cattle which made Gains in Weight at least Equal to the Normal, on the Minimum Intake of Protein

Experimental Station	Year	Designation of calf	Initial age	Duration of feeding trial	Average daily gain	Computed normal average daily gain	Average daily intake of digestible crude protein per 1000 pounds	Net energy value of average daily intake per 1000 pounds	Initial live weight
			Months	*Days*	*Pounds*	*Pounds*	*Pounds*	*Therms*	*Pounds*
Pennsylvania[1]	1921	982	7	133	1.58	1.14	1.3–1.69	11.33–14.88	470
Pennsylvania	1921–2	1006	7½	243	1.14	0.90	1.571	8.34	338
Pennsylvania	1922–3	1032	10	150	0.84	0.73	1.305	10.366	461
North Dakota	1919–20	A	7[2]	155	1.53	1.53	1.41	10.395	417
South Dakota	1919–20	307	6½[2]	408	1.12	0.92	1.399	11.426	339
South Dakota	1919–20	309	10[2]	408	1.29	0.82	1.399	11.426	468
Nebraska	1919–20	2	16	266	0.76	0.74	1.551	11.571	751
Nebraska	1919–20	4	14	266	1.01	0.67	1.346	11.960	804
Nebraska	1919–20	6	10½	407	1.05	0.71	1.170	11.860	855
Nebraska	1919–20	256	8⅔	252	1.25	0.82	1.377	12.455	848

[1] The data for protein and energy intake reported from the first year's experiment at Pennsylvania represent four disconnected balance periods and do not, therefore, cover exactly the entire feeding periods.

[2] Estimated; actual age not a matter of record.

It is our belief that the metabolism periods have not contributed results of unmistakable significance in relation to the protein requirements for growth, on account of diverse unfavorable conditions, but especially because of the brevity of the experimental periods.

The physiological facts of nitrogen metabolism which are prominent in this connection are that the adaptability of animals to different planes of protein intake, and the possible use of protein for purposes other than protein synthesis and repair, are so extensive, and final effects of a particular level of protein intake are attained so slowly, that it is impossible to obtain reliable results as to protein requirements of cattle from the nitrogen retention in ordinary digestion periods of a few days' duration.

Considered also as a study of methods of cooperative research, the results of this series of experiments point the way to further progress in the investigation of this subject by a similar course of procedure, with especial attention to the following particulars:

In the light of the evidence obtained, the protein intake in subsequent experiments should be so graduated in amount, with the experimental lots to be compared, that they "bracket" the probable optimum.

The nitrogen balance in digestion experiments being of uncertain value as a measure of the sufficiency of the protein intake, as applying to conditions of practice, the main reliance should be placed on the results of the growth experiments, which should be planned and conducted with critical attention to all details of method, as set forth in Bulletin No. 33 of the National Research Council, On the Formulation of Methods of Experimentation in Animal Production.

In view of the variability in the live weights of cattle, the minimum number of subjects in a lot, in the growth experiments, should be 10 to 15, even larger numbers being preferable.

The exact ages of all experimental subjects must be known, and must be satisfactorily uniform and comparable in the several experimental lots.

Digestion experiments should be conducted in all cases, using not less than 3 to 5 individuals on each treatment, the fore-period being not less than 7 to 10 days, and the collection period not less than 10 to 20 days in length.

All feeds must be sampled and analyzed, and the rations to be compared must in all cases conform exactly to the plan of experimentation.

It is essential that the experiments be conducted only in situations in which it is possible to carry them out in strict conformity to the plan; and it is desirable that the coordinating agency insure the character of the work by personal inspection during its progress.

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